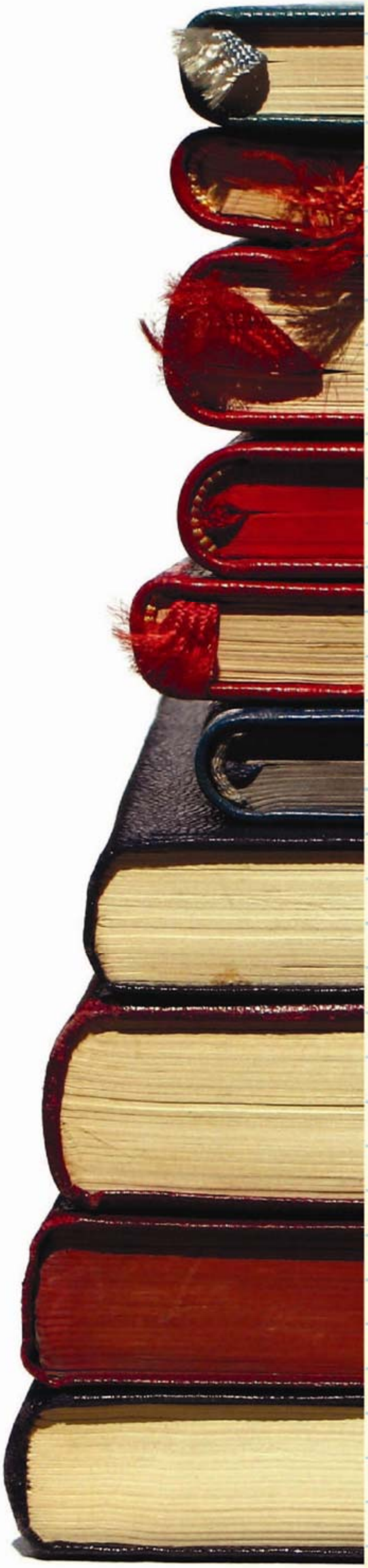


Comprehensive Curriculum

Revised 2008

Algebra I
Part 1

Louisiana Department of
EDUCATION
Paul G. Pastorek, State Superintendent of Education

Algebra I–Part 1

Table of Contents

Most of the math symbols in this document were made with *Math Type*® software. Specific fonts must be installed on the user's computer for the symbols to be read. It is best to use the *pdf* format of a document if a printed copy is needed.

To copy and paste from the Word document, download and install the *Math Type*® *for Windows Font* from http://www.dessci.com/en/dl/fonts/default.asp on each computer on which the document will be used.

Louisiana Comprehensive Curriculum, Revised 2008
Course Introduction

The Louisiana Department of Education issued the *Comprehensive Curriculum* in 2005. The curriculum has been revised based on teacher feedback, an external review by a team of content experts from outside the state, and input from course writers. As in the first edition, the *Louisiana Comprehensive Curriculum*, revised 2008 is aligned with state content standards, as defined by Grade-Level Expectations (GLEs), and organized into coherent, time-bound units with sample activities and classroom assessments to guide teaching and learning. The order of the units ensures that all GLEs to be tested are addressed prior to the administration of *i*LEAP assessments.

District Implementation Guidelines
Local districts are responsible for implementation and monitoring of the *Louisiana Comprehensive Curriculum* and have been delegated the responsibility to decide if

- units are to be taught in the order presented
- substitutions of equivalent activities are allowed
- GLES can be adequately addressed using fewer activities than presented
- permitted changes are to be made at the district, school, or teacher level

Districts have been requested to inform teachers of decisions made.

Implementation of Activities in the Classroom
Incorporation of activities into lesson plans is critical to the successful implementation of the Louisiana Comprehensive Curriculum. Lesson plans should be designed to introduce students to one or more of the activities, to provide background information and follow-up, and to prepare students for success in mastering the Grade-Level Expectations associated with the activities. Lesson plans should address individual needs of students and should include processes for re-teaching concepts or skills for students who need additional instruction. Appropriate accommodations must be made for students with disabilities.

New Features
Content Area Literacy Strategies are an integral part of approximately one-third of the activities. Strategy names are italicized. The link (view literacy strategy descriptions) opens a document containing detailed descriptions and examples of the literacy strategies. This document can also be accessed directly at http://www.louisianaschools.net/lde/uploads/11056.doc.

A *Materials List* is provided for each activity and *Blackline Masters (BLMs)* are provided to assist in the delivery of activities or to assess student learning. A separate Blackline Master document is provided for each course.

The *Access Guide to the Comprehensive Curriculum* is an online database of suggested strategies, accommodations, assistive technology, and assessment options that may provide greater access to the curriculum activities**.** The *Access Guide* will be piloted during the 2008-2009 school year in Grades 4 and 8, with other grades to be added over time. Click on the *Access Guide* icon found on the first page of each unit or by going directly to the url http://mconn.doe.state.la.us/accessguide/default.aspx.

Algebra I–Part 1
Unit 1: Variables and Numeric Relationships

Time Frame: Approximately seven weeks

Unit Description

This introductory unit consists of a thorough review of math topics from earlier grades. Topics include work with subsets of the set of real numbers including how to graph and perform operations on them; use of scientific notation; simplifying and estimating square roots; evaluating expressions; and representing real-life situations with numerical models and graphs. Also included in this unit is a review of geometric formulas.

Student Understandings

Students can use the order of operations and scientific notation, and work with rational and irrational numbers. Students write, evaluate, and simplify algebraic expressions in real-life situations and in mathematical formulas. They also recognize simple patterns in graphical, numerical, tabular and verbal forms.

Guiding Questions

1. Can students use order of operations and the basic properties (i.e., associative, commutative, and distributive) when performing computations and collecting like terms in expressions?
2. Can students correctly evaluate numeric and algebraic expressions involving rational numbers?
3. Can students use and apply scientific notation in representing numbers and solving problems?
4. Can students recognize functions in graphical, numerical, tabular, and verbal forms?

Unit 1 Grade-Level Expectations (GLEs)

GLE#	GLE Text and Benchmarks
Number and Number Relations	
1.	Identify and describe differences among natural numbers, whole numbers, integers, rational numbers, and irrational numbers (N-1-H) (N-2-H) (N-3-H)
2.	Evaluate and write numerical expressions involving integer exponents (N-2-H)

GLE#	GLE Text and Benchmarks
3.	Apply scientific notation to perform computations, solve problems, and write representations of numbers (N-2-H)
4.	Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H)
5.	Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H)
6.	Simplify and perform basic operations on numerical expressions involving radicals (e.g., $2\sqrt{3}+5\sqrt{3}=7\sqrt{3}$) (N-5-H)
Algebra	
8.	Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)
9.	Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
12.	Evaluate polynomial expressions for given values of the variable (A-2-H)
Data Analysis, Probability, and Discrete Math	
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
34.	Follow and interpret processes expressed in flow charts (D-8-H)

Sample Activities

Activity 1: Relationships in the Real Number System (GLE: 1)

Materials List: paper, pencil, Where do I Belong? BLM.

Review the real number system and discuss with students what the natural numbers, whole numbers, integers, rational and irrational numbers are. Draw a Venn diagram showing how the various sets of numbers within the real number system are related. After fully reviewing the real number system, make copies of the modified *word grid* (view literacy strategy descriptions) activity, Where do I Belong? BLM. A *word grid* is a learning strategy which allows the user to relate characteristic features among terms or items and helps the learner to compare and contrast similarities and differences between the items in a list. Let students get in pairs to complete this activity, and then discuss the results as a class.

Activity 2: Understanding Rational and Irrational Numbers (GLEs: 1, 4)

Materials List: paper, pencil, scientific calculators, four-function calculators (variety), Internet (optional)

Using calculators, let students explore the difference between rational and irrational numbers. To begin, have students input several rational numbers using the division key, and discuss why some rational numbers are finite (e.g., $\frac{4}{5}$, $\frac{3}{8}$, and $\frac{15}{32}$), while other rational numbers have non-terminating decimals that repeat (e.g., $\frac{11}{12}$ and $\frac{7}{11}$). Ask students to investigate which fractions will terminate and which will repeat by looking for a pattern. Students should see that fractions that terminate have denominators with factors of 2 and 5 only. Any rational number having factors other than 2 or 5 will result in non-terminating repeating decimal numbers.

Next, have students input several irrational numbers and let them see that although the numbers appear to terminate on the calculator, the calculator is actually rounding off the last digit. Students need to understand that irrational numbers don't terminate or repeat when converted to a decimal. There are some computer sites that show irrational values to many places. The website http://www.mathsisfun.com/irrational-numbers.html is one such site. It also explains in detail what an irrational number is along with values for *pi* and *e* and other "famous" irrational numbers.

Finally, discuss how different calculators handle numbers that do not terminate. The goal here is to help students to see that calculators can sometimes introduce calculation errors and how to handle this situation as it arises. For example, show students what error results from calculating with rounded values (i.e., multiply a number by 0.67 and then multiply it by 2/3). Present other examples of calculation error associated with rounding using calculators. Emphasize that in most cases, it is better not to round until the last step.

Activity 3: Estimating the Value of Square Roots (GLEs: 1, 4)

Materials List: paper, pencil, scientific calculators

Discuss with students how to estimate the value of irrational numbers involving square roots. Have the students determine which two whole numbers a particular square root would fall between. For example, if students know the square root of 49 is 7 and the square root of 64 is 8, then the square root of 51 would be between these two values (it would actually be closer in value to 7 than to 8, so an even better estimate might be 7.1). Once a thorough discussion takes place about estimation techniques, provide students with an opportunity to use their estimation skills by providing students with 10 square roots which are irrational and have them determine their approximate values. After students obtain their approximate values, have students share their reasoning with a partner first and then explain their reasoning to the class. Have students check their estimates with a calculator using the square root key.

Activity 4: Naming Numbers on a Number Line (GLE: 1)

Materials List: paper, pencil

As a class (with the teacher modeling and students working at their desks individually), construct a number line showing the integers from –4 to +4. Teacher and students should then identify and label the halfway points between each pair of integers (e.g., $-3\frac{1}{2}$, $-2\frac{1}{2}$). Next, identify and label where the following numbers would be placed on the number line they created: $-\pi, -\sqrt{3}, -\sqrt{2}, \sqrt{2}, \sqrt{3}, \pi$. First allow students to place the numbers on their own number lines. When students have completed their work, call individual students to the front of the room to place a number on a class number line and explain why he/she chose this particular position. Discuss the difference between an exact answer (such as the square root of 3) and an approximate answer (such as 1.73). Use this activity to reinforce the previous work done by having students name and identify the natural numbers, whole numbers, integers, rational, and irrational numbers. Use this opportunity to emphasize to students that a number line is made up of an infinite number of points. Many students think a number line consists only of integers. We want students to understand that between any two integers are "infinitely many" points. For example, between 2 and 3, there are points such as 2.000001, 2.0000002, and so on.

Activity 5: Many Ways to Solve a Problem (GLEs: 4, 5)

Materials List: paper, pencil, scientific calculators, teacher-made worksheets on operations with rational numbers, What Method Should I Use? BLM

Review paper and pencil operations with rational numbers (addition, subtraction, multiplication, and division) and include in the discussion how the calculations could be done using calculators. Also use this opportunity to discuss estimation and mental strategies with respect to operations with rational numbers. This work should be a review for students from 7th and 8th grades. Provide opportunities for students to develop proficiency in solving problems involving operations with rational numbers.

Afterwards, make copies of and have students work in pairs on What Method Should I Use? BLM. In this activity, students are given a problem in which they have to decide whether they should solve a problem using estimation, mental math, paper/pencil, or calculator. Students write their decisions in the math *learning log* (view literacy strategy descriptions) section of the BLM to defend the choice made. *Learning logs* are a literacy strategy designed to force students to put into words what they know or do not know. It offers students the opportunity to reflect on their understanding which can lead to further study and alternative learning paths. *Learning logs* are generally kept in a journal type of book, but because of the nature of Activity 5, the log is integrated into the BLM. Throughout the year, have students maintain a *learning log* in a central location, such as in a special place in a notebook or binder, in which to record new learning experiences. For Activity 5, have the students tape or paste the BLM into the log when completed.

After students have written their math *learning logs*, they should exchange math logs with another student to analyze one another's work and provide feedback to another student. Once this has taken place, the teacher should lead a discussion of the worksheet and discuss what students believe to be the best approach to each of the problems.

Activity 6: Using Exponents in Prime Factorization (GLE: 2)

Materials List: paper, pencil, teacher-made worksheet on using exponents or problems from a math textbook

Review the prime factorization process with whole numbers, and include in the discussion how exponents can be used to rewrite a particular prime factorization. For example, ask students to rewrite 136 as the product of primes. This should be a review for students from middle school work. Allow students the opportunity to prime factor different numbers using factor trees, and then have them write the numbers in factored form using exponents.

Include in the discussion the use of negative exponents (i.e., explain how ½ could be written as 2^{-1} or 1/16 could be written as 2^{-4}). Since this is the first time that students have exposure to negative exponents, develop the concept in the following way: Have students investigate a pattern starting with $2^2=4$; $2^1=2$; $2^0=1$; $2^{-1}=$?. Since each value is $\frac{1}{2}$ the previous value, then 2^{-1} would need to be $\frac{1}{2}$. Continue the process by using exponents of -2, -3, and so on, until the idea of using negative exponents is developed thoroughly. Once this is done, introduce the "rule" associated with negative exponents as is typically done in an algebra class. The goal of this activity is to help students to become comfortable with use of integer exponents. Teacher should provide a worksheet for practice as necessary for their particular class or use the math textbook as a resource for additional work on this topic.

Activity 7: Using Exponents In Scientific Notation (GLEs: 2, 3, 5)

Materials List: paper, pencil, scientific calculator, Internet (optional), Changing Forms BLM

Start by explaining what scientific notation is and what it is used for. Scientific notation is a method used by scientists to express numbers that are very small and very large (i.e., the mass of a hydrogen atom is $1.7x10^{-24}$ gram which if written out in standard form would look like .0000000000000000000000017).

Discuss the procedures used when writing numbers in scientific notation. Explain that when writing a number in scientific notation, the first part of the number, called the coefficient, must be greater than or equal to 1 and less than 10. The second part of the number, called the base, must be a power of 10. Explain the process for writing a number from one form to the next. For example, write the number 3.2 x 10^3 on the board. Explain how to use exponents in scientific notation to write the number 3200. Show students how 3.2 x 10^3 = 3.2 x10x10x10=3200.

When converting a standard number such as 3200 to scientific notation, the student must decide where to place a decimal to have a number between 1 and 10. Ask students where they think the decimal should go. Placing the decimal between 3 and 2 yields 3.2, which meets the criteria for the coefficient. Ask students to determine what power of 10 would need to be multiplied by 3.2 to yield 3200. 3.2 x 10 x 10 x 10 = 3.2 x 10^3 = 3200, so 10^3 is the base. A key idea is that a number written in scientific notation with a positive exponent indicates the standard number that is greater than 1. Likewise, a number written in scientific notation with a negative exponent indicates a standard number that is less than one. Refer to the two previous examples (i.e., 3.2 x 10^3 and $1.7x10^{-24}$).

Present numerous opportunities for students to convert from standard form to scientific notation (and vice versa) and to perform operations involving scientific notation by hand and with a scientific calculator (using the EXP key). The goal of this lesson is to help students understand and be able to compute using scientific notation. Since many Algebra I books don't contain the rules associated with converting between scientific and standard notation, http://www.nyu.edu/pages/mathmol/textbook/scinot.html is a good resource. Part of the site allows students to practice making conversions.

Next, provide students with a copy of the Changing Forms BLM. Have students work in pairs on the activity, then discuss answers as a class. Students are required to convert from standard form to scientific notation and solve two real life problems using scientific notation. There is also a website http://janus.astro.umd.edu/astro/scinote/ that allows students to try problems on their own for students who need additional help.

Activity 8: Astronomical Measurement and Scientific Notation (GLEs: 2, 3, 5)

Materials List: paper, pencil, scientific calculators, Internet (optional)

As an outside assignment have students use http://www.windows.ucar.edu/tour/link=/kids_space/distance.html or another resource to find the following astronomical distances and rates of speed. Write these measurements using scientific notation.

- Speed of light *(answer: 3 x* 10^8 *m/s)*
- Distance from Earth to the sun *(answer: about 1.5 x* 10^{11} *m)*
- Distance from Pluto to the sun *(answer: about 7.4 x* 10^{12} *m)*
- Distance from Mars to the sun *(answer: about 2.5 x* 10^{11} *m)*

Since the distances change due to the orbital nature of planets, have students determine the maximum distances. Using the information found, have students form groups and determine the answer to the questions below. Allow the use of scientific calculators, and instruct students to write the answers using scientific notation.

- Distance from Earth to Mars *(answer: about 1 x* 10^{11} *m)*

- Distance from Earth to Pluto *(answer: about 7.25 x* 10^{12} *m)*
- How much time does it take for the light from the sun to reach the Earth? *(answer: about 5 x* 10^{2} *sec which is about 8.3 minutes)*
- How much time does it take for the light from the sun to reach the planet Pluto? *(answer: about 2.467 x* 10^{4} *sec which is about 411 minutes or 6.8 hours)*
- If a manned space flight went from Earth to Mars and could travel at the speed of light, how long would it take to get there? *(answer: about 3.33 x*10^{2} *sec which is about 5.5 minutes)*
- If a manned space flight can travel at an average speed of 45,000 miles per hour, how fast is this speed in meters per second? If this craft went from the Earth to Pluto, approximately how long would it take to get there? *(answer: the speed is about 2 x* 10^{4} *m/s and it would take about 3.625 x* 10^{8} *sec to get from Earth to Pluto, which is about 10 years).*

It is important to note that these answers provided may differ slightly from those obtained by students. The goal of this activity is to have students work problems that involve the use of scientific notation. Have students share how they came up with their answers to the problems.

Activity 9: Order of Operations Activity (GLEs: 2, 5, 8)

Materials List: paper, pencil, calculators – more than one kind for comparison, math textbook

Review the order of operations with numerical applications. Explain why order of operations is necessary. For example, present the problem 4 + 5 x 9 to students, and let them obtain an answer on their own. In this case, if students work from left to right they get a solution of 81, which is not correct. If the multiplication is done first and then the addition, an answer of 49 is obtained, which is correct. This sets up a situation that in math is not okay…two solutions when there really should only be one. Students need to see the importance of having an order to follow when calculating. Also, have students use different calculators, if possible, to show how certain calculators perform the order of operations differently. Include in the discussion the importance of using parentheses when inputting data. For example, (4+5)/(6+2) would be a different result than 4+5÷6+2, and how to enter exponents in their calculators (e.g., 3^{3} - 4(3 + 6)). After the discussion, provide students with practice in using the order of operations using problems from their math textbook. Make sure problems include the use of parentheses and exponents.

Activity 10: Flow Charts—Not Just for Computers (GLEs: 2, 5, 8, 34)

Materials List: paper, pencil, Internet

Using the link http://deming.eng.clemson.edu/pub/tutorials/qctools/flowm.htm to show students examples of flow charts, what a flow chart is used for, and discuss with students how to make a flow chart using boxes and arrows and instructions. Make a flow chart as a whole group for the steps that would be followed when solving a simple algebraic equation so that students get an idea of how to create a flow chart in math. After students are ready, have students design a flow chart, (either in class or as an out of class assignment for homework) that demonstrates how to evaluate an expression using the order of operations. To design their flow chart, have students use the following procedures: questions go in the diamonds; processes go in the rectangles; *yes* or *no* answers go on the connectors. Students should be given several numerical expressions that involve powers, parentheses, and several operations. Have students exchange their flow charts with each other and use the flow chart to simplify the expressions. Students should provide feedback to one another on improvements that would be needed, if necessary. This concept of having students critique and question one another's work is a modified form of the literacy strategy known as *QtA* (view literacy strategy descriptions). Appropriate use of *QtA*, or Questioning the Author provides students the benefit of going beyond the words on a page of text and allows the student to construct meaning for the text. The strategy also encourages the learner to write In this particular activity, students are using the *QtA* method with one another's work and providing feedback to one another.

Activity 11: Writing and Simplifying Algebraic Expressions (GLE: 8, 9)

Materials List: paper, pencil, math textbook

Review with students how to write and simplify algebraic expressions for different real-life situations. For example, have students write an expression for the total weight of 24 cans of soft drink if each weighs k ounces (*Answer: 24k*); write an expression for the distance someone would travel if he/she went 40 miles per hour for t hours (*Answer: 40t*). Other examples can be found in any algebra textbook. Discuss simplifying algebraic expressions and combining like terms. For example, if a square has sides that are p units long, the perimeter can be expressed as $p + p + p + p$ or $4p$ units in length. The goal here is for students to become proficient at both writing expressions and simplifying them.

Activity 12: Evaluating Expressions and Using Geometry Formulas (GLEs: 2, 5, 8. 12)

Materials List: paper, pencil, real life objects for students to measure perimeter, area, volume and surface area, rulers, string, meter sticks

This activity has three parts:

1) Review of student's previous work with formulas used in middle school geometry
2) Paper/pencil computation and calculator computation using these formulas
3) Hands-on measurement applying the use of the formulas to real-life objects.

Begin the review by having students provide information on what they remember from previous work with geometry including terms, definitions, and formulas for areas and perimeters of various 2-D figures. Also ask students to provide correct mathematical names for various 3-D figures by presenting real-life examples (such a cereal box, a sugar ice-cream cone, or an oatmeal box). Ask them to give the correct name for each face. Facilitate the review by having students recall facts that they remember about formulas needed to find surface area and volume for rectangular solids and cylinders which were learned in grade 8. As part of the review, include an analysis of the derivation of the formulas (i.e., the surface area of a cylinder is $2\pi r^2 + Ch$ because the faces are composed of two circles, $2\pi r^2$, and a rectangle whose dimensions are the circumference of the circle and the height of the cylinder, Ch).

Next, provide students with practice problems which utilize the geometry formulas discussed in the review. This is another opportunity to include practice with fractions, decimals, and integers as well as exponents.

Once paper and pencil problems involving the geometry formulas have been completed, provide various 2-D and 3-D objects (boxes or cans) and require students to select the appropriate formula and measure in the appropriate places to determine the perimeters, areas, or volume. Allow students to work in small groups on this part of the activity. Once students have completed their work, discuss where measurements were taken in order to correctly determine the perimeter, area, or volume for each of the figures.

Finally, have students write a math *learning log* (view literacy strategy descriptions) entry about three things they learned while doing this activity that they didn't know how to do before their work. Remember that *learning log* entries should be kept in a specific place, such as a binder or the back section of the student's math notebook.

Activity 13: Simplifying Radical Expressions (GLEs: <u>6</u>, 8)

Materials List: paper, pencil, math textbook, scientific calculator, Operations with Radicals BLM

Begin the lesson by teaching students how to simplify square roots such as $\sqrt{25}$ to get a value of 5 (since 5 x 5 = 25). Begin with finding the square roots of perfect squares and then lead into simplifying square roots such as $\sqrt{50}$. Explain that since $\sqrt{50} = \sqrt{2}\ \sqrt{25}$, this can be simplified to $5\sqrt{2}$. When simplifying radicals such as $\sqrt{56}$ to the form $2\sqrt{14}$, have students use a calculator to show their equality with one another (both have a value of about 7.5). Use as many examples as are necessary for students to demonstrate mastery on this skill. Include problems which require multiplication of square roots and simplifying them, such as: $\sqrt{50}\ \sqrt{2} = \sqrt{100} = 10$. Division of radicals is not required by the GLEs in grade 9.

Next, relate the skill of simplifying variable expressions with that of simplifying numerical expression with radicals. For example, since the expression $2x + 4x$ can be simplified to $6x$ because they are like terms, likewise, the expression $2\sqrt{7} + 4\sqrt{7}$ can be simplified to $6\sqrt{7}$ because they are also like terms. When adding or subtracting radicals, what makes terms alike is that the radical part of the term must be the same. If the problem were $2\sqrt{5} + 3\sqrt{7}$ the expression could not be simplified any further because these two terms are not alike. Include problems where students have to simplify radicals first, and then combine like terms, such as $\sqrt{56} + \sqrt{14}$ (which results in $2\sqrt{14} + \sqrt{14}$ which equals $3\sqrt{14}$). After a thorough discussion has taken place on operations with radicals, provide students with a copy of Operations with Radicals BLM. Allow students time to work in small groups on the BLM then discuss the work as a class. Additional problems may be found in math textbooks.

Activity 14: Working with "$\sqrt{\ }$" (GLEs: <u>5</u>, <u>6</u>, 8)

Materials List: paper, pencil, math textbook

Review and discuss students' previous work with the Pythagorean Theorem, which should have been taught in grade 8. Have students solve for missing side lengths in right triangles using the Pythagorean Theorem in their math textbook. Make sure students understand that the Pythagorean Theorem is only valid for right triangles. Have students solve problems in which the side lengths are whole numbers or are irrational. However, none of the problems should require the student to divide by a radical. Be sure to include problems within the context of real-life situations. This activity will also allow students practice in simplifying square roots. Finally have students find the areas of the right triangles to reinforce the area of a triangle formula.

Activity 15: Patterns in the Real World (GLEs: 2, 8, 9, 15)

Materials List: paper, pencil, Patterns in the Real World BLM

In this activity, expose students to number patterns found in real-life examples, including patterns involving exponents. Have students describe the pattern in words, and then have them write an algebraic expression to represent the n^{th} term.

For example, suppose a new pizza shop opens in a shopping center. At the end of each day, a running total is kept for the total number of pizzas sold since the shop opened. On the first day, the shop sells 1 pizza. On the second day, the shop sells 3 more pizzas. On the third day, the shop sells 5 more pizzas. On the fourth day, the shop sells 7 more pizzas. On the fifth day, the shop sells 9 more pizzas. Have students describe the pattern in words, and then have them write an algebraic expression which could be used to express the number of pizzas sold on the *nth* day. Also, have students determine an algebraic expression which could be used to find the total number of pizzas sold since the shop opened by the end of the *nth* day. {*Solution: To represent the number of pizzas sold on the nth day, the expression 2n-1 could be used. To represent the total number of pizzas sold since the shop opened the expression* n^2 *could be used.*}

After doing several examples as a whole class, give students the opportunity to do some on their own. Afterwards, put students in groups of three and assign Patterns in the Real World BLM. When students have completed their work, have them write a math *learning log* (view literacy strategy descriptions) entry about a solution to a problem they did from the worksheet. They should explain how they did the problem, what approach they took to finding the solution, what was the hardest part in finding the solution (what gave them the most trouble when attempting the solution), and how they overcame that. Discuss both the problems and the writing as a class.

Activity 16: Matching Real-life Situations and Their Graphs (GLE: 15)

Materials List: paper, pencil, Internet, math textbook

One of the hardest things to help students visualize is the relationship between distance, time, and speed since it is hard to do this in an actual classroom. However, through the use of technology, students can better understand these relationships and how they can be interpreted in graphs. Begin this activity by using the interactive software found at http://standards.nctm.org/document/eexamples/chap5/5.2/. This computer simulation uses a context familiar to students, and the technology allows them to analyze the relationships more deeply because of the ease of manipulating the environment and observing the changes that occur. In the activity found on the website, three motion stories are presented.

Before the students use the simulation, have them physically simulate the motion stories (with their bodies). Then develop specific instructions (starting position and length of stride for each runner) to produce the action in the stories. Before actually doing the simulation, have students pair up to discuss what they think the graph for each situation presented would look like and have them sketch a graph. Then, try out the instructions using the computer simulation.

This technique of having students hypothesize is a form of a strategy called *SPAWN* (view literacy strategy descriptions) writing. *SPAWN* is used as a way to create writing prompts in order to elicit student writing and thinking. In this particular use of *SPAWN*, students are being asked to figure out the solution to a problem (P) and what will happen next (N). Since this is not formal writing, it should not be graded as such. Afterwards, have students create their own graphs and write a story to match the graph they have created.

As an extension of this activity, provide students with various numberless graphs (possibly from their math textbooks) and real-life situations that correspond to each graph. Have students match the graph with the situation. For example, have graphs of distance/time, and relate the act of moving toward home and away from home on a given day in reference to the time during the day. Provide students with many different situations and graph types which will require students to use analytical thinking.

Activity 17: Matching a Table of Values with a Graph (GLE: 9, 15)

Materials List: paper, pencil, Tables to Graphs BLM

Provide students with copies of Tables to Graphs BLM. In this activity, students are presented with a table of values resulting from a real-life situation: the price to rent a moving truck in relation to the number of miles it was driven. Students will complete the table by determining the missing data. After completing the table, students will determine which graph (chosen from the graphs provided with the activity) best fits the data shown in the table. Students will then write a math *learning log* (view literacy strategy descriptions) entry about why the graph they chose is the only graph that fits the data. Finally, students will come up with an expression or equation which represents the cost to rent a truck driving n miles.

Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will make a portfolio containing samples from various activities.
- The student will keep a math *learning log* (view literacy strategy descriptions) about the ideas and processes that are taught in class. The student can use the log as information or as a study guide, but it is also a good source of feedback to the teacher concerning questions the student has on a particular topic. Each week, the teacher picks up the learning log and examines it.
- For selected activities, the student will show his/her work, and use the work for assessment purposes.
- Paper/pencil tests which address student's ability to compute, write expressions, or evaluate expressions based upon the GLE's presented in this unit.

Activity-Specific Assessments

- Activity 4: The student will put 15 numbers on a number line ranging from –10 to 10. The teacher will provide the list of numbers with values of each type (natural, whole, integer, rational, and irrational) for the student to graph and have the student identify which subsets each number belongs to.

- Activity 5: The student will write his/her own problems that could best be solved using each technique (paper and pencil, estimation, technology, and mental math) along with an explanation of why this would be the best approach.

- Activity 8: The student will write explanations (mathematical and verbal explanations) of how the answer was found to each of the questions presented in the activity.

- Activity 13: The student will draw a flow chart for simplifying a square root.

Algebra I–Part 1
Unit 2: Measurement

Time Frame: Approximately three weeks

Unit Description

This unit is an advanced study of measurement. It includes the topics of precision and accuracy and investigates the relationship between the two. The investigation of absolute and relative error and how they each relate to measurement is included. Significant digits are also studied as well as how computations on measurements are affected when considering precision and significant digits. Use of indirect measurements is also covered.

Student Understandings

Students find the precision of an instrument and determine the accuracy of a given measurement. They know the difference between precision and accuracy. Students view error as the uncertainty approximated by an interval around the true measurement. They calculate and use significant digits to solve problems. They use proportions and other techniques to measure things that cannot be measured directly.

Guiding Questions

1. Can students determine the precision of a given measurement instrument?
2. Can students determine the accuracy of a measurement?
3. Can students differentiate between what it means to be precise and what it means to be accurate?
4. Can students discuss the nature of precision and accuracy in measurement and note the differences in final measurement values that may result from error?
5. Can students calculate using significant digits?
6. Can students use similar triangles or other techniques to find measurements directly?

Unit 2 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
4.	Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H)

GLE #	GLE Text and Benchmarks
5.	Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H)
Measurement	
17.	Distinguish between precision and accuracy (M-1-H)
18.	Demonstrate and explain how the scale of a measuring instrument determines the precision of that instrument (M-1-H)
19.	Use significant digits in computational problems (M-1-H) (N-2-H)
20.	Demonstrate and explain how relative measurement error is compounded when determining absolute error (M-1-H) (M-2-H) (M-3-H)
21.	Determine appropriate units and scales to use when solving measurement problems (M-2-H) (M-3-H) (M-1-H)
22.	Solve problems using indirect measurement (M-4-H)

Sample Activities

Activity 1: What Does it Mean to be Accurate? (GLEs: 4, 17)

Materials List: paper, pencil, three or more different types of scales from science department, three or more different bathroom scales, students' watches, Internet access, What Does It Mean To Be Accurate? BLM, sticky notes

This unit on measurement will have many new terms to which students have not yet been exposed. Have students maintain a *vocabulary self-awareness* (view literacy strategy descriptions) chart for this unit. *Vocabulary self-awareness* is valuable because it highlights students' understanding of what they know, as well as what they still need to learn, in order to fully comprehend the concept. Students indicate their understanding of a term/concept, but then adjust or change the marking to reflect their change in understanding. The objective is to have all terms marked with a + at the end of the unit. A sample chart is shown below.

Word	+	☑	-	Example	Definition
accuracy					
precision					
Relative error					
Absolute error					
Significant digits					

Be sure to allow students to revisit their *vocabulary self-awareness* charts often to monitor their developing knowledge about important concepts. Sample terms to use include accuracy, precision, significant digits, absolute error, and relative error.

Have students use the What Does It Mean To Be Accurate? BLM to complete this activity. Talk with students about the meaning of "accuracy" in measurement. Accuracy indicates how close a measurement is to the accepted "true" value. For example, a scale is expected to read 100 grams if a standard 100 gram weight is placed on it. If the scale does not read 100 grams, then the scale is said to be inaccurate. If possible, obtain a standard weight from one of the science teachers along with several scales. With students, determine which scale is closest to the known value, and use this information to determine which scale is most accurate.

Next, ask students if they have ever weighed themselves on different scales—if possible, provide different scales for students to weigh themselves. The weight measured for a person might vary according to the accuracy of the instruments being used. Unless "true" weight is known (i.e., there is a known standard to judge each scale), it cannot be determined which scale is most accurate. Generally, when a scale or any other measuring device is used, the readout is automatically accepted without really thinking about its validity. People do this without knowing if the tool is giving an accurate measurement. Also, modern digital instruments convey such an aura of accuracy and reliability (due to all the digits it might display) that this basic rule is forgotten—there is no such thing as a perfect measurement. Digital equipment does not guarantee 100% accuracy. Note: If some students object to being weighed, have them weigh their book bags or other fairly heavy items. Adjust the BLM if this is done.

Have all of the students who have watches to record the time (to the nearest second) at the same moment and hand in their results. Post the results on the board or overhead—there should be a wide range of answers. Ask students, "Which watch is the most accurate?" Students should see that in order to make this determination, the true time must be known. Official time in the United States is kept by NIST and the United States Naval Observatory, which averages readings from the 60 atomic clocks it owns. Both organizations also contribute to UTC, the world universal time. The website http://www.time.gov has the official U.S. time, but even its time is "accurate to within .7 seconds." Cite this time at the same time the students are determining the time from their watches to see who has the most accurate time.

Lead students in a discussion as to why their watches have different times (set to home, work, and so on) and how their skill at taking a reading on command might produce different readings on watches that may be set to the same time.

Ultimately, students need to understand that accuracy is really a measure of how close a measurement is to the "true" value. Unless the true value is known, the accuracy of a measurement cannot be determined.

Activity 2: How Precise is Your Measurement Tool? (GLEs: 4, 17, 18)

Materials List: paper, pencil, rulers with different subdivisions, four-sided meter sticks, toothpicks, What is Precision? BLM, wall chart, blue masking tape

Discuss the term "precision" with the class. Precision is generally referred to in one of two ways. It can refer to the degree to which repeated readings on the same quantity agree with each other. We will study this definition in Activity 4.

Have students use the What is Precision? BLM for this activity.

Precision can also be referred to in terms of the unit used to measure an object. Precision depends on the refinement of the measuring tool. Help students to understand that no measurement is perfect. When making a measurement, scientists give their best estimate of the true value of a measurement, along with its uncertainty.

The precision of an instrument reflects the number of digits in a reading taken from it—the degree of refinement of a measurement. Discuss with students the degree of precision with which a measurement can be made using a particular measurement tool. For example, have on hand different types of rulers (some measuring to the nearest inch, nearest $\frac{1}{2}$ inch, nearest $\frac{1}{4}$ inch, nearest $\frac{1}{8}$ inch, nearest $\frac{1}{16}$ inch, nearest centimeter, and nearest millimeter) and discuss with students which tool would give the most precise measurement for the length of a particular item (such as the length of a toothpick). Have students record measurements they obtain with each type of ruler and discuss their findings.

Divide students into groups. Supply each group with a four-sided meter stick. (This meter stick is prism-shaped with different divisions of a meter on each side. The meter stick can be purchased at www.boreal.com, NASCO, and other suppliers.)

Cover the side of the meter stick that has no subdivisions with two strips of masking tape and label it as side 1. (You need two layers of masking tape so the markings on the meter stick will not show through the tape.) Repeat this with the other sides of the stick such that side 2 has decimeter markings, side 3 has centimeter markings, and side 4 has millimeter markings. Have students remove the tape from side 1 and measure the length of a sheet of paper with that side and record their answers. Repeat with the other sides of the meter stick in numerical order. Post a wall chart similar to the one below and have each group record their measurements:

Length of Paper				
	Side 1	Side 2	Side 3	Side 4
Group 1				
Group 2				
Group 3				
Group 4				

Group 5				
Group 6				
Average				

Have students calculate the averages of each column. Lead students to discover that the measurements become closer to the average with the increase in divisions of the meter stick.

Help students understand that the ruler with the greatest number of subdivisions per unit will provide the most precise measure. Have students write a math *learning log* entry (view literacy strategy descriptions) explaining the difference between precision and accuracy at the end of this activity.

Activity 3: Temperature—How Precise Can You Be? (GLEs: 4, 17, 18)

Materials List: paper, pencil, thermometers

Have students get in groups of three. Provide each team with a thermometer that is calibrated in both Celsius and Fahrenheit. Have each team record the room temperature in both ^{o}C and ^{o}F. Have students note the measurement increments of the thermometer (whether it measures whole degrees, or tenths of a degree) on both scales. Make a class table of the temperatures read by each team. Ask students if it is possible to have an answer in tenths of a degree using their thermometers. Why or why not? It is important that students understand that the precision of the instrument depends on the smallest division of a unit on a scale. If the thermometer only has whole degree marks then it can only be precise to one degree. If the thermometer has each degree separated into tenths of a degree, then the measurement is precise to the nearest tenth of a degree. Regardless of the measurement tool being used, this idea of the precision of the instrument holds true.

Activity 4: Repeatability and Precision (GLE: 17)

Materials List: paper, pencil

As stated in Activity 2, precision can also refer to the degree to which repeated readings on the same quantity agree with each other.

Present students with the following situations:

- Jamaal made five different measurements of the solubility of nickel (II) chloride in grams per deciliter of water and obtained values of 35.11, 35.05, 34.98, 35.13, and 35.09 g/dL.
- Juanita made five different measurements of the solubility of nickel (II) chloride in grams per deciliter of water and obtained values of 34.89, 35.01, 35.20, 35.11, and 35.13 g/dL.

Have students work with a partner to discuss ways to determine which set of measurements is more precise. Have students come up with a method for determining which set of measurements is the most precise. Lead students to the determination that the set that has the smallest range is a more precise set of measurements.

Provide students with additional measurement situations so that they have the opportunity to practice determining the more precise set of measurements when given a group of measurements.

Activity 5: Precision vs. Accuracy (GLE: 17)

Materials List: paper, pencil, Target BLM transparency, Precision vs. Accuracy BLM, sticky notes

Student Questions for Purposeful Learning or *SQPL* (view literacy strategy descriptions) is a strategy designed to gain and hold students' interest in the material by having them ask and answer their own questions. Before beginning the activity, place the following statement on the board:

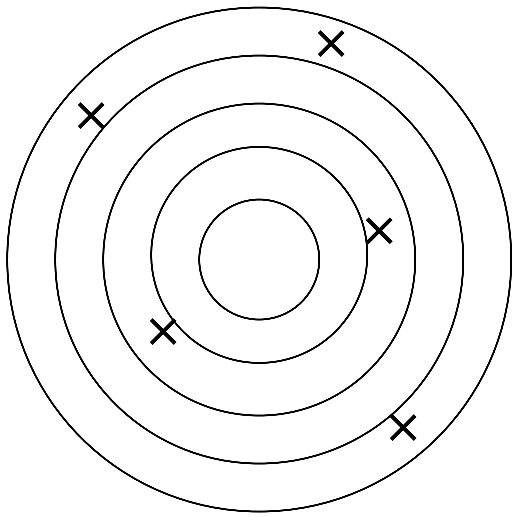
Accuracy is telling the truth. Precision is telling the same story over and over again.

Have students pair up and, based on the statement, generate two or three questions they would like answered. Ask someone from each team to share questions with the whole class and write those questions on the board. As the content is covered in the activity, stop periodically and have students discuss with their partners which questions could be answered and have them share answers with the class. Have them record the information in their notebooks.

Create a transparency of the Target BLM which includes the targets shown below, and have students determine if the patterns are examples of precision, accuracy, neither or both. Cover boxed descriptions with sticky notes and remove as the lesson progresses. After the lesson, provide students with Target BLM to include in their notes.

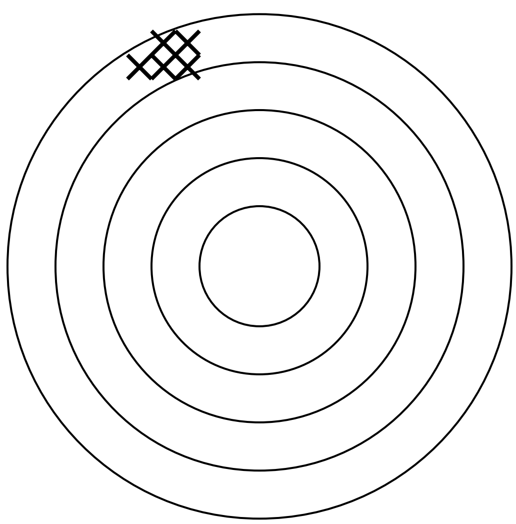
If you were trying to hit a bull's eye (the center of the target) with each of five darts, you might get results such as in the models below. Determine if the results are precise, accurate, neither or both.

Neither Precise Nor Accurate



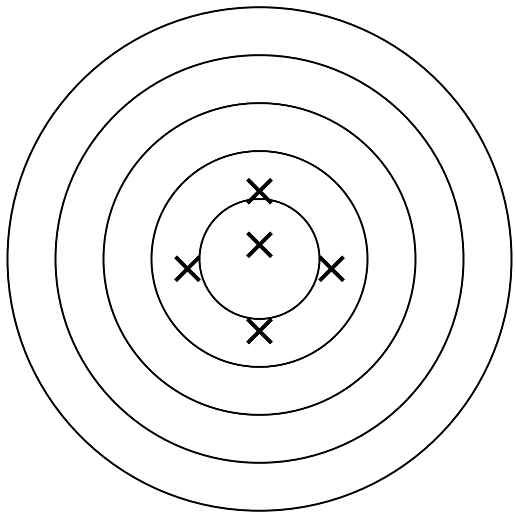
This is a random-like pattern, neither precise nor accurate. The darts are not clustered together and are not near the bull's eye.

Precise, Not Accurate



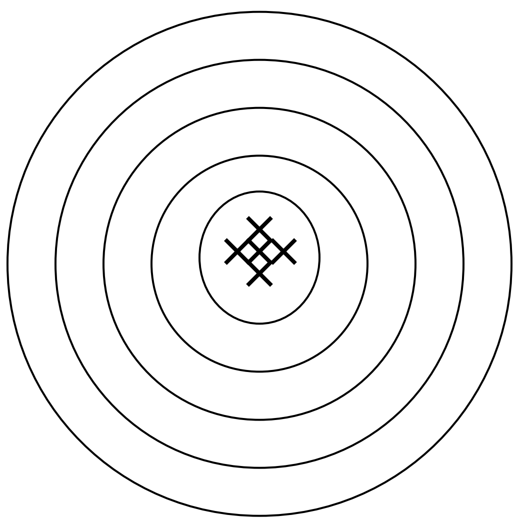
This is a precise pattern, but not accurate. The darts are clustered together but did not hit the intended mark.

Accurate, Not Precise



This is an accurate pattern, but not precise. The darts are not clustered, but their average position is the center of the bull's eye.

Precise and Accurate



This pattern is both precise and accurate. The darts are tightly clustered, and their average position is the center of the bull's eye.

Lead a class discussion reviewing the definitions of precision and accuracy, and revisit the class-generated questions.

Use the Precision vs. Accuracy BLM and present the examples to students. Lead a class discussion using the questions on the BLM.

Provide students with more opportunities for practice in determining the precision and/or accuracy of data sets.

Activity 6: Absolute Error (GLEs: 18, 20)

Materials List: paper, pencil, Absolute Error BLM, three different scales, 2 different beakers, measuring cup, meter stick, 2 different rulers, calculator, cell phone, wrist watch

In any lab experiment, there will be a certain amount of error associated with the calculations. For example, a student may conduct an experiment to find the specific heat capacity of a certain metal. The difference between the experimental result and the actual (known) value of the specific heat capacity is called absolute error. The formula for calculating absolute error is:

$$\text{Absolute Error} = |\text{Observed Value - Actual Value}|$$

Review absolute value with students and explain to them that since the absolute value of the difference is taken, the order of the subtraction will not matter.

Present the following problems to students for a class discussion:

Luis measures his pencil and he gets a measurement of 12.8 cm but the actual measurement is 12.5 cm. What is the absolute error of his measurement?
($\text{Absolute Error} = |12.8 - 12.5| = |.3| = .3\ cm$)

A student experimentally determines the specific heat of copper to be 0.3897 ^{o}C. Calculate the student's absolute error if the accepted value for the specific heat of copper is 0.38452 ^{o}C. ($\text{Absolute Error} = |.3897 - .38452| = |0.00518| = 0.00518$)

Place students in groups and have them rotate through measurement stations. Have students use the Absolute Error BLM to record the data. After students have completed collecting the measurements, present them with information about the actual known value of the measurement. Have students calculate the absolute error of each of their measurements.

Examples of stations:

Station	Measurement	Instruments	Actual Value
1	Mass	3 different scales	100 gram weight
2	Volume	2 different sized beakers and a measuring cup	Teacher measured volume of water
3	Length	Meter stick, rulers with 2 different intervals	Sheet of paper
4	Time	Wrist watch, calculator, cell phone	http://www.time.gov

Activity 7: Relative Error (GLEs: 4, 5, 20)

Materials List: paper, pencil

Although absolute error is a useful calculation to demonstrate the accuracy of a measurement, another indication is called relative error. In some cases, a very tiny absolute error can be very significant, while in others, a large absolute error can be relatively insignificant. It is often more useful to report accuracy in terms of relative error. Relative error is a comparative measure. The formula for relative error is:

$$Relative\ Error = \frac{Absolute\ Error}{Actual\ value} \times 100$$

To begin a discussion of absolute error, present the following problem to students:

> Jeremy ordered a truckload of dirt to fill in some holes in his yard. The company told him that one load of dirt is 5 tons. The company actually delivered 4.955 tons.
>
> Chanelle wants to fill in a flowerbed in her yard. She buys a 50-lb bag of soil at a gardening store. When she gets home she finds the contents of the bag actually weigh 49.955 lbs.
>
> Which error is bigger?
>
> *The relative error for Jeremy is 0.9%. The relative error for Chanelle is 0.09%. This tells you that measurement error is more significant for Jeremy's purchase.*

Use these examples to discuss with students the calculation of relative error and how it relates to the absolute error and the actual value of measurement. Explain to students that the relative error of a measurement increases depending on the absolute error *and* the actual value of the measurement.

Provide students with an additional example:

> In an experiment to measure the acceleration due to gravity, Ronald's group calculated it to be 9.96 m/s^2. The accepted value for the acceleration due to gravity is 9.81 m/s^2. Find the absolute error and the relative error of the group's calculation. (*Absolute error is .15 m/s^2, relative error is 1.529%.)*

Provide students with more opportunity for practice with calculating absolute and relative error.

Activity 8: What's the Cost of Those Bananas? (GLEs: 4, 17, 18)

Materials List: paper, pencil, pan scale, electronic scale, fruits or vegetables to weigh

The following activity can be completed as described below if the activity seems reasonable for the students involved. If not, the same activity can be done if there is access to a pan scale and an electronic balance. If done in the classroom, provide items for students to measure—bunch of bananas, two or three potatoes, or other items that will not deteriorate too fast.

Have the students go to the local supermarket and select one item from the produce department that is paid for by weight. Have them calculate the cost of the object using the hanging pan scale present in the department. Record their data. At the checkout counter, have the students record the weight given on the electronic scale used by the checker. Have students record the cost of the item. How do the two measurements and costs compare? Have students explain the significance of the number of digits (precision) of the scales and the effect upon cost.

Activity 9: What are Significant Digits? (GLEs: 4, 19)

Materials List: paper, pencil

Discuss with students what significant digits are and how they are used in measurement. Significant digits are defined as all the digits in a measurement one is certain of plus the first uncertain digit. Significant digits are used because all instruments have limits and there is a limit the number of digits with which results are reported. Demonstrate and discuss the process of measuring using significant digits.

After students have an understanding of the definition of significant digits, discuss and demonstrate the process of determining the number of significant digits in a number. Explain to students that it is necessary to know how to determine the significant digits so that when performing calculations with numbers, they will understand how to state the answer in the correct number of significant digits.

Rules For Significant Digits

1. Digits from 1-9 are always significant.
2. Zeros between two other significant digits are always significant
3. One or more additional zeros to the right of both the decimal place and another significant digit are significant.
4. Zeros used solely for spacing the decimal point (placeholders) are not significant.

Using a chemistry textbook as a resource, provide problems for students to practice in determining the number of significant digits in a measurement.

In their math *learning logs* (view literacy strategy descriptions) have students respond to the following prompt:

> Explain the following statement: The more significant digits there are in a measurement, the more precise the measurement is.

Allow students to share their entries with the entire class. Have the class discuss the entries to determine if the information given is correct.

Activity 10: Calculating with Significant Digits (GLEs: 4, 19)

Materials List: paper, pencil,

Discuss with students how to use significant digits when making calculations. There are different rules for how to round calculations in measurement depending on whether the operations involve addition/subtraction or multiplication/division. When adding, such as in finding the perimeter, the answer can be no more PRECISE than the least precise measurement (i.e., the perimeter must be rounded to the same decimal place as the least precise measurement). If one of the measures is 15 ft and another is 12.8 ft, then the perimeter of a rectangle (55.6 ft) would need to be rounded to the nearest whole number (56 ft). We cannot assume that the 15 foot measure was also made to the nearest tenth based on the information we have. The same rule applies should the difference between the two measures be needed.

When multiplying, such as in finding the area of the rectangle, the answer must have the same number of *significant digits* as the measurement with the fewest number of significant digits. There are two significant digits in 15 so the area of 192 square feet, would be given as 190 square feet. The same rule applies for division.

Have students find the area and perimeter for another rectangle whose sides measure 9.7 cm and 4.2 cm. The calculated area is (9.7cm)(4.2cm) = 40.74 sq. cm but should be rounded to 41 sq cm (two significant digits). The perimeter of 27.8 cm would not need to be rounded because both lengths are to the same precision (tenth of a cm).

After fully discussing calculating with significant figures, have students work computational problems (finding area, perimeter, circumference of 2-D figures) dealing with the topic of calculating with significant digits. A chemistry textbook is an excellent source for finding problems of calculations using significant digits.

Activity 11: Measuring the Utilities You Use (GLE: 19)

Materials List: paper, pencil, utility meters around students' households, utility bills

Have students find the various utility meters (water, electricity) for their households. Have them record the units and the number of places found on each meter. Have the class get a copy of their family's last utility bill for each meter they checked. Have students answer the following questions: What units and number of significant digits are shown on the bill? Are they the same? Why or why not? Does your family pay the actual "true value" of the utility used or an estimate? If students do not have access to such information, produce sample drawings of meters used in the community and samples of utility bills so that the remainder of the activity can be completed.

Activity 12: Which Unit of Measurement? (GLEs: 5, 21)

Materials List: paper, pencil, centimeter ruler, meter stick, ounce scale, bathroom scale, quarter, cup, gallon jug, bucket, water

Divide students into groups. Provide students with a centimeter ruler and have them measure the classroom and calculate the area of the room in centimeters. Then provide them with a meter stick and have them calculate the area of the room in meters. Discuss with students which unit of measure was most appropriate to use in their calculations. Ask students if they were asked to find the area of the school parking lot, which unit would they definitely want to use. What about their entire town? In that case, kilometers would probably be better to use. Provide opportunities for discussion and/or examples of measurements of mass (weigh a quarter on a bathroom scale or a food scale) and volume (fill a large bucket with water using a cup or a gallon jug) similar to the linear example of the area of the room. Use concrete examples for students to visually explore the most appropriate units and scales to use when solving measurement problems.

Activity 13: Using Indirect Measurement to determine the height of a Telephone Pole (GLEs: 4, 22)

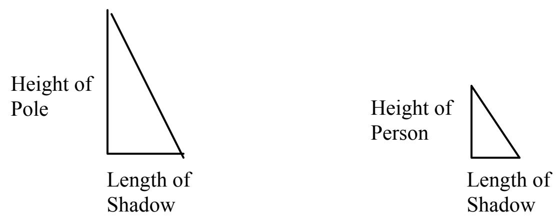
Materials List: paper; pencil; tape measures; meter sticks; math textbook

Review with students the use of proportions when solving similar triangle problems. Students should have solved problems involving proportions and similar triangles in grades 7 and 8 so this should not be new information.

After reviewing the topic of using proportions to solve such problems, present students with this challenge:

> Find the height of a telephone pole without actually measuring the pole.
> Have students get in groups to come up with ideas.

If no one discusses it, talk about how shadows are formed when the sun is shining at an angle with an object (shadows are longer during the evening hours and shorter during noon). Explain how comparing the height of a person to the length of the person's shadow could be used to calculate the height of the pole if the length of the pole's shadow is known. Have students do this activity either at school or at home, and make sure students make their measurements later in the day when a shadow is formed (not at noontime). In addition, provide students with additional problems using proportions to solve indirect measurement situations in the form of worksheets or the use of the math textbook.



$$\frac{\text{Height of Pole}}{\text{Length of Shadow Of Pole}} = \frac{\text{Height of Person}}{\text{Length of Shadow of Person}}$$

Activity 14: How thick is a Sheet of Paper? (GLE: 22)

Materials List: paper; pencil; rulers; calculators

It is often necessary to use an indirect measurement technique when measuring very large or very small things. For example, when measuring the thickness of a single sheet of paper, a ruler will not work. Present this dilemma to students and have them use *brainstorming* (view literacy strategy descriptions) to come up with ideas on how they could find the thickness of a sheet of paper by measurement. In this case, indirect measurement can help. If one measures the thickness of 400 sheets of paper (suppose these 400 sheets measured 40 mm) then divides that total measurement by the 400 sheets (40mm ÷ 400 sheets = 0.1 mm), the result will be the total thickness of a single sheet of paper. Provide calculators for students to do their computations.

Activity 15: What is the mass of a single grain of rice? (GLE: 22)

Materials List: paper; pencil; rice; scales

Have students come up with ideas on how they can determine the mass of a single grain of rice, and then actually perform the measurement using the approach they came up with. Afterwards, have each group report on its approach and on its findings. Students should be required to write a *math learning log* (view literacy strategy descriptions) discussing the approach they took to determine the mass of a single grain of rice. Discuss any discrepancies the students may have for the mass in the class. Have students determine the following from the whole group's data: the mean, median, mode, and range.

Sample Assessments

General Assessments

- Portfolio Assessment: The student will create a portfolio divided into the following sections:
    1. Accuracy
    2. Precision
    3. Precision vs. Accuracy
    4. Absolute error
    5. Relative error
    6. Significant digits

  In each section of the portfolio, the student will include an explanation of each, examples of each, artifacts that were used during the activity, and sample questions given during class. The portfolio will be used as an opportunity for students to demonstrate an understanding of each concept.

- The student will complete entries in their math *learning logs* using such topics as these:
    - Darla measured the length of a book to be $11\frac{1}{4}$ inches with her ruler and $11\frac{1}{2}$ inches with her teacher's ruler. Can Darla tell which measurement is more accurate? Why or why not? *(She cannot tell unless she knows which ruler is closer to the actual standard measure)*
    - What does it mean to be precise? Give examples to support your explanation.
    - What is the difference between being precise and being accurate? Explain your answer.
    - When would it be important to measure something to three or more significant digits? Explain your answer.

Activity-Specific Assessments

- <u>Activity 1</u>: The student will write a paragraph explaining in his/her own words what it means to be accurate. He/she will give an example of a real-life situation in which a measurement taken may not be accurate.

- <u>Activity 7</u>: The student will solve sample test questions, such as this:
  Raoul measured the length of a wooden board that he wants to use to build a ramp. He measured the length to be 4.2 m, but his dad told him that the board was actually 4.3 m. His friend, Cassandra, measured a piece of molding to decorate the ramp. Her measurement was .25 m, but the actual measurement was .35. Use relative error to determine whose measurement was more accurate. Justify your answer.

- Activity 12: The student will be able to determine the most appropriate unit and/or instrument to use in both English and Metric units when given examples such as these:

  How much water a pan holds
  Weight of a crate of apples
  Distance from New Orleans to Baton Rouge
  How long it takes to run a mile
  Length of a room
  Weight of a Boeing 727
  Weight of a t-bone steak
  Thickness of a pencil
  Weight of a slice of bread

Algebra I–Part 1
Unit 3: Solving Equations and Real-life Graphs

Time Frame: Approximately four weeks

Unit Description

This unit focuses on using algebraic properties to solve algebraic equations. The relationship between a symbolic equation, a table of values, a graphical interpretation, and a verbal explanation is also established.

Student Understandings

Students can solve linear equations graphically, from tables, with symbols, and through verbal and/or mental mathematics sequences. Students identify independent and dependent variables, slope as a "rate of change," and inverse and direct variation in graphs based on data from real-life situations.

Guiding Questions

1. Can students perform specified real-number calculations and relate their solutions to properties of operations?
2. Can students solve equations using addition, subtraction, multiplication, and division with variables?
3. Can students solve linear equations with rational (integral, decimal, and fractional) coefficients and relate the solutions to symbolic, graphical, and tabular/numerical representations?
4. Can students solve problems involving proportions and percentages?
5. Can students identify data as being directly or inversely related?
6. Can students distinguish the difference between independent and dependent variables in a real-life situation?
7. Can students understand how slope of a graph relates to a *rate of change* in a real-life situation?
8. Can students distinguish between a direct or inverse variation when analyzing a graph?

Unit 3 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Grade 8 (reinforcement)	
Algebra	
12.	Solve and graph solutions of multi-step linear equations and inequalities (A-2-M)
Grade 9	
Number and Number Relations	
4.	Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H)
7.	Use proportional reasoning to model and solve real-life problems involving direct and inverse variation (N-6-H)
Algebra	
8.	Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)
10.	Identify independent and dependent variables in real-life relationships (A-1-H)
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
16.	Interpret and solve systems of linear equations using graphing, substitution, elimination, with and without technology, and matrices using technology (A-4-H)
Geometry	
23.	Use coordinate methods to solve and interpret problems (e.g., slope as rate of change, intercept as initial value, intersection as common solution, midpoint as equidistant) (G-2-H) (G-3-H)
24.	Graph a line when the slope and a point or when two points are known (G-3-H)
25.	Explain slope as a representation of “rate of change” (G-2-H) (A-1-H)
Data Analysis, Probability, and Discrete Math	
34.	Follow and interpret processes expressed in flow charts (D-8-H)
Patterns, Relations, and Functions	
37.	Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)

Sample Activities

Activity 1: Review of basic concept of solving equations (GLE: 12—8th grade)

Materials List: paper, pencil, math textbook

Students coming into 9th grade should be very familiar with solving simple, one-step algebraic equations mentally. Review with students the basic premise behind solving simple equations building on the idea of equations as a “balance scale,” and discuss keeping both sides of the equation balanced. Have students solve a variety of real-life problems involving simple algebraic equations using their math textbook as a resource.

Have students create a *SPAWN* writing (view literacy strategy descriptions). Students should explain the steps they would use to solve an algebraic equation for an unknown value when there are variables on both sides of an equation. Allow students time to work in pairs to share their writing and provide feedback to one another. This writing incorporates the use of the P (Problem Solving) part of the strategy as described below.

SPAWN is a strategy which uses higher-level thinking prompts to elicit student writing. Each letter in SPAWN stands for a particular writing prompt. They are as follows:

S—Special Powers: students are given special powers to do things and then write about how they would use these powers.
P—Problem Solving: students write about the solution of a problem and how they would do this.
A—Alternative Viewpoints: students put themselves in the place of someone or something and write about it.
W—What If?: students are asked to write on what if something happened or changed.
N—Next: students are asked to write on what they think will happen next.

Allow students time to work in pairs to share their writing and provide feedback to one another.

Activity 2: Solving more complex equations (GLEs: 12—8th grade, 8—9th grade)

Materials List: paper, pencil, math textbook

Discuss the methods of solving multi-step equations that incorporate properties of equality (reflexive, symmetric, transitive, and substitution) to obtain a solution. Require students to solve equations which cannot be done mentally, and have students show the steps used when solving the equations. Have students solve equations that include integral and rational coefficients. Include real-world problem solving that requires writing and solving algebraic equations (i.e., perimeter applications, area problems, sum of angles in a polygon, distance/time relationships, percent increase/decrease, and proportions). Use the student math textbook as a resource for these types of problems.

Activity 3: Flow Charts for Solving Equations (GLEs: 8, 34)

Materials List: paper, pencil

Have students create a flow chart for solving equations. Assist students as necessary. Once the students have their flow charts developed, ask questions as they go through the flow chart steps with a practice problem. Repeat this activity several times by providing equations for the students to solve via the steps in the flow chart.

Activity 4: Independent vs. Dependent Variable (GLE: 10)

Materials List: paper, pencil, math textbook

Discuss the concept of *independent* and *dependent* variables using real-world examples. For example:

- The area of a square depends upon its side length
- The distance a person travels in a car depends upon the car's speed and the length of time it travels
- The cost of renting a canoe at a rental shop depends on the number of hours it is rented
- The number of degrees in a polygon depends on the number of sides the polygon has
- The circumference of a circle depends upon the length of its diameter
- The price of oil depends upon supply and demand
- The fuel efficiency of a car depends upon the speed traveled
- The temperature of a particular planet depends on its distance away from the sun

Present students with data from ten real-world contexts, and have the students work in groups to determine which is the dependent and the independent variable. Discuss each situation as a class. Explain that a two-dimensional graph results from the plotting of one variable against another. For instance, a medical researcher might plot the concentration in a person's bloodstream of a particular drug in comparison with the time the drug has been in the body. One of these variables is the *dependent* and the other the *independent* variable. The independent variable is the time lapsed since the drug was taken, while the dependent variable is the drug concentration. Explain to students that conventionally the *independent* variable is plotted on the *horizontal axis* (also known as the *abscissa* or *x-axis*) and the *dependent* variable on the *vertical axis* (the *ordinate* or *y-axis*). Relate this all pictorially with graphs.

Provide students with additional work on this topic using their math textbook as reference material. Have students write a math *learning log* (view literacy strategy descriptions) entry explaining the difference between an independent and dependent variable using a real-life situation of their own.

Activity 5: Graphing from a Table of Values (GLEs: 4, 10, 15, 24)

Materials List: paper, pencil, graph paper, math textbook, graphing calculator

Provide students with a table containing data that will form a line and have them construct a scatter plot on graph paper. For example, the table shown below displays the amount of oil being pumped from a well in relation to the number of days the well is operated. Let students first determine the dependent and independent variables, and have them use this information to appropriately graph the data (dependent variable on vertical axis and independent variable on horizontal axis). Afterwards, discuss what pattern students see in the data (i.e., it appears to form a linear function).

Have students answer questions based on the graph. For example, have students determine the amount of oil that was pumped after 4 days based upon the results of the graph. Ask them to determine if they think this answer is an exact value or an approximate value and why they think so. Next, have students draw a line through the data points and let them see that the line extends through the origin. Talk about the *initial value* and *intercepts* in real-world terms. For this example, the line intercepts the graph at (0,0) which means that the number of barrels pumped is 0 barrels after 0 days.

# of Days Pump is On	2	5	9	10
# of Barrels Pumped	12	30	54	60

Connect the paper and pencil work associated with this problem to using a graphing calculator to do the work. Demonstrate for students how to input data into lists, how to plot this data in a scatter plot, and how to determine a line of best fit for the inputted data. Provide students with additional work on this topic using their math textbook as a resource.

Activity 6: Direct and Inverse Relationships (GLE: 37)

Materials List: paper, pencil

Discuss with students what is meant by the terms *Directly Related* and *Inversely Related* in the context of real-life situations at an elementary level (Note: this is different than direct and inverse variation which will be discussed later in this unit). If two variables have a direct relationship, as one variable increases, the other will also increase in value. Likewise, as one variable decreases, the other also decreases in value. An example where a direct relationship exists is the cost to feed a family—as the number of members in the family increases, the cost to feed the family also increases.

In contrast to the direct relationship, in an inverse relationship, as one variable increases, the other variable decreases, and vice versa. An example of an inverse relationship is the relationship that exists between the number of workers to do a job and the time it takes to finish the job. For instance, suppose it takes 6 workers 1 day to paint a house. If the number of workers decreased, the time it takes to do the job would increase (an inverse relationship). Discuss several real-life examples with students. Have students write a math *learning log* entry (view literacy strategy descriptions) about a real-life situation that involves a direct relationship and an inverse relationship, and then discuss these as a class.

Activity 7: Graphs and Direct and Inverse Relationships (GLEs: 10, <u>37</u>)

Materials List: paper, pencil, graphs from science and other books, math textbook

Provide students with several line graphs relating different quantities (taken from science books, or their math textbook). Have students work in groups to obtain the following information for each graph:

1. What is the independent variable? What is the dependent variable?
2. Does the graph portray a direct or inverse relationship between the variables? Explain your reasoning.
3. Is the graph linear?

Afterwards, talk with students about increasing and decreasing functions and how they are related to direct and inverse relationships. For example, in an increasing function, a direct relationship exists. In contrast, for a decreasing function, an inverse relationship exists. Relate all of this information graphically. The goal here is to begin getting students to be able to analyze real-life relationships which are presented graphically and to connect this later with the equations which model these graphs.

Activity 8: Going on Vacation! (GLEs: 10, <u>15</u>, 23, 24, <u>37</u>)

Materials List: paper, pencil, graph paper, graphing calculators, Going on Vacation BLM

Have students work with a partner on the Going on Vacation BLM. The problem requires students to identify the independent and dependent variable, create a table of values relating the two variables, use the table to graph the relationship, write an equation to fit the graph, and then analyze the graph. After students create charts and graphs using pencil and paper, demonstrate for students how to use a graphing calculator to do the assigned work by inputting the table of values into lists, how to create a scatter plot of the data, and how to find an equation for the line of best fit. Compare the equation that the calculator produces with the equations that the students have written.

Activity 9: Analyzing Distance/Time Graphs (GLEs: 10, <u>25</u>, <u>37</u>)

Materials List: paper, pencil, math textbook, Analyzing Distance/Time Graphs BLM

Have students work in groups on Analyzing Distance/Time Graphs BLM. In this activity, each graph presented displays the distance three different cars traveled over a certain time period. Have students analyze the graphs, and discuss the questions related to the graphs. Be sure to discuss the difference between the dependent and independent variables, and relate the slope of a distance time graph with the speed or “rate of change” of the moving object.

After a full discussion of the activity has taken place, provide additional problems in which the student has to find the rate of change using the slope in real-life situations. Use the math textbook as a resource for additional problems.

Activity 10: Slope—What does it Tell Us About a Graph? (GLEs: 10, 15, 23, 24, 25, 37)

Materials List: paper, pencil, graph paper, What Does Slope Tell Us About a Graph? BLM

In this activity, students will interpret the meaning of slope as a *rate* as it applies to a real-life situation. Provide students with a copy of What Does Slope Tell us About a Graph? BLM. Let students work individually first, then allow students to pair with a partner to discuss their solutions. Afterwards discuss the results as a class. Students should understand that in a real life situation the slope tells the rate at which the values change. Students should also realize that a slope in a real-life situation has units associated with it. In this particular problem, the units associated with the graph are kilograms per year. The slope tells us how a person's weight changes over time.

Activity 11: When Will They Meet! (GLEs: 10, 15, 16, 23, 25, 37)

Materials List: paper, pencil, graph paper, colored pencils, When Will They Meet? BLM

Using the When Will They Meet? BLM, students will interpret the graph of a distance/time relationship and answer questions based upon the analysis of the graph. They will also use the point of intersection for two lines to answer a real-world problem.

Activity 12: Direct and Inverse Variation (GLE: 7, 15, 37)

Materials List: paper, pencil, graph paper, Direct and Inverse Variation-Part 1 BLM, Direct and Inverse Variation-Part 2 BLM, math textbook

Part 1: Provide students with copies of Direct and Inverse Variation-Part 1 BLM and let students work in pairs on this worksheet. It is a student-centered activity, and the teacher should let students try to figure out the work themselves and act as a facilitator. After students have been given the opportunity to do the work, discuss the worksheet and talk about what constitutes a direct variation situation. When the quotient of two quantities is related by a constant factor, there is a direct variation between the two variables. For example, $\frac{d}{t} = r$ represents a direct variation situation if the rate remains constant. The distance traveled divided by the time that is traveled represents this constant value. In more general terms, a direct variation is given by: $y = kx$, where k is any non-zero number. Provide students with examples and problems that represent a direct variation situation.

Part 2: Provide students with copies of Direct and Inverse Variation-Part 2 BLM. Again let students work on this first, and then discuss as a class. In an inverse variation, the product of two quantities is a constant. For example, $rt = d$ represents an indirect variation if the product of the rate and time represents a constant distance. Indirect variation is more commonly seen in math as given by the equation: $y = \frac{k}{x}$.

Part 3: Provide students with examples and problems from the student math textbook that involve direct and inverse variation situations and discuss these with students. Finally, have students create *vocabulary cards* (view literacy strategy descriptions) for the terms direct variation and inverse variation. Make sure they describe the difference between a direct and inverse relationship verbally, graphically, and algebraically. When students create vocabulary cards for terms that are related to what they are learning, students should see connections between words and critical attributes associated with the word. In the middle of a single card, the vocabulary word is written; and in the four corners of the card would be the definition, characteristics, examples, and illustrations. An example of a vocabulary card for direct variation is shown below:

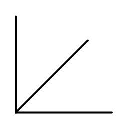
Definition
When the quotient of two quantities is related by a constant factor

Characteristics
Any equation of the form y = kx, where k is a constant is an example of direct variation.

Direct Variation

Examples
$\frac{d}{t} = r$ represents a direct variation situation if the rate remains constant. The distance traveled divided by the time that is traveled represents this constant value.

Illustrations



As an extension, there is an activity which utilizes direct and indirect variation which can be found at the following website: http://jwilson.coe.uga.edu/emt669/Student.Folders/Jeon.Kyungsoon/IU/rational2/Telescope.html. In this activity, students make a telescope using cardstock and perform investigations which demonstrate applications of direct and indirect variation.

Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will review magazines, newspapers, or journals (or recall personal experiences) for real-world relationships that can be modeled by linear functions (include function and graph).
- The student will compile a portfolio of work for Unit 3 to be handed in for a grade.
- The student will draw numberless graphs that relate to a situation in real-life, explaining the graph in words and relating it to the motion or situation depicted.
- The student will take paper and pencil tests related to the concepts that were taught in this unit.

Activity-Specific Assessments

- Activity 6: The student will write a short paragraph explaining what is meant by direct and indirect relationships and give examples.

- Activity 7: The student will write the relationship that exists between the variables for each graph provided him/her in the activity.

- Activity 11: The student will make an oral presentation of his/her findings and explain the processes that led to his/her conclusions.

Algebra I–Part 1
Unit 4: Linear Equations and Graphing

Time Frame: Approximately four weeks

Unit Description

This unit focuses on developing an understanding of graphing linear equations in the coordinate plane.

Student Understandings

Students recognize linear relationships, simplify linear expressions, and graph linear equations in two variables. They use a variety of techniques when graphing linear equations including input-output tables, two points, and slope and one point. They graph manually and using a graphing calculator.

Guiding Questions

1. Can students graph data from input-output tables on a coordinate graph?
2. Can students recognize linear relationships in graphs of input-output relationships?
3. Can students graph the points related to a direct proportion relationship on a coordinate graph?
4. Can students perform simple algebraic manipulations of collecting like terms and simplifying expressions?
5. Can students determine the slope of a line given a graph or two points?
6. Can students perform translations and line reflections on the coordinate plane?

Unit 4 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Algebra	
8.	Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)
GLE #	**GLE Text and Benchmarks**
9.	Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
10.	Identify independent and dependent variables in real-life relationships (A-1-H)

11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
12.	Evaluate polynomial expressions for given values of the variable (A-2-H)
13.	Translate between the characteristics defining a line (i.e., slope, intercepts, points) and both its equation and graph (A-2-H) (G-3-H)
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
Geometry	
23.	Use coordinate methods to solve and interpret problems (e.g., slope as rate of change, intercept as initial value, intersection as common solution, midpoint as equidistant) (G-2-H) (G-3-H)
24.	Graph a line when the slope and a point or when two points are known (G-3-H)
25.	Explain slope as a representation of "rate of change" (G-3-H) (A-1-H)
26.	Perform translations and line reflections on the coordinate plane (G-3-H)
Data Analysis, Probability, and Discrete Math	
29.	Create a scatter plot from a set of data and determine if the relationship is linear or nonlinear (D-1-H) (D-6-H) (D-7-H)
Patterns, Relations, and Functions	
37.	Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)
38.	Identify and describe the characteristics of families of linear functions, with and without technology (P-3-H)
39.	Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)
40.	Explain how the graph of a linear function changes as the coefficients or constants are changed in the function's symbolic representation (P-4-H)

Sample Activities

Activity 1: Lines on a Plane (GLEs: 10, 12, 13, <u>15</u>, 24, <u>38</u>, <u>40</u>)

Materials List: paper, pencil, graph paper, graphing calculators

Discuss with students how to create an *x/y* (input/output) table of values to graph the following two equations: $y = 2x$ and $y = -2x$. Have students graph the two equations on two different coordinate planes using paper/pencil and graph paper. Discuss the convention of using *x*-values as the independent variable and the *y*-values as the dependent variable when graphing general algebraic equations. Students should see that both equations form a line, thus they are linear equations. Have students individually write a two-column chart showing what is the same and what is different about the two graphs. Afterwards, have students exchange each other's list and provide feedback to one another. This analysis of one another's work is a form of *questioning the author* or *QtA* view literacy strategy descriptions) which is a literacy strategy designed to get students to question the author as they read and to write these questions down as they read text. In this particular case, as students read their fellow classmates' work, they should be asking themselves the following questions: Has the student identified that one of the similarities includes the fact that both equations form lines and both pass through the origin? Has the student observed that in the case of the first graph, $y = 2x$, the graph is increasing and in the second graph, $y = -2x$, the graph is decreasing? When going over this activity as a class, discuss what the terms increasing and decreasing mean mathematically and graphically (in an increasing graph, as the x-values increase, the y-values increase; in a decreasing graph, as the x-values increase, the y-values decrease). Finally, have students graph the two equations using a graphing calculator and discuss setting up a window to view graphs, as well as learning how to trace along the graphs using the trace feature

Activity 2: How Does "*m*" Affect the Graph of an Equation in the Form of *y* = *mx* (GLEs: 13, 38, 39, 40)

Materials List: paper, pencil, graphing calculators

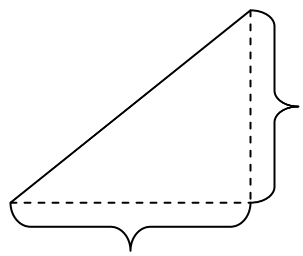
Build on the previous activity and students' prior work with slope in Unit 3 by providing students with graphing calculators, and have them work in pairs during this activity. Discuss the formula $d=rt$ which relates the distance traveled by an object as determined by the rate at which the object travels and the time it travels at this rate. Remind students about direct variation situations, and relate this formula for distance, rate, and time to the direct variation form of $y = kx$. Students should see they are of the same form; only the letters are different. Explain to students that instead of using the letters $d=rt$ to model the formula, the form $y=mx$ will be used by the graphing calculator to graph this relationship. In place of the variable r (the rate at which the object is moving) the variable m will be used. Have the students determine what the graph of $y = 5x$ looks like on their graphing calculators. Relate this graph and equation to moving at a rate of speed of 5 miles per hour, where the x-values represent the time in hours and the y-values represent the distance traveled in miles. Show students how the x and y values (which show up at the bottom of the calculator screen as the line is traced) indicate the time and distance values, respectively. Use the table function of the calculator to show students that the calculator also lists these values in a table with increments that can be changed by the user. Also, note that the graph passes through the origin.

Next, have students investigate how the graph of $y = mx$ would change if the value of m is changed to different positive integers. Students should see that the changes in m create changes in the graph's steepness, but that the graph is still increasing as x gets greater. Have students use negative integer values for m in the equation $y = mx$, and have students investigate what this change does to the original graph. Students should see that the graph is now a decreasing graph, but that the graph is still linear and still passes through the origin. Discuss the findings with students, and help students to realize that in the equation $y = mx$, the number m is the slope of the line. Students should see that positive values for m result in an increasing linear function, while negative values for m result in a decreasing linear function. Students should also see that the closer the value of *m* is to zero, the more the line becomes horizontal, and that making the value of *m* really large results in a more vertical graph. This will be important later when discussing horizontal and vertical lines.

Activity 3: Developing the Slope Formula (GLEs: 13, 23)

Materials List: paper, pencil, graph paper, math textbook

In this activity, begin helping students understand how to determine slope of a line using $\frac{rise}{run}$ and develop the slope formula when given two points. In this teacher-led activity, explain to students that when building a house, the pitch of a roof is usually given as a ratio between the rise of the roof and the run of the roof.



Provide students with graph paper and write the following roof pitches on the board. Have students create scale drawings of the roof pitches on coordinate graph paper:

Roof 1—rise 3 feet to a run of 1 foot;
Roof 2—rise of 8 feet to a run of 2 feet;
Roof 3—rise of 1 foot to a run of 1 foot;
Roof 4—rise of 4 feet to a run of 1 foot.

After students make sketches of each roof, discuss the steepness of each roof. Have the students compare which is the steepest and which is the least steep, and have them notice the fact that Roof 2 and Roof 4 have the same steepness. Relate this activity to the fact that the steepness of a line segment (or roof) is the slope of the line and is the ratio of $\frac{rise}{run}$. Discuss the fact that Roof 2 has a ratio of $\frac{8}{2}$ or 4 and Roof 4 has a ratio of $\frac{4}{1}$ or 4, hence the slope of both graphs is the same. Explain that the larger the ratio is numerically, the steeper the slope of the line segment will be. Provide students with problems where they have to determine the slopes of several lines on a coordinate grid by finding the $\frac{rise}{run}$. Finally, lead students to understand how the slope of a line can be calculated using the formula, $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ when given two points on a line. Using the same lines from the worksheet they just did, show students how the slope can be calculated with this formula using two points which are on the line. Provide additional practice for students on the skill of finding the slope given two points using the math textbook as a resource.

Activity 4: Slope as a Rate of Change (GLEs: 9, 10, <u>23</u>, <u>25</u>, 37)

Materials List: paper, pencil, graph paper, math textbook, Slope as a Rate of Change BLM

Provide students with graph paper and copies of the Slope as a Rate of Change BLM. Have students work in groups on the BLM which involves questions related to the concepts of slope and linear equations. Discuss the results as a class. After discussing the answers with students, guide them to understand the fact that the slope of a line is constant. In the activity, no matter what two points were chosen to determine slope, the slope was the same value. This is a BIG IDEA! Also, when determining slope in real-life contexts, it is important that students understand that there is not only a number associated with the slope, but there are also units associated with this slope. The slope represents a rate which has real-life meaning attached to it. Provide additional work having students determine the slope as a rate of change in real-life contexts. Use the math textbook as a resource for additional problems of this type.

Have students create a *vocabulary card* (view literacy strategy descriptions) for the term "slope" and have students exchange cards and provide feedback to one another. Discuss some of the cards that were created as a class. When students create vocabulary cards for terms that are related to what they are learning, students should see connections between words and critical attributes associated with the word. In the middle of a single card, the vocabulary word is written, and in the four corners of the card are the definition, characteristics, examples, and illustrations. An example of a *vocabulary card* for slope is shown below.

Definition

A measure of the steepness of a line

Characteristics

A line can have no slope (vertical), 0 slope (horizontal), positive slope (increasing) or negative slope (decreasing).

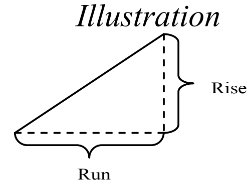
SLOPE

Examples

$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ represents the slope formula to be used when finding the slope given two points.

The slope of the line y = 5x – 3 is 5 since this is in slope intercept form (y = mx +b).

Illustration



Activity 5: Finding the Intercepts of a Linear Graph (GLEs: 13, 23, 24)

Materials List: paper, pencil, graphing calculators, graph paper

In this activity, the goal is to help students understand what intercepts are and how to determine the x- and y-intercepts for an equation and its graph.

Have students use a graphing calculator to graph the line $y = 2x + 6$. Discuss with students the definition of an intercept—the point where the graph intersects an axis. Ask students what they think the terms x-intercept and y-intercept mean, and clarify should there be misinterpretation of the terms.

Show students how to find the intercepts using both calculator technology and algebraically using paper and pencil methods. First, have students trace the graph and find the x-intercept. Ask what the y-value is when the graph intersects the x-axis. They should respond that the y-value is 0. Have students substitute 0 for y in the equation $y = 2x + 6$, and then solve for x. Thus the point (-3, 0) is the x-intercept for the graph of $y = 2x + 6$. Next, have students trace the calculator graph to the y-intercept and ask students to indicate to find the x-value. Have students substitute 0 for x in the same equation and solve for y to determine that the point (0, 6) is the y-intercept for the graph. Have students plot the intercepts on graph paper and draw a line through them. Ask students to compare this graph with the one drawn on the graphing calculator. It is important to remember that substituting $x = 0$ to find the y-intercept (and $y=0$ to find the x-intercept) is counter-intuitive to many students.

Next, have students determine the intercepts for the equation $y = -2x – 4$, and then have them use the intercepts to create a graph for the equation. Have students work individually and then form pairs to exchange answers and discuss how they determined the solutions. Discuss the results as a class.

Give students the equations $y = 3x - 6$ and $y = -3x + 6$. Have students write a math *learning log* (view literacy strategy descriptions) entry about the similarities and differences they should see in the graphs of these equations. Allow time for students to exchange their entries to check each other for accuracy and logic. Students should revise their entries based on peer feedback.

Activity 6: Is the Data Linear? (GLEs: 15, 23, 29, 39)

Materials List: paper, pencil, graphing calculator, math textbook

In this activity, provide students with data from input/output tables, and have students predict, without actually graphing, if the data is linear. Have students enter the data into their graphing calculators and graph to determine if their prediction is correct. Ask students to examine the data for patterns that might determine the answer to the question without the need to graph the data. It may be necessary to give them a hint by asking them to look for patterns within the x-values and then within the y-values of each chart. They should also compare the x values in one table to the x values in the second table, and they should do the same with the y-values. Students should see that the difference between any two x values in both tables is 1, so the data sets are alike in that respect. They will probably explain that that difference in any two y values in Table 1 is 4. Some may see the y-values in the second chart as being doubled each time; others may explain that the difference in any two y values is not constant or the same (the differences are doubled). For this example, the difference lies in the pattern of the y-values. Ask which table shows a linear graph. Explain to students that if data is linear, there are two things to determine – change in the x-values and the change in the y-values. Explain that linear relationships result when the change in y-values divided by the change in x-values for any two points is constant. Relate this to the slope formula used in Activity 4. For example, the data in the Table 1 is linear because there is a constant ratio (or slope) of 4 between any two consecutive points, while the data in Table 2 does not have this characteristic.

Data Table 1

x-value	y-value
1	4
2	8
3	12
4	16
5	20

Data Table 2

x-value	y-value
1	4
2	8
3	16
4	32
5	64

Provide other examples of data for students to analyze to become proficient at this skill using the math textbook as a resource. In addition, provide opportunities for students to actually collect data themselves, and then determine if the data is linear or non-linear. For example, students could investigate the relationship between the side length of a square and its perimeter where the side length is the independent variable and the perimeter is the dependent variable. Students could likewise investigate the relationship between the side length of a square and its area where the side length is the independent variable and the area of the square is the dependent variable. In these two cases, students should see that there is a linear relationship between side length and perimeter, but a non-linear relationship between side length and area.

Have students write a math *learning log* (view literacy strategy descriptions) entry relating the process they could use to determine whether a set of data is linear or non-linear without actually graphing the data. Let students exchange their logs and provide feedback to one another, then have students make any adjustments to their initial logs and turn them into the teacher.

Activity 7: Slope-Intercept Form of a Line (GLEs: 13, <u>40</u>)

Materials List: paper, pencil, graphing calculators, Match the Equation with the Graph BLM

In Activity 2, students learned that any equation in the form $y = mx$ forms a line. This activity extends to equations in slope-intercept form, $y = mx + b$. Students will investigate the effect of b on the graph of the line.

Students should already understand how the value of m affects the slope of the line based upon what they learned in Activity 2. Using the graphing calculator, let students discover what happens to a line when the value of m is held constant and the value of b is changed. Starting with the equation $y = 2x$, have students discover what happens to the graph when b is changed using equations such as $y = 2x +1$; $y = 2x +2$; and $y = 2x-1$. Make sure that students use both positive and negative values for b. Students should see that the effect of changing b translates the line $y = mx$ up or down the y-axis by b units. Students should also recognize that b is the y-intercept for the line. Once students have investigated thoroughly the effects of changing the values of m and b on a linear graph, provide students with copies of Match the Equation with the Graph BLM. Allow students the opportunity to work on this alone and then pair up with another student to compare their answers before going over the results as a class.

Activity 8: Graphing Using a Point and the Slope (GLEs: 13, <u>24</u>)

Materials List: paper, pencil, graph paper, math textbook

This activity is designed to use what has been learned about slope in order to graph lines on a coordinate plane. Have students plot the point (0, 4) on a coordinate grid. Have students draw some lines that pass through the point (0, 4). Students should realize that an infinite number of lines actually run through the point (0, 4).

Next, using a new coordinate grid, tell students to find the line that passes through (0, 4) but has a slope of $\frac{3}{2}$. Let the students work in groups to find other points on the line graph using the slope of $\frac{3}{2}$. (Remind students of the relationship between slope and the rise/run). Have students list some other points and ask them how many lines run through the point (0,4) and have a slope of $\frac{3}{2}$. Students should come to the realization that there is really only one distinct line that has those characteristics. They should also realize that there are an infinite number of points that could be produced in order to draw the line with these characteristics. Finally, have students graph other lines using a point and a slope with which to make their graphs. Use the math textbook as a resource for additional problems of this type.

Activity 9: Equivalent Forms of Equations (GLEs: 8, <u>11</u>, 15)

Materials List: paper, pencil, graph paper, math textbook

Provide students with the following pair of linear equations in two variables: $y = -2x + 4$ and $4x + 2y = 8$. Have students make a table of xy values for the two equations. After the table of values is found, have students graph the two lines using graph paper. Students should see that the two equations produce the same line. Provide students with additional pairs of equations, some which produce the same line and some which do not. Have students decide which equations and graphs are the same and which are different, and discuss what they did to figure this out. Refer back to the equations which were used initially: $y = -2x + 4$ and $4x + 2y = 8$. Remind students that both equations produced the same line. Explain that these two equations are equivalent forms of a linear equation. One of the equations is written in slope-intercept form, while the other is written in standard form and multiplied by a factor of 2. Discuss with students the two different forms of writing linear equations and how to translate a linear equation from standard form [$Ax + By = C$] to slope-intercept form [$y = mx + b$] using algebraic manipulation. Explain that the benefit of having an equation in $y = mx + b$ form is the ease with which one can determine the slope and y-intercept and then use that information to sketch the graph. The benefit of writing an equation in standard form is that there are no fractions, and one can easily see what the x- and y-intercepts are for the graph mentally. Provide students with the practice necessary to develop the skill of transforming an equation from standard form into slope-intercept form, and vice versa. Also, have students identify the y-intercept and slope for each equation and graph the line using paper and pencil. Use the math textbook as a resource for additional work on this concept.

Activity 10: Modeling Functions (GLEs: <u>9</u>, 10, 12, <u>15</u>, 23, 24, <u>25</u>, 37)

Materials List: paper, pencil, graph paper

This activity is intended to be a teacher-led activity. Introduce the following situation to students:

> *A family is going on a trip. They travel h hours in a day, averaging 50 mph. Write an equation to represent the distance traveled, d, in miles after traveling h hours.*

Allow students time to write an equation that matches the situation. Discuss this as a class. Have students explain how they named any variables they used to make their models. Next, have students construct a graph using paper and pencil to display the equation graphically. In the process, ask students to determine which of their variables represents the independent variable and which represents the dependent variable. In this case, the function used to model this situation is the distance traveled per day (d) equals the speed (50 mph) times the number of hours driven (h or t). So the equation they use to model the situation should be of the form: $d = 50h$. The x-axis (independent) is the

number of hours driven (scale from 0 to 24 hrs) and the *y*-axis (dependent) is the number of miles driven (scale from 0 to 1,200 miles). The graph would consist of a straight line starting at the origin (0,0) and ending at the point (24, 1200). Have the students answer questions concerning the graph. Some possible questions might be:
How far does the family travel in 3 hours? *Answer = 150 miles*; How many hours will it take to cover 200 miles? *Answer = 4 hours*. What does the slope of this graph represent in real life terms? *Answer: The rate of speed which is 50 miles per hour*. Help students to appreciate the value of representing an equation graphically and the ease with which information from the situation can be obtained.

Activity 11: Transformations of Figures on Graphs (GLEs: 26)

Materials List: paper, pencil, Internet, math textbook

In previous years, students should have been introduced to the three types of transformations—translations, reflections, and rotations. Review the three types of transformations with students, and then have students transform figures using a coordinate grid. Provide students with figures on a coordinate grid, have them transform the figures in different ways using all three types of transformations, and give the new location for the vertices of the transformed figures. Limit work with rotations to rotating the figure about the origin using multiples of 90° clockwise or counterclockwise. The following websites located at the NCTM Illuminations math site contain interactive lessons with translations and reflections. The sites are as follows:
http://illuminations.nctm.org/LessonDetail.aspx?id=L474
http://illuminations.nctm.org/LessonDetail.aspx?id=L475
Use the math textbook as an additional resource for work with transformations of figures on a coordinate grid.

Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will write a short paragraph explaining the connection between slope and rate of change.
- Project the following equations $y = x - 2$ and $y = -x + 2$, have students determine slopes, *x*- and *y*-intercepts, and explain what is the same and what is different about the two graphs.

- The student will take paper and pencil tests on all concepts studied during this unit.

Activity-Specific Assessments

- Activity 4: The student will write about a type of rate that occurs in real life that has not been discussed in class and explain what this rate means in real-life terms. For example, if a person grew 9 inches over a four year time span, that means that the person grew at an average rate of 2.25 or 2 ¼ inches per year over that four years.

- Activity 5: The student will find the *x*- and *y*-intercepts for the linear equation $3x + 5y = 12$, and use them to graph the equation on grid paper.

- Activity 6: The student will create two data tables—one that will produce a linear function and one that is not linear.

- Activities 9: The student will write two equivalent forms of a linear equation. For example, the equation $3x + 5y = 10$ can also be written as $5y = -3x + 10$ or $y = -\frac{3}{5}x + 2$.

- Activity 11: The student will draw a polygon on a coordinate grid and show transformations of the figure using all three types—translation, reflection, and rotation. The student will explain what type of transformation was made in each case.

Algebra I–Part 1
Unit 5: Graphing and Writing Equations of Lines

Time Frame: Approximately three weeks

Unit Description

In this unit, the emphasis is on writing and graphing linear equations in both real-life and abstract situations. Interpretation of the real-life meaning of equations and the relationship between the values of coefficients in the linear equation and their effect on graphical features is reinforced.

Student Understandings

Students understand the meanings of slope and y-intercept and their relationship to the nature of the graph of a linear equation. They write and interpret equations of lines using the slope-intercept, point-slope, and standard form for the equation of a line. Given equations, students can graph and interpret information from the graphs.

Guiding Questions

1. Can students write the equation of a linear function given appropriate information about the slope and intercept?
2. Can students use the basic methods for writing the equation of a line (i.e., two-point, slope-intercept, point-slope, and standard form) and translate from one form to another?
3. Can students perform the algebraic manipulations on the symbols involved in a linear equation to find its solution and relate its meaning graphically?
4. Can students discuss the meanings of slope and intercepts in the context of an application problem?
5. Can students interpret and analyze the results from a linear equation which models real-life situations and apply meaning to this equation in terms of its slope and y-intercept?

Unit 5 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Algebra	
9.	Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
10.	Identify independent and dependent variables in real-life algebraic relationships (A-1-H)

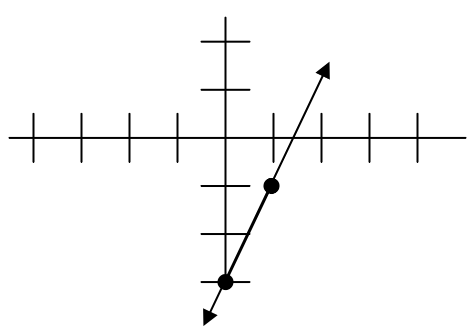
GLE #	GLE Text and Benchmarks
11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
12.	Evaluate polynomial expressions for given values of the variable (A-2-H)
13.	Translate between the characteristics defining a line (i.e., slope, intercepts, points) and both its equation and graph (A-2-H) (G-3-H)
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
Geometry	
23.	Use coordinate methods to solve and interpret problems (e.g., slope as rate of change, intercept as initial value, intersection as common solution, midpoint as equidistant) (G-2-H) (G-3-H)
24.	Graph a line when the slope and a point or when two points are known (G-3-H)
25.	Explain slope as a representation of "rate of change" (G-2-H)
Patterns, Relations, and Functions	
37.	Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)

Sample Activities

Activity 1: Writing a Linear Equation Given Slope and Y-intercept (GLEs: 11, 13, 23, 24)

Materials List: paper, pencil, graph paper, math textbook

In Unit 4, students were taught to graph a line given its slope and *y*-intercept. In this activity, students will write the equation of a line given its slope and *y*-intercept. Discuss with students how to write and graph an equation in slope-intercept form (i.e., $y = mx + b$) given the slope, m, and *y*-intercept, b. For example, if a line has a slope of 2 and a *y*-intercept of -3, the equation would be written as $y = 2x - 3$ and its graph would be as shown:



Explain to students than anytime the slope and *y*-intercept for a line are known, it is very simple to write its equation using this format. Provide additional examples for students, and make the connection between how to write, as well as how to graph, linear equations when given a slope and a *y*-intercept. In addition, lead students through the process for changing an equation from slope-intercept form into standard form ($Ax + By = C$) by the use of algebraic manipulation. Because students are not solving for a value, this is a very hard process for many students. Provide ample opportunity for students to show proficiency in this skill. Use the textbook as a resource for additional problems. Students should be able to convert equation from one form to the other with ease.

Activity 2: Writing a Linear Equation Given a Point and a Slope (GLEs: 11, 13, 23, 24)

Materials List: paper, pencil, graph paper, math textbook

Introduce the point-slope form of a linear equation and how it can be used to find the equation of a line. Indicate that this is especially useful when a portion of the graph of the line is given, but the y-intercept is not included in the portion of the graph shown.

The point-slope form of a line is derived from the slope formula: $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$. By cross multiplication: $(y_2 - y_1) = m\,(x_2 - x_1)$. Assuming (x_1, y_1) is the known point, $(y_2 - y_1) = m\,(x_2 - x_1$ becomes $(y - y_1) = m\,(x - x_1)$. Demonstrate to students how this formula can be used to write an equation. For example, if the slope is 5 ($m = 5$) and the line contains the point (3, 4), the point-slope equation would be given as follows: $(y - 4) = 5\,(x - 3)$. Emphasize how the point and slope are replaced in their respective places in the formula.

Have students work to rearrange or transform into either the slope-intercept form ($y = 5x - 11$) or standard form ($5x - y = 11$). Continue to have students draw the graph of an equation. Knowing the point and slope, students can plot the point and count off the slope to graph. Emphasize the connection between the point-slope and slope-intercept forms of the equation for graphing. Both provide a point (the intercept or another point) and both provide a slope; therefore, plotting and counting slope to find a second point can be used to graph each of these. Note that the standard form of the equation might be most useful in determining the x and y intercepts for the graph since one of the terms in the equation is eliminated when 0 is substituted for either x or y.

Provide additional examples and practice using the math textbook as a resource for students to achieve proficiency in writing equations of lines given a point and a slope.

Activity 3: Determine an Equation for a Line Given Two Points (GLEs: 11, 13, 15)

Materials List: paper, pencil, math textbook, What's the Equation of the Line? BLM

Make copies of the What's the Equation of the Line? BLM. Allow students to work in groups of three to four on this BLM. In this activity, students must use what they learned in Activity 2 (determining the equation of a line given its slope and a point on the line) and extend that knowledge to be able to determine the equation of a line given two points on the line. The intent of the activity is to develop the understanding that to find the equation for a line, two things must be known about the line in question—the slope and a point that lies on the line. Students should first realize that the two points must be used to determine the slope of the line that contains them. Then, using this slope and one of the two data points given, students use the procedures previously discussed to determine the equation of the line in all three forms—point-slope, slope-intercept, and standard form.

Have students create a *SPAWN* writing (view literacy strategy descriptions) using the letter *P* from *SPAWN* to explain how they found the solution to the problem 4 on the BLM.

Provide students with additional problems of this type using the math textbook as a resource.

Activity 4: Equations of Vertical and Horizontal Lines (GLE: 13)

Materials List: paper, pencil, math textbook

In this activity, we extend writing equations of lines to include equations of vertical and horizontal lines. In a teacher-led discussion, explain to students why the slope of a horizontal line is 0, then use the point-slope formula to help students see that the equation for a horizontal line is $y = a,$ where a is actually the y-coordinate for any point on the line. Repeat the same process using points on vertical lines. Lead students to see that since division is undefined, the slope of a vertical line is undefined. Ask students to write the equation of a vertical line based on what they know about the equations of horizontal lines. Lead students to see that the equation of a vertical line is $x = a$, where a is the x-coordinate for any point on the line. Provide students additional examples and problems using the math textbook as a resource.

At the end of this activity, have students participate in a version of *professor know-it-all* (view literacy strategy descriptions). This strategy can be used once coverage of a particular content has been completed. In this particular use of the strategy, form groups of 3 students to come up to the board and act as the professor. Call on groups randomly. Students should quiz the professors on how to determine the equation for a line given a slope and a point, two points, or a vertical or horizontal line. Students ask the questions, the professors answer the questions and aren't allowed to sit until the class feels its question has been answered satisfactorily. Each "professor" in the group should be required to answer some part of the question. *Professor know-it-all* is a fun way to review a concept and also to see if the students really grasp the material being covered.

Activity 5: Fahrenheit and Celsius—how are they related? (GLEs: 9, 10, 12, 13, 15, 23, 25, 37)

Materials List: paper, pencil, Fahrenheit and Celsius—How Are They Related? BLM

In this activity, students will work in groups on the Fahrenheit and Celsius—How Are They Related? BLM. In order to accomplish this task, students will have to extend what they have learned in determining equations of lines and apply that knowledge to solving a real-life problem. The relationship between temperatures measured in degrees Fahrenheit and Celsius is linear and can be obtained by knowing that water freezes at 32˚F and 0˚C while water boils at 212˚F and 100˚C. Discuss these important benchmarks with students, and have students work in groups of three to perform tasks associated with the BLM. In this activity, students must also interpret the meaning of slope and the *x*- and *y*-intercepts in a real-life setting. After students have had an opportunity to work on this task, discuss the results as a class.

Activity 6: Writing an Equation from a Table of Values (GLEs: 9, 10, 12, 13, 15, 23, 37)

Materials List: paper, pencil, graph paper, From Tables to Equations BLM

In this activity, students apply the skill of determining equations of lines for a real-life situation and interpreting its meaning.

As a precursor to this activity, draw a picture of a triangle on the board, and begin by asking if students know what a polygon is and if they know what type of polygon this is. Let students *brainstorm* (view literacy strategy descriptions) about their thoughts on the term polygon with their fellow classmates, and have students share what they think a polygon is. Students should be led to understand that a polygon is a closed geometric figure with straight sides joined at the endpoints of those segments. Ask students if they remember the sum of the interior angles of a triangle, and review with students the fact that the sum of the three angles of any triangle is 180˚. This should have been learned in previous math classes.

Provide copies of the From Tables to Equations BLM. The table on the BLM shows the relationship between the number of sides of an n-sided polygon and the sum, S, of its interior angles. Have students use the data table provided to answer the questions presented on the BLM. Let students work in pairs to perform the indicated tasks. After students have had an opportunity to perform the tasks, discuss the results as a class. Again, students are required to find the equation and to interpret the real-life meanings of the slope and *x*- and *y*-intercepts.

Activity 7: Wages vs. Hours Worked (GLEs: 9, 10, 13, 15, 25, 37)

Materials List: paper, pencil, graph paper, Wages vs. Hours Worked BLM

Provide students with copies of the Wages vs. Hours Worked BLM. Allow students to work in groups on this BLM. This activity is another opportunity to have students analyze a real-life situation, make a table of values that match the situation, and use the graph the data generated in the table. Students then create an equation to match the data and interpret information from the graph. Afterwards, students use the equation to help them answer other questions based on the situation. After students have had the opportunity to answer the questions from the BLM, fully discuss the problems that were presented as a class.

Activity 8: The Stock Is Falling! (GLEs: 9, 10, 13, 15, 23, 25, 37)

Materials List: paper, pencil, graph paper, graphing calculator, The Stock is Falling! BLM

Provide students copies of The Stock is Falling! BLM. Allow students to work in groups on the BLM. In this particular activity, students make the mathematical connection between a verbal situation, a table relating the situation and its graph, and the algebraic representation of the situation. Fully discuss the questions presented as a class. Follow up the activity by showing students how to input the equation they found into a graphing calculator and how to use it to determine the x- and y-intercepts and the value of the stock at 10 weeks to integrate technology into this activity.

Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will write a *learning log* (view literacy strategy descriptions) entry that explains how to determine slope, x-intercept, and y-intercept, given an equation.
- The student will write a letter to an absent classmate explaining the connection between slope and rate of change.
- The student will create portfolios containing samples of their activities.

Activity-Specific Assessments

- Activity 1: Provide the student with a graph of a line. The student will write the equation in slope-intercept form and then translate the equation into standard form.

- Activities 2 and 3: The student will write the equations and sketch the graphs when given a point and the slope of the line or when given two points on a line.

- Activity 8: The student will create a problem for a real-world context in which a decreasing linear situation exists. The student will make a graph and write an equation to represent the situation.

Algebra I–Part 1
Unit 6: Inequalities and Absolute Values in One Variable

Time Frame: Approximately two weeks

Unit Description

In this unit, an examination is made of the nature of solving linear inequalities in one variable and graphing their solutions on a number line. The unit also includes an introduction to absolute value equations and inequalities.

Student Understandings

Students recognize and distinguish between strict inequality (< and >) statements and relaxed inequality/equality (≤ and ≥) statements. Students solve linear inequalities in one variable and graph their solutions on the number line. Students graph simple absolute value inequality relationships on the number line.

Guiding Questions

1. Can students perform the symbolic manipulations needed to solve linear inequalities in a single variable and graph their solutions on the number line?
2. Can students interpret and graph simple absolute value equalities on the number line?
3. Can students relate absolute value inequalities in one variable to real-world settings (i.e., measurement, absolute value distances) and graph their solutions on the number line?

Unit 6 Grade-Level Expectation (GLE)

GLE #	GLE Text and Benchmarks
Algebra	
9.	Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
14.	Graph and interpret linear inequalities in one or two variables and systems of linear inequalities (A-2-H) (A-4-H)

Sample Activities

Activity 1: Solving Inequalities with a Single Variable (GLEs: 11, 14)

Materials List: paper, pencil, math textbook

Students should have been introduced to solving inequalities and graphing the solutions in 7th grade. Use this activity to review the basic steps in solving an inequality and expressing the resulting graphs for the solution sets. Relate the solving of inequalities to its counterpart—solving equations in one variable. To visually represent the concept of an inequality (in one variable), on the board or overhead projector, draw an out-of-balance scale. Make the left side the lower side and write $x-4$ on this side. On the other side, write -3. Tell the students that the picture represents the inequality *some number minus four is greater than negative three* and write $x-4>{}^{-}3$ above the scales. Relate how the solution to the inequality becomes $x>1$ by adding four to both sides of the inequality (just as would be done if it were an equation being solved). Discuss how to graph such an inequality on a number line. Provide additional examples for students to model with the classroom including problems which involve flipping the inequality symbol when multiplying or dividing both sides by a negative. Use each of the inequality symbols (<, >, ≥, and ≤) in the problems, and explain how each affects the way in which the graphical solution is expressed using parentheses, brackets, dots, and open holes, as appropriate. When students understand how to solve and graph inequalities with a single variable, provide additional practice using the math textbook as a resource. Pick one of the inequality problems that is assigned and using the *P* from *SPAWN writing* (view literacy strategy descriptions) as a writing prompt, have students write and explain verbally how they solved the problem. Use the *SPAWN writing* as a way to check student understanding of the concept.

Activity 2: Real-life Inequalities (GLEs: 9, 11, 14)

Materials List: paper, pencil, math textbook, Real-life Inequalities BLM

Present the following situation to students:

> Problem: The math club raised \$6800 to go on a trip to the math competition in Chicago. A travel agent charges a \$300 fee to organize the trip and an additional charge of \$600 per person for each student attending the competition.
>
> Let students work in groups of three to find the following information:
>
> - Write an inequality showing the relationship between the money that was raised and the cost for sending *x* people on the trip, assuming the amount raised has to be more than the amount needed to go on the trip.
>   *Solution: \$6800 > \$300 + \$600x*
> - Solve the inequality to determine the maximum number of people that can go on the trip.

Solution: $x < 10\ 5/6$; the maximum number of people that can go on the trip is 10—more money would be needed to send 11 people.

Allow students the opportunity to work in their groups on the problem, and then discuss the solution as a class. After doing this problem and discussing it thoroughly, provide students a copy of the Real-life Inequalities BLM. Allow students to work on these problems in their groups, and then go over the BLM as a class. Once students are comfortable with real-life inequality problems, provide additional work on this topic using the math textbook as a resource.

Activity 3: Absolute Value as Distance (GLE: 9)

Materials List: paper, pencil

To begin this activity, have students *brainstorm* (view literacy strategy descriptions) why they think the absolute value of a number is always positive. Allow students to debate this topic with one another in small groups then as a class. Explain that the absolute value of a number is really the distance the number is from zero on the number line. For example, write $|x| = 3$ on the board. Explain to students how to interpret the equation. In this particular example, we want the number or numbers, x, which are a distance of 3 units from zero on the number line. Have students find numbers on the number line that meet these criteria. Connect this solution mathematically with the following statements: $|-3| = 3$ and $|3| = 3$. When students see the absolute value symbol, they should immediately think of *distance*. Since distance is associated with a positive value, the absolute value of a number is positive. Provide additional examples in which students practice this concept. This concept of absolute value as distance is laying the groundwork to actually have students solve more complex absolute value equations and inequalities in the next few activities.

Activity 4: Solving Absolute Value Equations (GLEs: 11)

Materials List: paper, pencil, math textbook

The previous activity should have laid the foundation for what is next—solving an absolute value equation. Using what they learned about absolute value as a distance, relate this to solving the following absolute value equation: $|x - 3| = 7$. Conceptually, lead students to understand that there are actually two situations that need to be considered. One is when $x - 3 = 7$ (when the expression inside the absolute value is positive) and the other is when $x - 3 = -7$ (when the expression inside the absolute value is negative). Since both situations will result in solutions that provide a distance of 7, students should see that $|x - 3| = 7$ has two solutions for x. In this particular problem, the two values of x that make the equation true are 10 and -4. These solutions are derived from solving each individual equation (i.e., $x - 3 = 7$ and $x - 3 = -7$). Provide students with additional examples and practice from their math textbook, have students explain in words what each equation means geometrically in terms of distance on a number line, and solve the equations for the unknown variable.

Activity 5: Absolute Value Inequalities I (GLE: 9, 11, 14)

Materials List: paper, pencil, math textbook

Lead the students through the following activity using whole-group instruction.

Begin by asking students to identify points on a number line that are located less than a specified distance from zero. For example, put up the absolute value inequality $|x| < 2$. Write the following statement on the board: "The solution to this problem is all the numbers on the number line less than 2" and use it to form a student *opinionnaire* (view literacy strategy descriptions). An *opinionnaire* is designed to promote deep and meaningful understandings of the content by activating and building on prior knowledge while building interest in the topic being presented or learned about. In this particular case, have students individually decide if the statement is true or false and why they agree or disagree with the statement. Take a poll of the students to see who agrees and who disagrees.

Form one group of students who agree and another group of those who disagree. Ask students to discuss in their groups what they think this particular problem means in terms of distance, and then come up with a graphical solution to the problem through debating their thinking on the problem. When the groups are ready to discuss the problem further, use the classroom discussion to drive the lesson. For example, in this particular problem students are asked to locate all points on the number line that are less than two units from zero. That is what $|x| < 2$ means. Students should write the inequality $-2 < x < 2$ or ($x > -2$ and $x < 2$) as the solution to the problem, and then graph the inequality that results. Writing an inequality this way may be new for students since it is a compound inequality. Provide additional problems of this type for students to solve. Include absolute value inequalities with *greater than*, *less than or equal*, and *greater than or equal.* When working with a distance *greater than* some amount, students should understand that "visually" this becomes a compound inequality. For example, to show *all of the points which are more than 2 units from 0 on the number line* this would be expressed algebraically as $|x| > 2$. Relating this graphically on a number line, students should see that to express this information, two inequality statements would have to be written: $x > 2$ or $x < -2$. This is critical to students' understanding and solving absolute value inequalities of this type.

Use the math textbook as a resource for additional work on this topic.

Activity 6: Absolute Value Inequalities II (GLEs: 9, 11, 14)

Materials List: paper, pencil, math textbook

Extend student understanding of absolute value inequalities by considering expressions such as $|3x+2|<7$. Lead students to understand how this problem, like the absolute value equations solved earlier, has two cases to consider—when $3x + 2$ is less than positive 7 but greater than -7. Show this graphically on a number line. Students should recognize that it can be written in the same fashion as the earlier inequality (i.e., $-7<3x+2<7$). Ask students to use previously learned rules to solve the complex inequality. Provide additional examples as are necessary for students to fully understand the skill of solving absolute value inequalities. Use the math textbook for additional work on this topic.

Activity 7: Real-life Absolute Values (GLEs: 9, 11)

Materials List: paper, pencil, math textbook, Real-life Absolute Values BLM

Provide students with copies of Real-life Absolute Values BLM. Allow students to work in small groups on the BLM. Discuss the results as a class. If possible, provide additional work on this topic using the math textbook as a resource.

Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will use sample work from the activities to place in a portfolio that would showcase knowledge of inequalities.
- The student will describe the difference between an equation and an inequality in words and give an example of each using real-world examples.
- The student will write a letter to a classmate explaining what an inequality is and how to solve inequalities and graph them.
- The student will create absolute value inequality statements and share with the class on a math bulletin board.

Activity-Specific Assessments

- Activity 1: The student will solve and graph inequalities in one variable.

- Activity 3: The student will explain in words why there is no solution to the absolute value equation $|x| = -3$.

  Answer: Since absolute value represents a distance, there are no numbers, x, that are a distance of -3 units from zero, since distance is always positive. Hence, there is no solution to this problem.

- Activity 5: The student will explain (using distance on a number line) what the inequality $|x| \geq 4$ means and find its solution.

  Answer: This inequality represents the values that are a distance greater than or equal to 4 from zero on the number line. The solution is $x \leq -4$ *or* $x \geq 4$.

Algebra I–Part 1
Unit 7: Systems of Equations and Inequalities

Time Frame: Approximately three weeks

Unit Description

This unit examines the nature and mathematical procedures used to find and interpret solutions for real-life and abstract problems involving system of equations and inequalities.

Student Understandings

Students graph and interpret the solution of a system of two linear equations or inequalities. With regard to systems of equations, students relate the existence or non-existence of solutions to the slope of the two lines. They develop algorithmic ways of determining the solutions to a system of linear equations or inequalities.

Guiding Questions

1. Can students explain the meaning of a solution to a system of two linear equations or inequalities?
2. Can students determine the solution to a system of two linear equations by graphing, substitution, or elimination?
3. Can students relate the solution, or lack of solution, for a system of equations to the slopes of the lines in the system?
4. Can students identify coincident lines by their slopes and *y*-intercepts and relate this to the possibility of an infinite number of solutions?
5. Can students graph and interpret linear inequalities in two variables?
6. Can students solve a system of inequalities and represent the solution graphically?

Unit 7 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
4.	Distinguish between an exact and an approximate answer, and recognize errors introduced by the use of approximate numbers with technology (N-3-H) (N-4-H) (N-7-H)

Algebra	
9.	Model real-life situations using linear expressions, equations, and inequalities (A-1-H) (D-2-H) (P-5-H)
11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
14.	Graph and interpret linear inequalities in one or two variables and systems of linear inequalities (A-2-H) (A-4-H)
16.	Interpret and solve systems of linear equations using graphing, substitution, elimination, with and without technology, and matrices using technology (A-4-H)
Geometry	
23.	Use coordinate methods to solve and interpret problems (e.g., slope as rate of change, intercept as initial value, intersection as common solution, midpoint as equidistant) (G-2-H) (G-3-H)
25.	Explain slope as a representation of "rate of change" (G-3-H) (A-1-H)
Patterns, Relations, and Functions	
37.	Analyze real-life relationships that can be modeled by linear functions (P-1-H) (P-5-H)
38.	Identify and describe the characteristics of families of linear functions, with and without technology (P-3-H)
39.	Compare and contrast linear functions algebraically in terms of their rates of change and intercepts (P-4-H)

Sample Activities

Activity 1: Is There a Point of Intersection? (GLEs: 11, <u>16</u>, 38, 39)

Materials List: paper, pencil, graphing calculators, Is There a Point of Intersection? BLM

In this activity, students use graphing calculators to explore whether two lines will intersect. Provide students with copies of Is There a Point of Intersection? BLM and graphing calculators to work in small groups on the BLM investigation. For this activity, students are only to determine whether or not there is a point of intersection, not to actually determine the point of intersection which will be done in the next activity. The purpose is to get students to analyze equations in slope-intercept form and determine which linear equations will intersect based upon their slopes and y-intercepts. Through the investigation, students should realize that there are three possibilities when two linear equations are graphed on the same coordinate grid:

1) Exactly 1 point of intersection if the slopes are different
2) No point of intersection if the slopes are the same but different y-intercept
3) Infinitely many points of intersection if the slope and y-intercept are identical

Activity 2: Finding a Point of Intersection by Graphing Method (GLEs: 4, 16, 23, 39)

Materials List: graph paper, pencil (two colors), graphing calculators, math textbook, ruler

In this activity students determine the actual point of intersection for the graphs of two lines using the graphing method. Before graphing, ask students to decide whether or not there should be a point of intersection based upon what they learned in Activity 1 by rewriting the equations in slope-intercept form and comparing slopes. For example, present the following two equations: $x + y = 6$ and $3x – 4y = 4$. By writing the two equations in slope-intercept form, students should see that the two equations have different slopes so there should be exactly one point of intersection.

Next, have students graph the two lines on the same coordinate graph using two different colored pencils and graph paper. Have the students determine "visually" where the point of intersection appears to be on the graph. Ask students if they are absolutely certain that this is the point where the two lines intersect, or if this is an approximate value. Point out the fact that using graphing to determine where two lines intersect has limitations as the actual point of intersection for the two lines is approximated.

Discuss with students that a point of intersection for two linear equations is a common solution to both equations (the point makes both equations true). To verify that a point is a solution to both equations, the x and y values for the coordinates can be substituted into both equations and should make both equations true.

Provide additional examples for students including examples in which there are no solutions and infinitely many solutions. After the examples are done by hand, demonstrate for students how to use the graphing calculators to determine the point of intersection by using the "trace," "zoom," and "intersect" features.

Provide additional work on this topic using the math textbook as a resource.

Activity 3: Using Substitution to Solve a System of Equations (GLE: 16)

Materials List: paper, pencil, math textbook

In this activity, extend the idea of finding a point of intersection to the mathematical language of *solving a system of equations*. Discuss this new terminology with students. Using the previous example in Activity 2, (e.g., $x + y = 6$ and $3x – 4y = 4$), remind students that the point of intersection (4, 2) was found graphically. The drawback to using a graphical approach is that the exact point of intersection is not always certain, especially if the coordinates are not whole numbers. If this is the point of intersection, the x- and y-coordinate in each of the two equations can be replaced with $x = 4$ and $y = 2$, and they should make both equations true, which in this case they do. Therefore, the point (4,

2) is referred to as the "solution to the system". No other values for *x* and *y* will make both equations true at the same time because this is the only point that the two equations have in common.

At the point of intersection for the lines, the *x* in one equation is equal to the *x* in the other equation; likewise for the *y* coordinates. This idea is the basis for the technique of solving systems by "substitution." In the example problem, the equations in slope-intercept form are $y = -x + 6$ and $y = \frac{3}{4}x - 1$. Since the *y* values are equal at the point of intersection, by substitution $-x + 6 = \frac{3}{4}x - 1$. This can then be solved for the variable *x*, giving the value of $x = 4$, which is the *x* value for the point of intersection. The *y* value for the point of intersection, $y = 2$, can be found by substituting the *x* value found into either of the original equations.

Provide additional examples for students to become proficient at the substitution method of solving systems of equations. Use the math textbook for additional work on this topic.

Activity 4: Solving Systems by Elimination (GLE: 16)

Materials List: paper, pencil, graphing calculators, math textbook

Demonstrate how to solve systems by using the elimination method. Point out that the elimination method is used when the two equations are put in standard form. In the elimination method, the goal is to eliminate one of the variables and solve for the one that remains. Include examples where there is no solution (no point of intersection) and examples where there are infinitely many points of intersection. In these two special cases when elimination is used, both the *x* and *y* terms are eliminated concurrently. When this occurs, one of two things will happen. In one case, there will be an equation that makes sense, such as, 0 = 0, which would indicate the two equations are exactly the same and thus have infinitely many points of intersection. In the second case, an equation will remain that does not make sense, such as 0 = 8. This would indicate the two equations have the same slope and different *y* intercepts and are parallel to one another thus having no point of intersection. This can be related to what was done in Activity 1 by rewriting the equations in slope-intercept form and comparing the slopes and *y*-intercepts for the two equations.

Provide additional systems for students to solve using the elimination method. Have students check their solutions by graphing the equations using graphing calculators. Provide additional practice using the math textbook as a resource in which students must determine which method, elimination or substitution, best fits the problem.

To complete work with this activity, have students participate in a version of *professor know-it-all* (view literacy strategy descriptions). In this particular use of the strategy, form groups of 3 students to come up to the board and act as the professor. Call on groups randomly to come to the front of the class to answer questions from other students. Students at their desks should ask questions related to the topics they have learned with solving systems of equations. The *professors* take turn answering the questions and aren't allowed to sit until the class feels their question has been answered satisfactorily.

Activity 5: Break-even Point! (GLEs: 9, 11, 16, 23, 25, 37)

Materials List: paper, pencil, Starting a Business BLM**,** graph paper, ruler

Provide students with copies of Starting a Business BLM and allow students to work on the problem in small groups. Students may have problems when working through some of the questions, so assist groups if they need additional help. When completed, discuss the work as a class.

Activity 6: Pizza Parlor (GLEs: 9, 16, 23, 25, 37)

Materials List: paper, pencil, Pizza Parlor BLM**,** graph paper, ruler, colored pencils

In this activity, another opportunity is provided for students to explore the real-world meaning of a point of intersection for two lines for a given situation. Have students work on Pizza Parlor BLM with their group members. As groups complete the BLM, have students exchange their work with another group. Utilizing the literacy strategy of *questioning the author* (view literacy strategy descriptions) or *QtA,* have students review the work that was done by that group. As they read through the work, they should ask the following questions:

- Is the work on writing cost equations correct? If not, what recommendations can you make to correct what is wrong?
- Is the explanation clear and concise regarding the price that Mr. Moreau should charge? If is unclear, write a note to explain what is not clear.
- When creating the graph, does the group use proper graphing techniques, labels, and appropriate scales? If not, what is incorrect?
- Does the group understand what the point of intersection means in real-life terms and is the explanation clear? If not, what is incorrect?

Questioning the author is a great way to let students evaluate one another's work. After this process has occurred, groups should get their original BLMs and make any modifications needed to their answers or explanations. Discuss the results as a class.

During this activity, students should discover that the point of intersection for this particular graph tells us where the cost and revenue are the same. At that point, the cost to make the pizzas is equal to the revenue that was made selling the pizzas. Before this point, the revenue is less than the cost. After this point, the cost is less than the revenue and a profit is made. Students need to understand the value of making a graph. In addition to being a visual representation of a problem, the graph provides useful information. This activity also uses many of the concepts learned thus far with graphing and writing equations.

Activity 7: Which is the Better Offer? (GLEs: 9, 16, 23, 25, 37)

Materials List: pencil, graphing calculator, Which is the Better Offer? BLM

Provide copies of Which is the Better Offer? BLM to students and have them work in pairs on the activity. The activity deals with renting a van and involves a real-life application of systems of equations. Discuss the BLM when students have completed the activity.

Next, have students get into groups of four. Have each group of students create a real-life problem whose solution would require solving a system of equations. Students should come up with a situation and several questions to accompany their problem. Each group should then create an answer key for its problem. Once each group has created a problem and the accompanying solution key, have each group exchange problems with one another. Each group then should work the other group's problem that was created and provide feedback on the problem and questions that were created.

This activity is a modified form of the literacy strategy known as a math *story chain* (view literacy strategy descriptions). In a *story chain*, students write story problems on concepts being learned and then solve the problem. Typically, what is done is in a group of four, one student writes the first sentence of the story problem; the second student writes the second sentence of the story problem; the third student writes a question based on the first two sentences; and finally, the fourth student solves the problem and the other three check the work. In this particular use of the strategy, each group creates the problem and the questions to accompany the problem. They work together to come up with the answers and then exchange with another group.

Activity 8: Linear Inequalities (GLE: 14)

Materials List: graph paper, pencil, math textbook, graphing calculators (optional), ruler

In this activity, students compare graphing a linear equation with a linear inequality. In a linear equation, points that form the actual line are solutions to the linear equation. In a linear inequality, an area associated with all the coordinate points that make the inequality true is shaded and the line itself is a boundary for the region shaded.

Begin the activity by having students graph the equation $3x + 4y = 12$. Next, discuss the difference and the similarity between this linear equation and the following linear inequalities:

- $3x + 4y > 12$
- $3x + 4y \geq 12$
- $3x + 4y < 12$
- $3x + 4y \leq 12$

Discuss how each graph differs—whether the line is dashed (if > or <) or solid (if ≥ *or* ≤), and where the shading is located (to the right or to the left of the line). To aid in helping students understand which side of the line gets shaded, have students pick points and replace their coordinates in the inequality to see if they make the inequality true. If the particular points make the inequality true, then that is the side of the line that gets shaded.

Provide additional examples using the math textbook for a resource to help students become proficient at this skill. Include vertical and horizontal lines and inequalities such as $x > 2$ *or* $y < 5$. You may also want to show students how a graphing calculator can be used to graph inequalities at this time.

Activity 9: System of Inequalities (GLE: 14)

Materials List: graph paper, two different colored markers, math textbook**,** ruler

Discuss with students how to solve a system of inequalities by graphing both inequalities on a single coordinate grid and looking at the intersection of areas as the solution to the system. For example, have students graph the two inequalities shown below:

$$y > 2x - 5$$
$$3x + 4y < 12$$

Using two different color markers, have students shade in both inequalities and discuss the intersection of the two areas as being the solution to the system of inequalities. Point out that in order to be a solution, the coordinates must satisfy both inequalities. Provide students with additional practice on this skill using the math textbook as a resource.

Sample Assessments

General Guidelines

Performance and other types of assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The students will take paper and pencil tests on the skills associated with this unit.
- The student will write a short paragraph explaining his/her algorithm for determining the number of solutions, given the equations for two lines.
- The student will explain, in writing and using a graph, what it means for a certain point to be a solution to a set of equations.

Activity-Specific Assessments

- Activity 1: Provide students with pairs of equations. Without the aid of graphing calculators, students will determine whether there will be one, none, or infinitely many points of intersection based upon the slope and intercept of each equation.

- Activity 2: The student will graph two linear equations using paper and pencil and use the graph to approximate the point of intersection.

- Activity 3: The student will determine the exact point of intersection for two linear equations using the elimination method.

- Activity 4: The student will create three problems and solve them using elimination which show one solution, no solution, and an infinite number of points of intersection.

- Activity 5: Provide a real-world problem that requires the student to come up with cost and revenue equations, and use the equations to determine the break-even point for a particular situation.

Algebra I–Part 1
Unit 8: Solving Using Matrices

Time Frame: Approximately two weeks

Unit Description

In this unit, the solving of systems of equations is extended to include the use of matrices. The unit also presents matrices and shows how they can be utilized in real-life applications.

Student Understandings

Students develop the concept of a matrix and matrix operations of addition and multiplication and the relationship between solving matrix equations $ax = b$ and $Ax = B$ as $x = a^{-1}b$ and $x = A^{-1}B$ respectively. They apply matrices to the solution and interpretation of a system of two or three linear equations.

Guiding Questions

1. Can students explain what a matrix is and how it is applied in real-life situations?
2. Can students perform operations with matrices (addition, subtraction, multiplication)?
3. Can students determine the solution to a system of two or three linear equations with two or three variables by matrix methods?
4. Can students use matrices and matrix methods by hand and calculator to solve systems of equations $Ax = B$ as $x = A^{-1}B$?

Unit 8 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
5.	Demonstrate computational fluency with all rational numbers (e.g., estimation, mental math, technology, paper/pencil) (N-5-H)
Algebra	
11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
16.	Interpret and solve systems of linear equations using graphing, substitution, elimination, with and without technology, and matrices using technology (A-4-H)

Sample Activities

Activity 1: Matrices—an Introduction (GLEs: 5, 16)

Materials List: paper, pencil, graphing calculator, Vocabulary Self-Awareness for Matrices BLM, Movie Cinema Matrix BLM

Begin the activity by administering the Vocabulary Self-Awareness for Matrices BLM whereby students analyze what they know about matrices. The use of *vocabulary self-awareness* (view literacy strategy descriptions) as a literacy strategy has as its goal to help bring all students to a comfortable level with the vocabulary being used. In this particular use of the strategy, terms and processes involved with matrices are introduced at the beginning of a unit, in this case using the BLM. This is a variation of the typical use of the *vocabulary self-awareness* strategy whereby only vocabulary is assessed. In this modification of the strategy, both vocabulary and processes are being assessed to determine students comfort level with the material that will be taught in the activity.

Have students complete the Vocabulary Self-Awareness for Matrices BLM, and use the results to target the vocabulary which students feel least comfortable with to help guide instruction throughout this entire unit. After students have completed the BLM, continue on with the activities presented. At the end of the unit, have students go back through the Vocabulary Self-Awareness for Matrices BLM and have them assess their comfort level with the topics again. The goal is to have all students replace any O or – marks with + marks to indicate their understanding of what was taught on matrices.

Discuss that a matrix is a rectangular array of numbers. Provide students with copies of the Movie Cinema Matrix BLM which displays the items sold at different times at a movie cinema on a Monday. Discuss with students how the chart can be written as a rectangular array and enclosed with brackets or parentheses. The enclosed array is called a *matrix*. The advantage of writing the numbers as a matrix is that the entire array can be treated as a single mathematical entity. A matrix can be named with a single capital letter as shown.

The numbers that make up a matrix are called the entries, or elements, of the matrix. The entries of matrix M are all numbers, but the matrix itself is not a number, just as a multiplication table is not a number. Various operations can be used on matrices (addition, subtraction, multiplication) and will be discussed in the other activities.

A matrix is often classified by its order or dimension; that is, by the number of rows and columns that it contains. For example, matrix M has 4 rows (rows run across horizontally) and 3 columns (columns run up and down vertically). This means that matrix M is a *4* x *3* matrix. When a matrix has the same number of rows as columns, it is called a *square matrix*. Relate the rows and columns of the chart with the matrix. Introduce students to the matrix function on the calculator, and show them how to enter the data in this BLM into a matrix.

Activity 2: Adding Matrices (GLEs: 5, 16)

Materials List: paper, pencil, graphing calculator

Provide students with several pairs of matrices that can be combined by addition. An example is provided below. Using matrix, M, from the previous activity which shows the sales on a Monday at a movie cinema, and matrix T, which displays the same items sold on a Tuesday, have students find M + T and describe what this new matrix describes.

$$M = \begin{pmatrix} 4 & 20 & 25 \\ 8 & 24 & 18 \\ 10 & 34 & 28 \\ 34 & 38 & 55 \end{pmatrix} \qquad T = \begin{pmatrix} 6 & 15 & 24 \\ 5 & 8 & 22 \\ 8 & 25 & 15 \\ 13 & 22 & 16 \end{pmatrix}$$

$$M + T = \begin{pmatrix} 10 & 35 & 49 \\ 13 & 32 & 40 \\ 18 & 59 & 43 \\ 47 & 60 & 71 \end{pmatrix}$$

Students should see that the new matrix, M + T, really describes the total sales of items at the concession stand at the movie cinema for Monday and Tuesday. To add two matrices such as this, the only things that can be added are "like terms." In this case, recall that snacks are in the first column, and the movie times are given in each row. Because the two matrices contain the same types of information at the intersection of each row and column, the corresponding values in matrix M and matrix T are alike and can be added. Adding the numbers from row1, column 1 in each matrix gives 6 +4 =10, which means that the total snacks sold at the 1:00 show on Monday and Tuesday is $10. The complete new matrix that results from the addition of M + T is shown. Discuss with students the results of the new matrix and how to interpret its meaning.

Have students find M-T and have them explain in real-world terms what this new matrix represents (the difference between Monday and Tuesday sales). Follow this by showing students how this can be done using the graphing calculator. Provide additional problems of this type for students to become proficient at this skill.

Activity 3: Multiplying a Matrix by a Scalar (GLEs: 5, 16)

Materials List: paper, pencil, graphing calculator

In this activity, the work with matrices is extended to include multiplying a matrix by a scalar. Continuing the work from the movie theater example, present the following situation to students:

> *How can we find $\frac{1}{2}$ the sales of the concession stand on Monday at the cinema using a matrix?*

Discuss with students how each sale would then be "halved." This would result in a new matrix to display the results. Have students get into small groups to *brainstorm* (view literacy strategy descriptions) how to find $\frac{1}{2}$ M and what the results would mean. Once students have had the opportunity to discuss how this could be accomplished, have a class discussion on the ideas students came up with. Compare their ideas with the process shown below.

Explain that this is what is referred to as "*multiplying a matrix by a scalar.*" In this case, all elements in the original matrix are multiplied by the scale factor of ½.

$$M = \begin{pmatrix} 4 & 20 & 25 \\ 8 & 24 & 18 \\ 10 & 34 & 28 \\ 34 & 38 & 55 \end{pmatrix} \qquad \frac{1}{2} M = \begin{pmatrix} 2 & 10 & 12.50 \\ 4 & 12 & 9 \\ 5 & 17 & 14 \\ 17 & 19 & 27.50 \end{pmatrix}$$

Show students how to multiply a matrix by a scalar using the graphing calculator, and then provide additional practice for students on this skill.

Activity 4: Multiplying Two Matrices (GLEs: 5, 16)

Materials List: paper, pencil, graphing calculator

Multiplication of two matrices is not defined so straightforwardly as addition, subtraction, and multiplication of matrices by a scalar. The process is much more complex.

Write on the board the two matrices, P and Q as shown below. Both are 2 by 2 matrices. Demonstrate how to do the matrix operation of multiplication to find the product. Explain that matrix multiplication is a *row times column* process where the entry of a row in the first matrix multiplies the corresponding entry in the column of the second matrix. The sums of the products are found until the entries of a row are all used. The process is repeated for the next row until all elements are accounted for.

For example, look at matrix P and Q shown below.

$$P = \begin{bmatrix} 10 & 15 \\ 15 & 20 \end{bmatrix} \quad Q = \begin{bmatrix} 8 & 6 \\ 10 & 5 \end{bmatrix} \quad PQ = \begin{bmatrix} 230 & 135 \\ 320 & 190 \end{bmatrix}$$

The process begins by multiplying each entry in the first row of P by the corresponding entry in the column of Q. The two products are then added together to form the first element in the new matrix called PQ. In this example, in row 1 of matrix P multiply the number 10 by the first entry in column 1 matrix Q, which is 8. This product is 80. Add to this the product of the second element in row 1 of matrix P, which is 15, by the second entry in column 1 of Q, which is 10, giving a product of 150. When the two individual products are added, the result is 80 + 150 for a sum of 230. Thus, 230 is the first entry in the new matrix PQ. The complete new matrix PQ is shown. Discuss with students how the entire matrix was formed.

Repeat this activity reversing the matrices to show students that matrix multiplication is not a commutative operation. Instead of P times Q, multiply Q times P, and prove to students that the resulting matrix, QP, is different than PQ. Matrix QP is shown below.

$$QP = \begin{bmatrix} 170 & 240 \\ 175 & 250 \end{bmatrix}$$

Provide additional opportunities for students to gain experience at this skill. Then show students how this can be done easily using graphing calculator technology.

Activity 5: The Inverse Matrix (GLEs: 5, 11, 16)

Materials List: paper, pencil, graphing calculators

In Activity 4, students were introduced to the concept of multiplying one matrix with another. Students discovered that the order in which matrices are multiplied results in different answers (unlike multiplication of real numbers). In this activity, the idea of an *inverse matrix* is introduced.

The inverse matrix of matrix A, symbolized by A^{-1}, is the matrix that will produce an identity matrix when multiplied by A. In other words: $AA^{-1} = I$ and $A^{-1}A = I$. The identity matrix, symbolized by I, is one of a set of matrices that do not alter or transform the elements of any matrix A under multiplication, such that AI = A and IA = A. Show students that the identity matrix for any 2 x 2 matrix is as follows:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Demonstrate, using paper and pencil method as well as the graphing calculator, that any 2 x 2 matrix, when multiplied by this identity matrix, results in the original matrix. Connect this idea with that of multiplying any real number by 1 will result in the original number.

Next, show students how solving a matrix equation is related to solving linear equations. For example, if $ax = b$, to solve for x we can multiply both sides of the equation by $\frac{1}{a}$, which is the multiplicative inverse of a. We can also write $\frac{1}{a}$ as a^{-1}. Show students the inverse key on a calculator, and relate finding the multiplicative inverse of 5 as $\frac{1}{5}$ or 0.2. Use several examples to demonstrate this to students. To solve the equation ax = b, we then would have: $a\ a^{-1}x = ba^{-1}$, and since $aa^{-1} = 1$, we get $x = ba^{-1}$.

Just as normal equations can be solved using this approach, so too can matrix equations. For example, suppose AX = B, where A, B, and X are all matrices. To find X when A and B are known, multiply both sides of the matrix equation by the inverse of A, or A^{-1}. Thus: $A\ A^{-1}X = A^{-1}B$ which when solved for X gives us $X = A^{-1}B$.

Provide the following example to students.

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

Show students how to use a graphing calculator to find A^{-1}.

$$A^{-1} = \begin{pmatrix} .8 & -0.6 \\ -0.2 & 0.4 \end{pmatrix}$$

Have students prove that $AA^{-1} = I$ (Identity Matrix) by multiplying each matrix by the other using paper and pencil as well as a calculator. Provide additional 2x2 matrices for students to determine their inverse using the graphing calculator. This skill will be utilized when solving systems using matrices in the next two activities.

Activity 6: Solution of Two Equations: Found Five Ways (GLEs: 5, 11, <u>16</u>)

Materials List: graph paper, pencil, graphing calculators, ruler, paper

Write the following pairs of equations on the board: $2x + 3y = 5$ and $2x + 4y = 9$. Have students solve the system of equations first by graphing using graph paper, then by graphing on a graphing calculator using the intersect function. Next, have students solve the problems using the substitution and elimination methods.

Finally, show students how the two equations can be solved using matrices. Discuss how to write the two equations into matrices as follows:

Let $A = \begin{pmatrix} 2 & 3 \\ 2 & 4 \end{pmatrix}$ $X = \begin{pmatrix} x \\ y \end{pmatrix}$ $B = \begin{pmatrix} 5 \\ 9 \end{pmatrix}$

Point out to students that matrix A comes from the coefficients associated with the *x* and *y* terms from the two equations. Matrix X contains the values we are trying to determine. Matrix B is created from the constants (5 and 9) from the two linear equations.

The idea here is to find *x* and *y*, which are the values for matrix X. Since AX = B, to solve for X take the inverse matrix of A or A^{-1} of both sides; therefore, $X = A^{-1}B$. Show students how this can be done using the calculator. Verify that the point of intersection is the same regardless of the method. The point of intersection for the two lines is (-3.5, 4). Provide additional examples for students to become proficient at using matrices to solve such equations. Repeat this activity using systems that have no solutions, one solution, or an infinite number of solutions.

Activity 7: Solving Three Equations with Three Unknowns Using Matrices (GLEs: 5, 11, <u>16</u>)

Materials List: paper, pencil, graphing calculator, math textbook, Solving Systems Using Matrices BLM

Provide students with copies of the Solving Systems Using Matrices BLM. Have students work in groups on the activity. When students have had an opportunity to work through the activity, discuss fully as a class. Since the goal of the activity is to have students extend their knowledge of matrices, this activity may challenge the students. Another option would be to do the BLM with the class as a teacher-led activity. Discuss with students how matrices can be used to solve systems with 2, 3, or even more unknowns utilizing the procedures they have learned. For additional practice on this skill, find problems which have three unknowns for students to solve using matrices. Use the math textbook as a resource.

At the end of this activity, remember to have students re-assess their comfort level using the Vocabulary Self-Awareness for Matrices BLM from Activity 1. Discuss what the students have learned during the course of this unit and what they still don't feel comfortable with. Then use the results from the *vocabulary self-awareness* (view literacy strategy descriptions) activity to guide any additional instruction.

Sample Assessments

General Guidelines

Performance and other type of assessments can be used to ascertain student achievement. Following are some examples:

General Assessments

- The student will research and write a one-page report on the history of matrices.
- The student will show proficiency in operations with matrices using paper and pencil tests.
- The student will use matrices to solve systems of equations with two and three unknowns.

Activity-Specific Assessments

- Activity 2: Provide the student with real-life data similar to the data shown in the activity, and then have the student create two matrices for the data. Once the matrices have been created, the student will find the sum of the two matrices and interpret its meaning.

- Activity 3: Provide the student with two 2 x 2 matrices to multiply by hand. Afterwards, the student will use technology to check to see if the answers are correct.

- Activity 5: Provide the student with pairs of matrices to determine if they are inverse matrices of one another using paper and pencil as well as using graphing calculator technology.

- Activity 6: The student will find the solution to a system of equations in two variables using five different methods.