



Comprehensive Curriculum

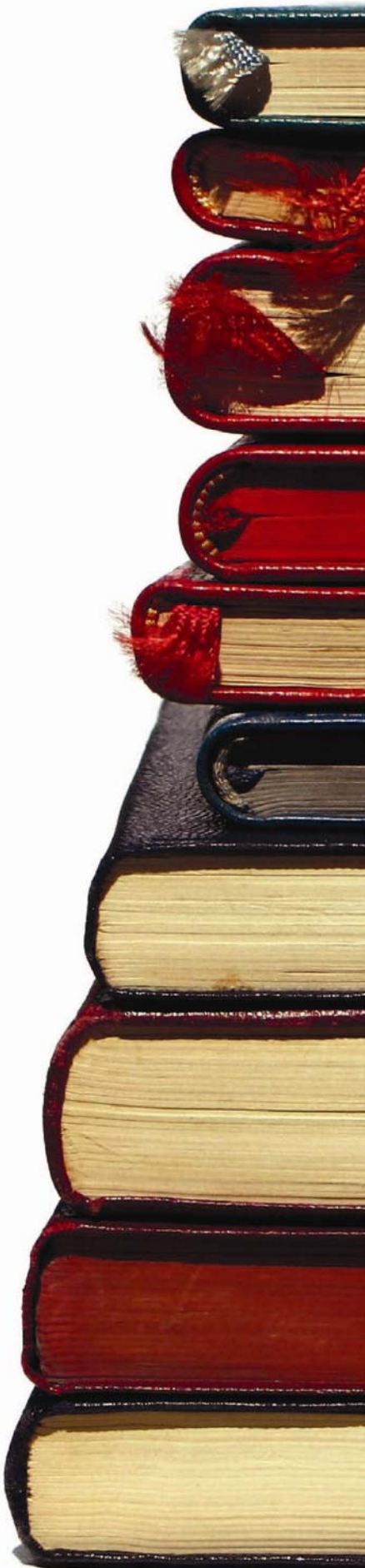
Revised 2008

Geometry



Louisiana Department of
EDUCATION

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Geometry

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Louisiana Comprehensive Curriculum, Revised 2008 **Course Introduction**

The Louisiana Department of Education issued the *Comprehensive Curriculum* in 2005. The curriculum has been revised based on teacher feedback, an external review by a team of content experts from outside the state, and input from course writers. As in the first edition, the *Louisiana Comprehensive Curriculum*, revised 2008 is aligned with state content standards, as defined by Grade-Level Expectations (GLEs), and organized into coherent, time-bound units with sample activities and classroom assessments to guide teaching and learning. The order of the units ensures that all GLEs to be tested are addressed prior to the administration of *iLEAP* assessments.

District Implementation Guidelines

Local districts are responsible for implementation and monitoring of the *Louisiana Comprehensive Curriculum* and have been delegated the responsibility to decide if

- units are to be taught in the order presented
- substitutions of equivalent activities are allowed
- GLEs can be adequately addressed using fewer activities than presented
- permitted changes are to be made at the district, school, or teacher level

Districts have been requested to inform teachers of decisions made.

Implementation of Activities in the Classroom

Incorporation of activities into lesson plans is critical to the successful implementation of the Louisiana Comprehensive Curriculum. Lesson plans should be designed to introduce students to one or more of the activities, to provide background information and follow-up, and to prepare students for success in mastering the Grade-Level Expectations associated with the activities. Lesson plans should address individual needs of students and should include processes for re-teaching concepts or skills for students who need additional instruction. Appropriate accommodations must be made for students with disabilities.

New Features

Content Area Literacy Strategies are an integral part of approximately one-third of the activities. Strategy names are italicized. The link ([view literacy strategy descriptions](#)) opens a document containing detailed descriptions and examples of the literacy strategies. This document can also be accessed directly at <http://www.louisianaschools.net/1de/uploads/11056.doc>.

A *Materials List* is provided for each activity and *Blackline Masters (BLMs)* are provided to assist in the delivery of activities or to assess student learning. A separate Blackline Master document is provided for each course.

The *Access Guide to the Comprehensive Curriculum* is an online database of suggested strategies, accommodations, assistive technology, and assessment options that may provide greater access to the curriculum activities. The *Access Guide* will be piloted during the 2008-2009 school year in Grades 4 and 8, with other grades to be added over time. Click on the *Access Guide* icon found on the first page of each unit or by going directly to the url <http://mconn.doe.state.la.us/accessguide/default.aspx>.



Geometry

Unit 1: Geometric Patterns and Reasoning

Time Frame: Approximately three weeks



Unit Description

This unit introduces the use of inductive reasoning to extend a pattern, and then find the rule for generating the n th term in a sequence. Additionally, counting techniques and mathematical modeling, including line of best fit, will be used to find solutions to real-life problems.

Student Understandings

Students apply inductive reasoning to identify terms of a sequence by generating a rule for the n th term. Students recognize linear versus non-linear sets of data and can justify their reasoning. Students can apply counting techniques to solve real-life problems.

Guiding Questions

1. Can students give examples of correct and incorrect usage of inductive reasoning?
2. Can students use counting techniques with patterns to determine the number of diagonals and the sums of angles in polygons?
3. Can students state the characteristics of a linear set of data?
4. Can students determine the formula for finding the n th term in a linear data set?
5. Can students solve a real-life sequence problem based on counting?

Unit 1 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Algebra	
5.	Write the equation of a line of best fit for a set of 2-variable real-life data presented in table or scatter plot form, with or without technology (A-2-H) (D-2-H)
Geometry	
17.	Compare and contrast inductive and deductive reasoning approaches to justify conjectures and solve problems (G-4-H) (G-6-H)
Data Analysis, Probability, and Discrete Math	

GLE #	GLE Text and Benchmarks
20.	Show or justify the correlation (match) between a linear or non-linear data set and a graph (D-2-H) (P-5-H)
22.	Interpret and summarize a set of experimental data presented in a table, bar graph, line graph, scatter plot, matrix, or circle graph (D-7-H)
24.	Use counting procedures and techniques to solve real-life problems (D-9-H)
25.	Use discrete math to model real-life situations (e.g., fair games, elections) (D-9-H)
Patterns, Relations, and Functions	
26.	Generalize and represent patterns symbolically, with and without technology (P-1-H)
27.	Translate among tabular, graphical, and symbolic representations of patterns in real-life situations, with and without technology (P-2-H) (P-3-H) (A-3-H)

Sample Activities

Activity 1: Inductive Reasoning (GLE: 17)

Materials List: pencil, paper

The purpose of this activity is to provide students with the definition of inductive reasoning and to have them recognize when inductive reasoning is used in real-life situations. Provide the definition of inductive reasoning and give an example of inductive reasoning that students may encounter on a day-to-day basis (e.g., the mailman came to my house every day at noon for five days in a row. I deduce that the mailman will come today at 12 P.M.). Discuss the fact that one counter-example is sufficient to disprove a conjecture made when using the inductive reasoning process (e.g., the mailman came today at 3 P.M.). Ask students to give other real-life examples. Provide students with a variety of scenarios in which students can make a conjecture using inductive reasoning. Have students identify situations in which inductive reasoning might be used inappropriately (e.g., matters of coincidence rather than a true pattern).

Activity 2: Using Inductive Reasoning in Number and Picture Patterns (GLE: 17)

Materials List: pencil, paper, Extending Number and Picture Patterns BLM

Before discussing patterns, have students complete a modified *SPAWN* writing ([view literacy strategy descriptions](#)) based on their knowledge of patterns from previous courses. *SPAWN* is an acronym that stands for five categories of writing options—*Special Powers*, *Problem Solving*, *Alternative Viewpoints*, *What If?* and *Next*. Using these categories, teachers can create prompts that promote critical thinking related to the topic. If teachers want students to anticipate what will be learned, they could use the *Problem*

Solving or *Next* prompts. If teachers want students to reflect critically on the topic just learned, they would use *Special Powers*, *Alternative Viewpoints*, or *What If?* prompts.

In this particular activity, using the *Next* category, give students the following prompt:

Given the pattern _____, -6, 12, _____, 48, ...answer the following exercises:

- a. Fill in the missing numbers.
- b. Determine the next two numbers in this sequence.
- c. Describe how you determined what numbers completed the sequence. Be sure to explain your reasoning.
- d. Are there any other numbers that would complete this sequence? Explain your reasoning.

Students will have to think critically to determine which numbers make the sequence work. Some will create a linear pattern while others will create a non-linear pattern. Having students complete this pattern requires them to anticipate what they will be learning in the lesson about patterns and sequences. It will help the teacher demonstrate the difference between linear and non-linear data. Students should include these writings in their math *learning logs* ([view literacy strategy descriptions](#)). A *learning log* is a notebook that students keep in order to record ideas, questions, reactions, and new understandings. Documenting ideas in a log about the content being studied forces students to “put into words” what they know or do not know. This process offers a point of reflection and can help the teacher determine whether there are misunderstandings or if students grasp the material. Students should keep their learning logs in a separate section of their binders or composition notebooks.

Solution: There are two patterns. First solution: a.) 3, -6, 12, -24, 48; b.) the next two numbers are -96 and 192; c and d.) See students’ explanations. The descriptions should include a discussion about using opposite operations to find the missing numbers. Second solution: a.) -24, -6, 12, 30, 48; b.) the next two numbers are 66 and 84; c and d.) Same as the first solution.

After completing the *SPAWN writing*, allow students to use inductive reasoning to find the next number or picture in a sequence. Additionally, students will indicate verbally or in writing the process for generating the next item. Use the Extending Number and Picture Patterns BLM to provide practice exercises in each of these strategies, starting with fairly simple problems and progressing to more challenging problems.

Take time at the end of the activity to review the students’ responses to the *SPAWN writing* to help students see the connection between their answers and the activity.

Activity 3: Recognizing Linear Relationships in Table Formats (GLEs: 5, 20, 22, 26, 27)

Materials List: pencil, paper, graphing calculator or access to Microsoft *Excel*[™], graph paper, Linear or Non-linear BLM, Using Rules to Generate a Sequence BLM

Teacher note: Information for activities 3 and 4 can be found in most Algebra I and/or Algebra 2 textbooks. While this skill should have been mastered in Algebra 1, the review is used to help students distinguish the difference between inductive and deductive reasoning (GLE 17).

Using the Linear or Non-linear BLMs, have students complete a modified *opinionnaire* ([view literacy strategy descriptions](#)) before discussing the definition of linear. *Opinionnaires* are used to promote critical understanding of content area concepts by activating and building on relevant prior knowledge. They are used to build interest and motivation to learn more about the topic. *Opinionnaires* are used to force students to take positions and to defend their positions. The emphasis is not on the correctness of their opinions but rather on the students' points of view.

For this activity, the *opinionnaire* has been modified to present students with different representations of patterns which are both linear and non-linear. The patterns on the Linear or Non-linear BLM are given as rules (equations), tables, and sequences. Have each student complete the modified *opinionnaire* by placing a check in the column indicating whether he/she believes the given sequence is linear or non-linear. This should happen before any discussion of the definition of linear begins. The goal is to have the students express their ideas about what it means for a pattern to be linear. This could lead to the students developing their own definitions that the teacher can build upon throughout the following lessons. Do not discuss whether students are correct or not at this point. The focus is on giving them a voice about the content, not whether their answers are correct. Have students retain the BLMs in their math *learning logs* ([view literacy strategy descriptions](#)). The students will need the completed BLMs for a discussion in a later activity.

After completing the modified *opinionnaire*, have students work in groups to generate terms in a sequence using a given rule or function.

The purpose of this activity is to develop the strategy of looking for common differences between values to determine if a relationship is linear. This strategy will be used in future activities to generate the rule for finding the n th term in relationships that are linear. Use the Using Rules to Generate a Sequence BLM to provide students with practice.

After performing several exercises, groups should determine that the common difference is the same as the coefficient for n .

Discuss the differences between the data sets to determine what makes a data set linear or not linear.

The skills listed in the following activity will review concepts that were to be mastered in Algebra I. Have students work in pairs and provide each pair of students with a graphing calculator or access to a Microsoft *Excel*TM on a computer.

- Using the available technology, have students
 - plot the terms and values as ordered pairs for each of the examples, using the term numbers as the x -coordinates and the values as the y -coordinates (term, value).
 - generate the equation of the line of best fit.
 - recognize the relationship between the terms and values to be linear or not linear.
 - explain the relationship between the rule or function in the original problem and the equation of the line.
 - recognize the relationship between the common difference, the coefficient of n , and the slopes of the lines.
 - determine the next two or three values using the common difference rather than the function or rule.
- Ask students to perform similar tasks using pencil and paper so that they may review manual methods of writing linear equations for a data set.

For additional practice, provide small groups of students with different data sets. Some data sets should be non-linear. During a reporting session, have groups explain how they determined whether or not their data set was linear. For linear data sets, students should give the equation of the line and indicate the steps used in determining the equation.

Activity 4: Use a Formula to Find the n^{th} Term in a Pattern (GLEs: 20, 26, 27)

Materials List: pencil, paper, graphing calculator or access to Microsoft TM, graph paper, Generating the n^{th} Term for Picture Patterns BLM

This activity ties activities two and three together. Present various number patterns that are linear in nature to the class, but do not give them a table or formula. Using the techniques from the previous activities, ask students to generate the formulas which describe the relationship of the linear data.

Examples are:

- 1, 3, 5, 7, 9, ... Find the 20th term. *Solution: Students should realize that writing out terms through the 20th will take a while. If they assign each term a number to represent n (1 for first term, 2 for second term, 3 for third term, etc.) they can then apply the technique of plotting points, generating the equation for the line of best fit, then finding the 20th term. The formula is $2n - 1$. The 20th term is 39.*
- 4, 8, 12, 16, 20 ... Find the 100th term. *Solution: Formula $4n$; 100th term 400*

- 4, 9, 14, 19, 24, ... Find the 67th term. *Solution: Formula $5n-1$; 67th term 334*
- Students should also be required to develop the formulas without the use of technology.

Students should also be required to generate the n^{th} term for picture patterns. Use the Generating the n^{th} Term for Picture Patterns BLM for examples.

Activity 5: Figurate Numbers (GLEs: 5, 20, 22, 26, 27)

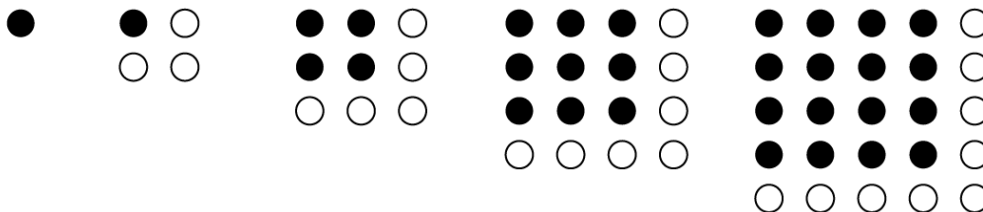
Materials List: pencil, paper, graphing calculator, Square Figurate Numbers BLM, Rectangular Figurate Numbers BLM, Triangular Figurate Numbers BLM

In this activity, students will generate the formulas for finding the n^{th} term in square, rectangular, or triangular number patterns. Each of these is a non-linear number pattern. Figurate numbers are numbers that can be represented by a regular geometrical arrangement of equally spaced points. They may be in the shape of any regular polygon, or other geometric arrangements. Each set of figurate numbers represents a distinct non-linear pattern. This activity concentrates on geometrical figures students are familiar with which aid in finding the algebraic rule for finding the n^{th} term.

Teacher note: More information can be found through a search on Yahoo! or Google.

Square Numbers

Use the Square Figurate Numbers BLM to present the following diagram.



First, have students translate the picture pattern into a number pattern by counting the number of dots in each figure. The number pattern is 1, 4, 9, 16, 25.... Ask the students if the pattern is a linear one. They should tell you that the data cannot be linear since the difference between values is not constant. Some students may recognize immediately that the numbers are perfect squares, but many will not unless the teacher provides leading questions for class discussion. If needed, ask students why the picture pattern is called a square number pattern. Lead students to recognize that the dots form squares, and that the number of dots in each square is the same as the area of the square. It may be necessary to ask them what is meant by the term *perfect square*. The students will understand that the numbers in the number sequence are the squares of the counting numbers ($1^2, 2^2, 3^2, 4^2, \dots$). The formula for generating the n^{th} term is n^2 . Have students recognize that it is important to know the characteristics of linear data sets (common difference between each two terms) in order to quickly identify those that are non-linear.

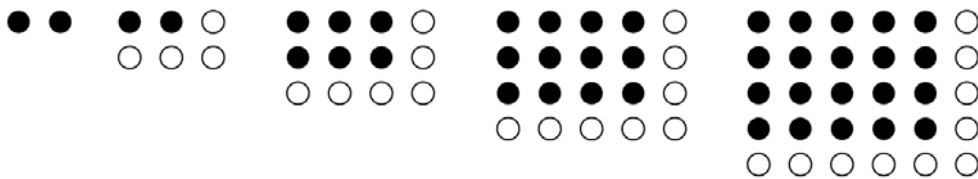
Have students enter the data from the picture pattern into their graphing calculators and create a scatter plot [i.e., (1,1), (2,4), (3, 9) (4, 16) (5, 25) (6, 36)]. Students may need to make a chart like the one below to determine what ordered pairs to use.

Figure	1	2	3	4	5
# of Dots	1	4			

Determine that students understand that the scatter plot is the graphical representation of the non-linear data set in the same way that the graph of a line is the graphical representation of a linear data set. Guide students through the process for generating the regression equation for the data set. Help students make the connection between the regression equation, $y = x^2$, and the rule for generating the nth term in the square number pattern, n^2 .

Rectangular Numbers

Use the Rectangular Figurative Numbers BLM to present the following diagram.

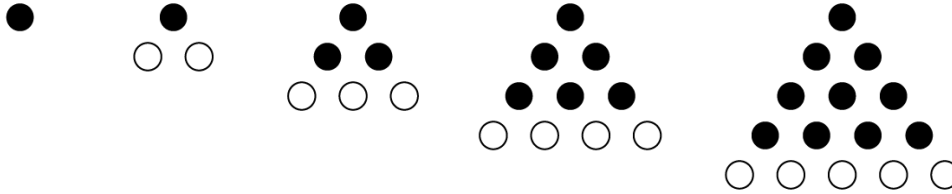


Have students:

- Write the number pattern that is created when counting the dots in each figure. *Solution: (2, 6, 12, 20, 30)*
- Determine if the number pattern is linear or non-linear by using the characteristics of linear data sets. Do not allow students to use the scatter plot feature on their calculators to determine this. Instead, have students indicate that the differences between each pair of numbers is not the same (i.e., the differences are 4, 6, 8, 10...); therefore, the data cannot be linear.
- Use their graphing calculators to determine the regression equation once they have determined that the pattern is non-linear. *Solution: $y = x^2 + x$*
- Indicate how the equation relates to the number pattern and how the equation can be used to determine the number of dots for any figure in the picture pattern. *Solution: If n represents a given figure, the number of dots for that figure is $n^2 + n$ or $n(n+1)$. Each rectangle has a width the same as the figure number and a length which is one greater than the width; therefore, the number of dots needed for any figure is the same as the area of the rectangle, $n(n+1)$, where n is the width and the length is one more than the width.*

Triangular Numbers

Use the Triangular Figurate Numbers BLM to present the following diagram.

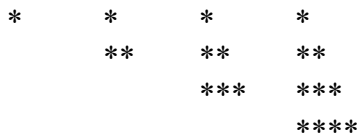


Have students make a chart similar to the one below in which the student enters the number of dots needed to make each figure. Ask them to follow the procedure outlined above to determine whether the relationship is linear or non-linear and then to generate the regression equation.

Figure	1	2	3	4	5
# of Dots (Triangular #)					

Teacher Note: The number pattern is 1, 3, 6, 10, 15.... Some students may recognize that they can add 2 to the first value to get the second value, add 3 to the second value to get the third value, etc. They will want to say that the rule is to add the next whole number to the previous one. They need to understand that this indicates that the pattern is not linear, since the difference between values is not the same. Lead them to understand that this pattern cannot be the formula or rule for generating the n^{th} term, since the pattern they see is based upon knowing a previous term.

The formula for generating the n^{th} term is $\frac{n(n+1)}{2}$. The calculator will show the regression equation as $0.5n^2 + 0.5n$. Notice that this is half of $n(n + 1)$ which was the rule for the rectangular numbers. Show students that the triangular number pattern could also be drawn as



Each of the patterns above is one-half of each of the rectangles below.



Therefore, if the area of the rectangle is $n(n+1)$, the area of the triangle would be half as much.

Provide students with a variety of number patterns, some linear and some non-linear, with which to practice their skills. Activity 6 gives an example of some geometric situations in which these skills must be applied.

At this point, have students refer to the Linear or Non-linear BLM completed in Activity 3. The teacher should have the students decide whether their first instincts were correct. The teacher should lead a discussion about which patterns are linear, and how the students know they are linear using the terminology and strategies presented in Activities 3, 4, and 5.

Activity 6: Applying Patterns and Counting to Geometric Concepts (GLEs: 5, 20, 22, 26, 27)

Materials List: pencil, paper, graphing calculator

Have students engage in a discussion about the sum of angles in various polygons, recognizing that if all possible diagonals are drawn from one vertex, the sum of the angles in the resulting triangles is the same as the sum of the angles in the polygon. Have students identify a pattern and use the pattern to write the formula for finding the sum of the angles in an n -gon, $S = 180(n - 2)$. This is a linear relationship. Ask students to determine the formula without a graphing calculator and allow them to verify their results using the calculator.

Students should also discuss the total number of diagonals which can be drawn in a polygon. Have students draw the diagonals in a triangle, quadrilateral, pentagon, hexagon, and heptagon. Ask students to identify a pattern and use the pattern to determine the number of diagonals in other polygons. They should recognize that it is not linear and explain how they know it is not linear. Have each student create a graph using the data collected and generate the formula using the regression equation function on a graphing calculator.

Activity 7: Round-Robin Tournaments (GLEs: 20, 22, 24, 25, 26, 27)

Materials List: pencil, paper, graphing calculator

The purpose of this activity is to use modeling or counting principles to determine answers to real-life problems. Examples:

- How many games need to be scheduled for six teams to play each other once?

- How many handshakes would take place among ten people if each person shakes hands with every other person?
- How many phone calls can be made between two people from among a group of five friends?

Encourage students to use various strategies for solving these problems. One technique is to model the situation by drawing and counting the diagonals in a polygon with the same number of sides as the number of teams or people. A second technique is to make lists showing all the possible combinations. Another technique is generating a formula by making a chart based upon how many handshakes would be needed for two people, three people, four people . . . n people. Prompt students to determine that reasoning is a valid process—each team plays every team except itself, but division by 2 will eliminate the duplicates (A playing B is the same as B playing A).

When determining the formula for answering each question, have students determine if the data is linear or not without using the graphing calculator. The regression equation function on the graphing calculator should be used to determine the formula only if the data is non-linear.

Activity 8: Permutations and Combinations (GLEs: 24, 25)

Materials List: pencil, paper, scientific calculator (minimum)

Teacher note: Information on permutations and combinations can be found in most Algebra 1 and/or Algebra 2 textbooks. This activity reviews combinations and permutations and extends the students prior knowledge to include circular permutations which have not been included in prior grades.

The purpose of this activity is for students to apply the concepts of permutation and combination to geometric situations.

For example:

A. How many ways can 3 books be arranged on a shelf if they are chosen from a selection of 8 different books? *Solution: 336*

B. How many committees of 5 students can be selected from a class of 25?
Solution: 53, 130

First, review simpler problems whose answers can be determined by making lists or tree diagrams. For example, how many different ways can you write the name of a triangle whose vertices are A, B, and C. The possibilities are:

ABC ACB
BAC BCA
CAB CBA

One way to think about this is that for any vertex, there are two different possible names. So three vertices times two names each is six possibilities.

Another way to think about this is that there are 3 positions to fill when naming the triangle. There are 3 vertices from which to choose for the first position, but only 2 remain as choices for the second position. Once the second position is filled, there is only one vertex remaining with which to fill the last position. Review with students that $3!$ is $3 \times 2 \times 1 = 6$ which is the same as the number of possible names. This is a concept taught in Algebra I. Give a few more examples in which the total number of possibilities can be determined.

Relate the idea of determining how many choices one has to name a triangle to Problem A: *How many ways can 3 books be arranged on a shelf if they are chosen from a selection of 8 different books?*

There would be 8 ways to fill the first position, 7 ways to fill the second position, and 6 ways to fill the third position. $8 \times 7 \times 6 = 336$. In situations in which order is important (e.g., ABC is different than ACB), the number of possibilities is called a permutation.

For situations in which order is NOT important (i.e., ABC and ACB would be considered duplicates since they are the same three letters), the number of possibilities is called a combination. To know the number of combinations of 3 books that can be put on the shelf, take into account how many arrangements would be considered to be the same for each set of 3 books. This is $3!$ or 6, so dividing 336 by 6 is 56. There would be 56 different combinations to put on the shelf. In other words, one could display a different combination of 3 books for 56 ways before he/she would have to repeat a set.

Have students discuss problem B. First, have them determine if the problem requires a permutation or combination and then solve the problem accordingly. Ask students if a committee of John, Sue, and Mary is the same committee as Sue, Mary and John. (*yes*)

It may be appropriate to use the permutation formula, $\left(P(n, r) = \frac{n!}{(n-r)!} \right)$, and

combination formula, $\left(C(n, r) = \frac{n!}{(n-r)!r!} \right)$, with some classes; however, the teacher

should guide the students through the development of these formulas using counting, listing, etc. The use of the formulas can be an extension of the lesson.

Provide students a variety of problems to work. It is better for some students to think through the position process. For those who have had more experience, the use of the formula is acceptable when solving such problems. As stated above, whether the formula is introduced and/or used should depend upon prior experience and knowledge of students in the class.

Introduce the class to circular permutations to answer such questions as, “How many ways can n people sit at a round table?”

If one of the chairs is designated as the "head" of the table, then the answer is $n!$. Any of n people sits at the head of the table, and the permutation proceeds in a clockwise direction. In this situation, it doesn't matter who sits at the head. In this case everyone could be sitting in the same relative order (ABCD, BCDA, CDAB, DABC for four people at the table) seated in different chairs. A sits in position 1, then position 4, then position 3, then position 2 but A is always next to B, who is next to C, who is next to D. Therefore, for n people there would be n duplicate arrangements. So $n!$ divided by n duplicate arrangements results in $(n-1)!$ permutations if there is no designated head position in the circle.

Have students draw all arrangements for some simple problems to help them understand the process. For example, how many permutations are there for 3 people sitting at a round table? for 4 sitting at a round table? for 5? Then repeat the same process with the idea that one place is designated as the head position.

The following example shows a real-life application of a circle permutation.

A disk jockey is setting up some CDs to play during his shift. He can put 6 different CDs on the tray. How many different ways can the discs be arranged?

In this instance, once the discs are arranged in a circle, that same arrangement can be rotated. The discs are in different positions, but the arrangement is the same. If you label them ABCDEF and rotate so that it is now FABCDE, the discs are still in the same relative order. Lead the students in a discussion to find that, in this case, 6 of the arrangements are the same, so the permutation is $\frac{6!}{6}$ or $(6-1)!$ possible arrangements of the discs.

Students should then generalize the concept so that any circular permutation without a fixed point is $(n-1)!$; with a fixed point, the permutation is $n!$.

Examples:

- A. How many ways can 8 campers be seated around a campfire? *Solution: 5040*
- B. How many ways can 3 books be placed on a shelf if chosen from a selection of 7 different books? *Solution: 210*
- C. Find the total number of diagonals that can be drawn in an octagon. *Solution: 20 (this is a combination taking 8 points, 2 at a time—however, since 8 segments are the sides of the figure, those 8 must be subtracted from 28 which is the number obtained from the formula).*

- D. Given 7 distinct points in a plane, how many line segments will be drawn if every pair of points is connected? *Solution: 21*
- E. Suppose there are 8 points in a plane such that no three points are collinear. How many distinct triangles can be formed with 3 of these points as vertices? *Solution: 56*
- F. How many pentagons can be formed by joining any 5 of 11 points located on a circle? *Solution: 462*

At the conclusion of this activity, students should respond to the following prompt in their math *learning logs* ([view literacy strategy descriptions](#)):

Describe the difference between a permutation and a combination. In your description, you should discuss the formulas, as well as, how you decide when to use a permutation or a combination. Include an example of each type and show how you would solve the problem. Explain why you chose to work each problem as either a permutation or combination.

Sample Assessments

General Assessments

- The student will create a variety of scenarios in which he/she makes conjectures using inductive reasoning.
- The student will create portfolios containing samples of his/her activities. He/she should create some of his/her own patterns and exchange them with the other students in class. He/she should include them in the portfolio stating whether the other students were able to determine their patterns, and if they were able to figure out patterns made by others.
- The student will respond to prompts to be answered in the math *learning logs* ([view literacy strategy descriptions](#)) and explain his/her ideas. For instance:
 - Given the following pattern, explain how you would determine the formula for the pattern, and how you would find the 35th term.
 - How are triangular, square, and rectangular numbers related to each other?
 - What is the difference between a permutation and a combination?

Activity-Specific Assessments

- Activities 2 and 4: Have the student create a variety of number or pictorial sequences. Each sequence should require the use of inductive reasoning to find the next number or picture in the sequence. Additionally, the student will

indicate, orally or in writing, the process for generating the next item. The students will also state the rule for generating the n^{th} term in each sequence.

- Activity 3: The student will use a graphing calculator to plot table entries for a given non-linear sequence in order to determine the regression equation for the data set.
- Activity 7: The student will participate in a simulation exercise to determine a tournament schedule for his/her district, regional, or state high school baseball team, basketball team, etc.

Geometry Unit 2: Reasoning and Proof

Time Frame: Approximately four weeks



Unit Description

This unit introduces the development of arguments for geometric situations. Conjectures and convincing arguments are first based on experimental data, then are developed from inductive reasoning, and, finally, are presented using deductive proofs in two-column, flow patterns, paragraphs, and indirect formats.

Student Understandings

Students understand the basic role proof plays in mathematics. Students learn to distinguish proofs from convincing arguments. They understand that proof may be generated by first providing numerical arguments such as measurements, and then by replacing the measurements with variables.

Guiding Questions

1. Can students develop inductive arguments for conjectures and offer reasons supporting their validity?
2. Can students develop short algorithmic-based proofs that generalize numerical arguments?
3. Can students develop more general arguments based on definitions and basic axioms and postulates?

Unit 2 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Geometry	
10.	Form and test conjectures concerning geometric relationships including lines, angles, and polygons (i.e., triangles, quadrilaterals, and n -gons), with and without technology (G-1-H)(G-4-H)(G-6-H)
11.	Determine angle measurements using the properties of parallel, perpendicular, and intersecting lines in a plane (G-2-H)
17.	Compare and contrast inductive and deductive reasoning approaches to justify conjectures and solve problems (G-4-H) (G-6-H)
19.	Develop formal and informal proofs (e.g., Pythagorean theorem, flow charts, paragraphs) (G-6-H)

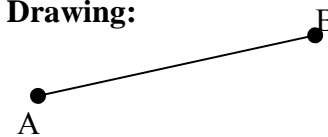
GLE #	GLE Text and Benchmarks
Data Analysis, Probability, and Discrete Math	
23.	Draw and justify conclusions based on the use of logic (e.g., conditional statements, converse, inverse, contrapositive) (D-8-H) (G-6-H) (N-7-H)

Sample Activities

Daily Warm-ups with Vocabulary

Teacher note: These warm-ups will review the basic vocabulary and basic concepts students need to be successful in Geometry. However, these vocabulary/concepts should have been mastered through GLEs in grades 3, 6, 7, and 8. Because of their importance, they have been included in these warm-ups to provide a way to make sure students have an understanding of the concepts before completing proofs.

Using 3 x 5 or 5 x 7 index cards, teachers should have students create *vocabulary cards* ([view literacy strategy descriptions](#)) for each term listed below. *Vocabulary cards* help students see connections between words, examples of the word, and critical attributes associated with the word. Have students complete one or two vocabulary cards per day. All vocabulary cards should be completed by the beginning of Activity 8 so students will have reviewed the terms/concepts necessary for the remaining activities. Students should place the term in the middle of the card. It is best if the class develops a complete definition that is mathematically correct to avoid misconceptions. Have students use prior knowledge to offer their definitions in a whole group setting, and then have the students create one definition as a class. In the upper left corner, the students should put a definition/explanation of the term. In the upper right corner, students should give examples of names and symbols for the term. In the lower left corner, students should give examples of real life objects that could represent each term. In the lower right corner, students should draw an illustration of what this term would look like including how it is correctly labeled. See the example card below.

<p>Definition/explanation: a part of a line with two endpoints; has measure</p>	<p>Ways to name: \overline{AB} or \overline{BA} means Segment AB or Segment BA; also, points A and B are the endpoints of the segment</p>
<p>line segment</p>	
<p>Real-life objects: pencil, piece of dry spaghetti, flag pole</p>	<p>Drawing:</p> 

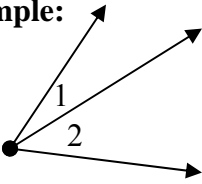
Students should be encouraged to use the cards to study and to refer to them as they encounter various symbols throughout the rest of the course. Students can buy rings to keep the cards together, or they can punch holes in the cards in order to keep them in their binders. Another option for organization is to have students keep the cards in a zippered bag which could also be kept in their binders. Remember, students should use these cards to help them study and when they are doing assignments, so it is important to check that the students are creating their cards correctly.

Have students create vocabulary cards for the following terms (please include other terms as necessary):

- point
- line
- plane
- line segment
- ray
- angle
- congruent
- parallel
- perpendicular
- adjacent angles
- vertical angles
- linear pair
- complementary
- supplementary

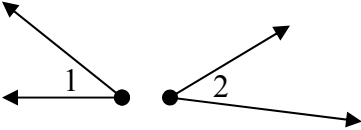
For the last five terms in the list, modify the vocabulary card to include examples, non-examples, and relationships between the angles. See the modified card below.

Definition: two angles that share a common vertex and a common side, but no common interior points

Example: 
 $\angle 1$ and $\angle 2$ are adjacent angles

adjacent angles

Relationship: All linear pairs are adjacent angles.

Non-example: 

Be sure the following relationships are included:

- Vertical angles are congruent.
- Two angles which form a linear pair are supplementary.
- Complementary angles which are also adjacent form a right angle.

Daily Warm-ups With Technology

Using a computer drawing program such as *The Geometer's Sketchpad*®, have students review basic terminology by constructing various figures such as angles, segments, segment bisectors, angle bisectors, etc. Give students characteristics of the figure they are to draw, such as the measure of the angle or segment. Have students measure segments and angles you provide using the drawing program. When measuring segments be sure students have the opportunity to measure in both English and metric units. It is also a good idea to have students use the drawing program to investigate the special angle pairs formed by two intersecting lines. If there is no access to a computer drawing program, use other materials such as rulers, protractors, and patty paper to construct and explore the same concepts.

Activity 1: Deductive Reasoning Skills (GLE: 17)

Materials List: pencil, paper, Logic Puzzle BLM, Logic Puzzle with Grid BLM, other logic puzzles

Remind students of the deductive reasoning skills used by Sherlock Holmes to solve mysteries by reading some excerpts (or providing excerpts for students to read) from Sherlock Holmes stories.

Ask students to work in small groups to solve the deductive reasoning or logic puzzles on the Logic Puzzle BLM. Give groups no help in solving the puzzle. After the students have worked for a while, have class members discuss the strategies employed to solve the puzzle. Then give the students a copy of the Logic Puzzle with Grid BLM and allow them to work for a while longer. Discuss the use of charts and how they might help.

Discuss what skills or strategies students think are needed to solve the problem and what tools will help them solve the problem.

Once the better strategies have been determined, give the students another puzzle in their groups and allow them to work it. Teachers can search the Internet for logic puzzles or purchase puzzle magazines from local stores. Teachers can include various types of logic puzzles including Sudoku and word games.

Over a time period of one or two weeks, give the students puzzles of varying degrees of difficulty. Allow them to use teacher provided help charts, but have them develop the ability to produce their own charts to facilitate their problem solving. Logic puzzles should be used throughout the year as fillers and/or warm-up materials because this is a skill not easily learned by some students.

Activity 2: Comparing Reasoning: Inductive vs. Deductive (GLE: 17)

Materials List: examples of inductive and deductive reasoning

Review the definition of inductive logic and provide the students with the following scenario.

Judy's Problem/Solution

"My dad is in the Navy and he says that food is great on submarines," offered Judy. "My mom," added Bobbie, "works for the airlines and she says that airline food is notoriously bad." "My mom is an astronaut trainee," added Greer, "and she says that their food is the worst imaginable." "You know," concluded Judy, "I bet no life exists beyond earth!" Bobbie and Greer both looked at her, puzzled. "What?" "Sure," explained Judy, "At extreme altitudes, food must taste so bad that no creature could stand to eat; therefore, no life exists out there."

What do you think of Judy's inductive reasoning? What possible other conjectures could be made? What are the problems with her conclusion?

Students should recognize that Judy has identified a pattern—each of Judy's friends' parents works at a different altitude, and as the altitude increases (below sea level, to airplane altitudes, to outer space) the food gets worse. Using this pattern, Judy makes a conjecture which she does not state—as the altitude increases the food gets worse. She uses that conjecture to make the statement that no life exists beyond earth. The conclusion is invalid because it does not come directly from the pattern Judy has observed. Students should be able to identify the process of inductive reasoning as identifying a pattern to make a conjecture, but they should realize Judy's conjecture is invalid.

After this discussion, define deductive reasoning and offer the following example for the students to analyze.

Alex's Grades

Alex's math teacher always tells him that homework is practice at home. She also tells him that the more he practices his math, the better his grades will be. Alex did all of his homework this week. When he gets to class before the test, he tells his teacher, "I'm going to do well on the test today."

What do you think of Alex's deductive reasoning? Are there any problems with his conclusion?

The teacher should then lead a discussion about the differences between inductive and deductive reasoning. Students should be given other examples of reasoning and be asked to determine if the reasoning used was inductive or deductive.

Activity 3: Distinguishing Between Inductive and Deductive Reasoning (GLE: 17)

Materials List: Internet access or presentation equipment for classroom use; pencil; paper

Have students visit

<http://www.sparknotes.com/math/geometry3/inductiveanddeductivereasoning/> for a presentation on inductive versus deductive reasoning. The site provides real-life examples of these types of reasoning and asks students to answer questions based on their reading. There are problems for students to solve as well. If access to a computer lab is not available, print the material from the website to be used as worksheets or show the information on a single computer with a projection system.

Activity 4: Finding Segment and Angle Measures Analytically (GLEs: 10)

Materials List: pencil, paper

Review the correct symbol for denoting the measure of a line segment. Be careful to point out the differences between the symbols (\overline{AB} , \overline{AB} , \overline{AB} and AB) and their meanings. Introduce the Segment Addition Postulate which states “If A , B , and M are collinear points and M is between A and B , then $AM + MB = AB$.” Also introduce the Midpoint theorem which states “If M is the midpoint of \overline{AB} , then $\overline{AM} \cong \overline{MB}$.” Students should be given various opportunities to find the measures of segments involving algebraic expressions by employing the Segment Addition Postulate and the Midpoint theorem. For example:

If A is between C and T , $CA = 2x + 5$, $AT = 5x - 2$, and $CT = 8x - 2$, find x and AT . *Solution: Using the Segment Addition Postulate, we know that $CA + AT = CT$, so $x = 5$, and $AT = 23$ units.*

In addition to finding measures of segments analytically, students should work with the Angle Addition Postulate which states “If R is in the interior of $\angle PQS$, then $m\angle PQR + m\angle RQS = m\angle PQS$.” Students should work problems in which they are to find the measures of various angles with and without algebra. Also, have students use the definition of angle bisector (If \overline{PQ} is an angle bisector of $\angle RPS$, then Q is in the interior of $\angle RPS$ and $\angle RPQ \cong \angle SQP$) to find angle measures using algebra.

Activity 5: Conditional Statements (GLE: 23)

Materials List: pencil, paper, newspapers, magazines

Define the term *conditional statement* and discuss the parts of a conditional statement (hypothesis and conclusion). Lead a discussion about the truth value of a conditional statement. It is important for students to know that a true hypothesis does not mean that

the conditional statement is also true, and likewise, a false hypothesis does not mean that the conditional is false. Show students the following truth table and have them determine if various conditionals are true or false based on a given set of conditions.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	F

Display the following conditional statement for all students to see: “If two angles have the same measure, then they are congruent.” Lead a brief discussion about the truthfulness of the statement. Next, display the converse: “If two angles are congruent, then they have the same measure.” Lead a discussion about how these two statements are related and about the truthfulness. Now, display the inverse of the conditional statement: “If two angles do not have the same measure, then they are not congruent.” Lead another discussion about how the conditional and the inverse statements are related and about the truthfulness. Finally, display the contrapositive of the conditional: “If two angles are not congruent, then they do not have the same measure.” Lead a discussion about how the contrapositive is related to the converse and about its truthfulness.

Define the term *logically equivalent*. Using the conditional “If two angles form a linear pair, then they are supplementary,” have students write the converse, inverse, and contrapositive of each statement and then determine the truthfulness of each statement. Using these statements and the definition of logically equivalent, have the students determine which statements are logically equivalent. Students should see that a conditional and its contrapositive are logically equivalent as are the converse and inverse of a conditional. Show the students the truth table below and give students more practice writing the converse, inverse, and contrapositive and determining the truthfulness of each statement.

p	q	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	Inverse $\sim p \rightarrow \sim q$	Contrapositive $\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

After students have demonstrated an understanding of the relationships above, then display other conditional statements. Have students work in pairs to write the converse, inverse, and contrapositive statements and to determine their truthfulness. Be sure to select conditional statements for which the converse is not a true statement.

Ask students to find examples of conditional statements in magazine or newspaper articles and discuss whether they are true or false. Have each student write and present

the converse of his or her conditional statements to the class and explain why the converse is true or false.

Activity 6: Laws of Syllogism and Detachment (GLE: 23)

Materials List: pencil, paper

Display statements similar to the following: “All dogs are mammals. Buster is a dog.” These statements illustrate the law of detachment. Ask students to determine a logical conclusion from these statements. Have students rewrite the statements in a conditional format if it helps them to “see” the conclusion better. To illustrate the law of syllogism, present students with statements like the following: “If I study for tests, I will make good grades. If I make good grades, I will be on the honor roll.” Ask students to form a logical conclusion based on these statements. Discussion should ensue about the validity (truthfulness) of these statements.

Present students with situations that do not lead to logical conclusions. Ask students to write their own pairs of conditionals that lead to logical conclusions and pairs that do not lead to logical conclusions. This activity will help students develop their deductive reasoning skills.

Have students apply the laws of syllogism and detachment to algebraic and geometric concepts in preparation for proofs.

Activity 7: Algebraic Proofs (GLE: 19)

Materials List: paper, pencil, Proof Process Guide BLM

In this activity, students will work in cooperative groups to correctly order the steps and reasons in an algebraic proof. Each group will complete a modified *process guide* ([view literacy strategy descriptions](#)). *Process guides* are designed to stimulate students’ thinking during or after their reading or listening to information presented in any content area. This *process guide* is modified because students will not be reading a text or passage in order to complete the proofs. To create this modified process guide, write the steps (statements) and reasons (algebraic properties) used to solve an algebraic equation as shown on the Proof Process Guide BLM. Next cut on the dotted lines to separate all the pieces and place them in an envelope or baggie. Give the envelope to a group of students and ask them to arrange the steps in a logical order. Depending on the ability of the class, it may be easier if the statements are in one envelope and the reasons (algebraic properties) are in a separate envelope. Four algebraic proofs are provided in the Proof Process Guide BLM. Additional proofs may need to be created if the class has a large number of students. This activity forces students to think through the process of solving an equation in order to arrange the steps in a logical order. Once students are satisfied with their ordering, they should copy the final result on paper.

Once groups have had the opportunity to complete their proofs, have them present the proofs to the class. Using a modified *questioning the author (QtA)* ([view literacy strategy descriptions](#)) technique, have students ask questions about the proofs to clarify their own understanding. The goal of *QtA* is to help students construct meaning from text. Instead of students asking the questions during reading, in this activity students will be asking questions after they have reviewed the proofs of other groups. Some possible questions might be:

- Does the flow of the proof make sense logically?
- Is the correct reason given for the statement presented?
- Are the statement and reason necessary to complete the proof?
- Is there a step missing that would help the reasoning sound more logical?

Teachers should direct students to think about their classmates' presentations and develop questions that may highlight incorrect logic or missing information. Teachers should set rules that create an environment conducive to this process. Students should ask and answer most of the questions; however, where necessary the teacher should offer his/her own questions/explanations to avoid misconceptions and incorrect answers.

Once students have adjusted to organizing the proofs, introduce proofs with unnecessary information. Require that students use only information that is relevant to the proof and organize the information into a logical order. Provide students with the opportunity to progress from basic algebraic proofs to basic geometric proofs based on algebraic concepts (definition of congruence, angle and segment addition postulates, properties of equality). Both of the strategies employed earlier in this activity (*process guides* and *QtA*) can also be used here to promote higher order thinking and understanding.

Activity 8: Proofs (GLE: 19)

Materials List: pencil, paper

The algebraic proofs were two column proofs, but some students find flow proofs or paragraph proofs easier to follow. Emphasis should be placed on providing a convincing, easy-to-follow argument with reasons rather than on using one particular format. The content for these proofs should focus on basic geometric concepts (segment and angle addition, congruent segments, and angles). While these proofs may be similar to those in Activity 7, these proofs are different because students have to come up with the statements and reasons as opposed to just arranging them in the correct order. Students must determine the arguments and reasons with their classmates. Facilitate students' work with proofs by having the students create a modified math *story chain* ([view literacy strategy descriptions](#)). A *story chain* typically has a group of students create a story based on content that has already been presented. Each member of the group adds a line to the story until the story is completed. In this activity, the *story chain* has been modified by having students complete a proof based on the algebraic and geometric concepts learned in this unit.

- Have students work in small groups of three to four students.

- In each group, each member should take a turn writing one statement and reason for the proof. The first member will write the first statement and reason. The second member will read the first person's statement and reason and decide if it is logical. Then he/she will add his/her own statement and reason. This process will continue until the entire proof is written. Each time a member receives the proof, he/she should read the entire proof to be sure he/she agrees with the logic and flow of the proof. If any person in the group has a concern with any of the previous information, he/she should help his/her classmate correct the statement then add his/her new information. Groups should be allowed to use a two-column proof, a paragraph proof, or a flow proof.
- Look for correct proofs. When most groups have completed their proofs, encourage them to discuss their ideas with other groups. At this point, students should question each other if they feel as though there are errors in any of the work.
- Choose three different groups to write a particular (correct) proof on the board. As a class, discuss variations and similarities of the three proofs, and talk about extra steps that could be added or omitted.

Activity 9: Fun with Angles (GLEs: 11, 19, 23)

Materials List: pencil, paper

Review the relationships among angles formed by the intersection of two parallel lines and a transversal that were learned in grade 8. Provide students with a graphic similar to Diagram 1 in which lines a and b are parallel. First, provide a number that represents the measure of angle 1. Have students find the measures of all the other numbered angles in the diagram and provide a justification for each measurement found (e.g., if the measure of angle 1 is 105° , the measure of angle 5 is 105° because angles 1 and 5 are corresponding angles).

Next, have students provide a convincing argument that pairs of angles are either congruent or supplementary (e.g., given that lines a and b are parallel, prove that angles 1 and 7 are supplementary), without using angle measures. (*Solution: If lines a and b are parallel, then angles 1 and 5 are congruent corresponding angles. Angles 5 and 7 are supplementary because they form a linear pair. If angles 5 and 7 are supplementary and angle 1 is congruent to angle 5, then angles 1 and 7 must also be supplementary since angles which are congruent can be substituted for one another.*)

Slightly more difficult proofs can be devised using diagrams similar to Diagram 2.

Use activities that require students to provide proofs or convincing arguments for answers throughout the year.

Diagram 1

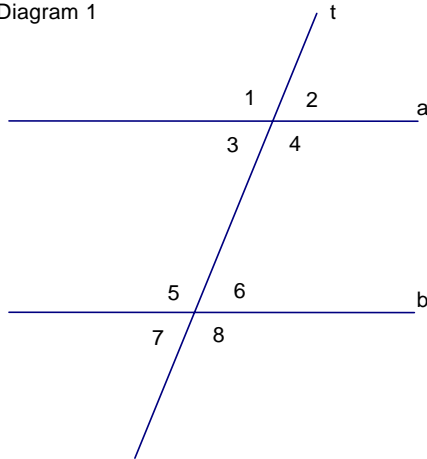
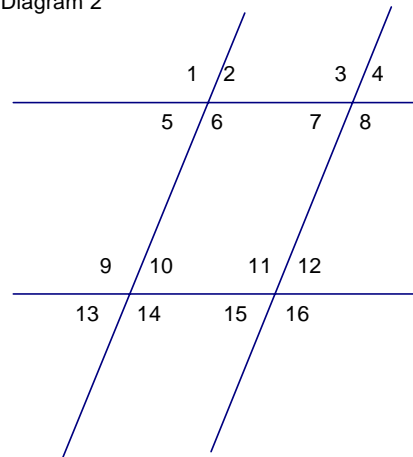


Diagram 2



Sample Assessments

General Assessments

- The student will answer prompts that include the following concepts in his/her math *learning logs* ([view literacy strategy descriptions](#)):
 - Comparing inductive and deductive reasoning.
 - Describing a situation in which he/she had several experiences that led him/her to make a true conjecture. The student will describe a situation in which he/she had several experiences that led to a false conjecture.
 - Responding to an advertisement, such as the following: “Those who choose Tint-and-Trim Hair Salon have impeccable taste, and you have impeccable taste.” This shows misuse of the Law of Detachment making a reader come to an invalid conclusion.
 - a. What conclusion does the ad want to imply?
 - b. Write another example that illustrates incorrect logic.
- The student will create a portfolio containing samples of his/her activities. For instance, the student will select the logic puzzle he/she liked best, explain how it was solved, and why he/she likes it.
- The student will write the inverses, converses, and contrapositives of given conditional statements, organize information for a proof, write his/her own proofs for basic algebraic and geometric concepts, and draw conclusions based on the laws of syllogism and detachment.

Activity-Specific Assessments

- Activity 1: The student will create his/her own logic problems. The students should then solve each other’s problems turning in the solutions for assessment.

- Activity 2: The student will find instances in real-life in which logical conclusions have been made. He/she can use newspapers, magazines, experiences at home, etc., and will write a paragraph explaining whether the logic used was inductive or deductive and if the conclusions are true or false. In this explanation, the student will demonstrate a concrete understanding of the difference between inductive and deductive reasoning.
- Activity 2: The student will read a book or watch a movie or TV show in which deductive logic is used to solve a mystery. He/she will detail the facts used in the deductive process in a short synopsis of the book or movie.
- Activity 7: The student will write proofs independently using basic algebra concepts. The student will explain the reasons for each step in the process of solving the problem.
- Activities 7 and 8: Provide students with proofs—some that are accurate and some that have flaws. The student will evaluate the proofs and identify and correct any flaws that exist.
- Activity 9: Provide students with one measurement in a diagram using parallel lines and transversals (possibly three parallel lines and one or two transversals). The student should find all the missing angle values in the diagram and provide an explanation of how each value was determined.

Geometry

Unit 3: Parallel and Perpendicular Relationships

Time Frame: Approximately three weeks



Unit Description

This unit demonstrates the basic role played by Euclid's fifth postulate in geometry. Euclid's fifth postulate is stated in most textbooks using the wording found in Playfair's Axiom: *Through a given point, only one line can be drawn parallel to a given line.* This axiom and several others are considered by some mathematicians to be equivalent to Euclid's fifth postulate. The focus is on basic angle measurement relationships for parallel and perpendicular lines, equations of lines that are parallel and perpendicular in the coordinate plane, and proving that two or more lines are parallel using various methods including distance between two lines.

Student Understandings

Students should know the basic angle measurement relationships and slope relationships between parallel and perpendicular lines in the plane. Students can write and identify equations of lines that represent parallel and perpendicular lines. They can recognize the conditions that must exist for two or more lines to be parallel. Three-dimensional figures can be connected to their 2-dimensional counterparts when possible.

Guiding Questions

1. Can students relate parallelism to Euclid's fifth postulate and its ramifications for Euclidean Geometry?
2. Can students use parallelism to find and develop the basic angle measurements related to triangles and to transversals intersecting parallel lines?
3. Can students link perpendicularity to angle measurements and to its relationship with parallelism in the plane and 3-dimensional space?
4. Can students solve problems given the equations of lines that are perpendicular or parallel to a given line in the coordinate plane and discuss the slope relationships governing these situations?
5. Can students solve problems that deal with distance on the number line or in the coordinate plane?

Unit 3 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
1.	Simplify and determine the value of radical expressions (N-2-H) (N-7-H)
Algebra	
6.	Write the equation of a line parallel or perpendicular to a given line through a specific point (A-3-H) (G-3-H)
Geometry	
10.	Form and test conjectures concerning geometric relationships including lines, angles, and polygons (i.e., triangles, quadrilaterals, and n -gons), with and without technology (G-1-H) (G-4-H) (G-6-H)
11.	Determine angle measurements using the properties of parallel, perpendicular, and intersecting lines in a plane (G-2-H)
12.	Apply the Pythagorean theorem in both abstract and real-life settings (G-2-H)
16.	Represent and solve problems involving distance on a number line or in the plane (G-3-H)
19.	Develop formal and informal proofs (e.g., Pythagorean theorem, flow charts, paragraphs) (G-6-H)
Data Analysis, Probability, and Discrete Math	
22.	Interpret and summarize a set of experimental data presented in a table, bar graph, line graph, scatter plot, matrix, or circle graph (D-7-H)

Sample Activities**Activity 1: Slopes of Perpendicular Lines (GLEs: 10, 11, 22)**

Materials List: pencil, paper, graphs of lines for work in pairs

Have students work in pairs. Provide each pair of students with three graphs of lines, each on a separate coordinate grid system. On one set of axes have a graph of parallel lines; on the other two sets of axes have single lines with different slopes and y -intercepts. Be sure to use a different set of lines for each pair of students. Provide at least two points on each line by marking the points on the graphs. Review how to find the slope of a line from two points, and then have students determine the slope of each of the lines provided.

Next, have students carefully fold each of the single lines onto itself and crease the paper along the fold line. Have students measure the angle formed by the line and the “crease” line to confirm that it is a right angle. Be sure to engage students in a discussion that helps them see that the “crease” line is perpendicular to the original line. Students should develop a convincing argument that the “crease” line is perpendicular to the original lines (i.e., The two angles are congruent because they are the same size since the angles

“match” when folded over one another. Because a line measures 180 degrees, the measures of the two angles are 90 degrees each).

Students will then determine the slope of the “crease” line and compare it to that of the original line. All data from the class should be recorded in a chart. The chart should include a column for the slope of the original line and a column for the slope of the “crease” line. Using the class data, student pairs should make a conjecture about the slope of perpendicular lines.

Using the graph with the given parallel lines, have students fold the graph so each line lies on itself to create a “crease” line which passes through the pair of parallel lines. Then have students discuss how the slopes of the parallel lines are related, and whether or not the “crease” is perpendicular to one or both of the given lines. This should lead to a discussion about the theorem that states “if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.”

Activity 2: Parallel and Perpendicular Lines (GLE: 6)

Materials List: pencil, paper, graph paper, computer drawing program (optional)

Provide students with several equations of pairs of lines that are parallel, and several equations of pairs of lines that are perpendicular, but don’t give the students the relationships. Have students graph the lines and determine the characteristics of the equations of the lines that are parallel, and the characteristics of those that are perpendicular (i.e., parallel lines have the same slopes and different y-intercepts, perpendicular lines have slopes that are opposite reciprocals of one another). As an alternative, have students use a computer software program like *Geometer’s Sketchpad* to draw a pair of perpendicular lines and a pair of parallel lines. Have the program generate the equations of those lines and then determine the characteristics of these equations.

Once the characteristics are determined, review with students the process for developing the equation of the line if two points on the line are given. Provide students with graphs of lines that are parallel or perpendicular (several of each). Have students apply the characteristics of parallel or perpendicular lines to write the equations for the given lines. Next, have students write the equation of lines that are parallel or perpendicular to a line through a given point on the line.

Examples:

Given that a line passes through $(-2,3)$ and $(4,6)$, write the equation of a line that is parallel to the given line and passes through $(1,-2)$. Write the equation of the line that is perpendicular to the original line through $(1,-2)$. *Solution: Parallel:*

$$y = \frac{1}{2}x - \frac{5}{2}; \text{ Perpendicular: } y = -2x.$$

To end the activity, have students working alone write equations of any two lines that are parallel and any two lines that are perpendicular. Do not provide them with any information such as slopes or y-intercepts.

Activity 3: Proving Lines are Parallel (GLEs: 10, 11, 19)

Materials List: pencil, paper, diagrams for discussion, learning log

Teacher note: The names of the special angle pairs formed by two lines and a transversal, and the relationships of these special angle pairs are found in the Grade 8 GLEs so they are not discussed in detail here. A review may be necessary depending on the students in the class.

Give students diagrams of parallel lines and transversals, and diagrams of lines that are not parallel with transversals. Lead a discussion to determine what characteristics of parallel lines will guarantee that two lines are parallel (e.g., two lines are parallel if corresponding angles are congruent, alternate interior angles are congruent, alternate exterior angles are congruent, consecutive interior angles are supplementary, parallel lines are everywhere equidistant).

Have students form conjectures that lead to the converses of the parallel lines theorems (e.g., if alternate interior angles are congruent when a transversal intersects two lines, then the two lines are parallel). Remind students that the statement “If two parallel lines are cut by a transversal, then corresponding angles are congruent” is a postulate which is accepted as true without proof. The converse “If two lines in a plane are cut by a transversal so that corresponding angles are congruent, then the two lines are parallel” is also a postulate accepted as true without proof. Using these postulates as truth, students can prove the other theorems and converses. Allow students to initially use angle measures to write proofs for specific sets of lines to prove these theorems, but also require them to use general proofs that prove lines parallel through generalities. These proofs can take any form (informal, paragraph, two-column, flow). Provide opportunities for students to prove the other theorems which are based on the postulate for corresponding angles. Have students complete a proof (or proofs) which involves the theorems and/or their converses in their math *learning logs* ([view literacy strategy descriptions](#)). Students should be allowed to complete any type of proof they wish as long as they demonstrate logical thinking and reasoning and provide evidence for their statements. The diagrams should be more than just a pair of parallel lines cut by a transversal. The diagrams used in this proof should incorporate parallel lines in other figures such as triangles and quadrilaterals (in a trapezoid, the angles along one leg are supplementary; opposite angles of a parallelogram are congruent; if a segment is drawn parallel to any side of a triangle, the corresponding angles in the similar triangles are congruent; etc.). Students should not be asked to prove two triangles congruent at this time, but this can serve as a precursor to the proofs they will write in Unit 4.

Provide diagrams of two lines that are perpendicular to one line. Have students form a conjecture that if two lines are perpendicular to the same line, they must be parallel. Then, have students write a proof of this theorem.

To end the activity, have students complete *SPAWN Writing* ([view literacy strategy descriptions](#)) using the *What If?* category. Have students answer the following prompt:

Think about your favorite outing (a trip to the mall, football games, a trip to Walt Disney World, etc.). What might your day be like if there were no way to ensure lines are parallel?

After students have had time to construct their responses, the teacher may allow a few students to share their responses with the class. Students may add their own ideas to other students' responses or question their reasoning. These writings could be included in a portfolio of the student's work.

Activity 4: Distance in the Plane (GLEs: 1, 12, 16)

Materials List: pencil, paper, graph paper

Have students explore the distance between two points in the rectangular coordinate system. Give attention to the idea that distance in the plane between two points can be thought of as the length of a hypotenuse of a right triangle. Thus, the Pythagorean theorem can be used to determine these distances. Have students apply the concept of distance on a number line to find the length of the legs of the right triangle. Once students understand how the Pythagorean theorem applies to distance on a coordinate plane, guide students as they develop the formula for distance on the coordinate plane through the use of arbitrary points (x_1, y_1) and (x_2, y_2) . When using the Pythagorean theorem and the distance formula, have students simplify radical solutions as well as use the calculator to estimate the solution.

Activity 5: Parallel Lines and Distance (GLEs: 10, 16)

Materials List: pencil, paper, computer drawing program (optional), learning log

Provide different sets of two lines. Some sets should be parallel; others should not be parallel. These lines should be drawn on lineless paper. Ask students how they visually determine which of the sets of two lines are parallel. Have a discussion which leads to an understanding that parallel lines are always the same distance apart.

Lead students in a discussion about the definition of *distance between a point and a line* or *distance between two parallel lines*. One way to do this is to draw two parallel lines on the board and use a ruler to determine distances. Use a drawing program, such as *Geometer's Sketchpad*[®], as an alternate way to demonstrate the same concept. The

discussion should reveal that the distance is always the shortest line segment between two points (or the shortest distance between a point and a line). Have students realize that distance between a point and a line is the same as the length of a line segment which starts at the point and is perpendicular to the line. To find the distance between two parallel lines, identify a point on one of the lines and draw a segment from this point perpendicular to the second line.

Give students diagrams on the coordinate plane and ask them to find the distance between lines and points not on the lines. Review the concept of the Parallel Postulate here (If there is a line and a point not on the line, then there is exactly one line that can be drawn through the given point that is parallel to the given line).

Have students apply the concept of distance between a point and a line to polygons. Relate the distance between a point and a line to finding the length of an altitude in a triangle (i.e., an altitude is the perpendicular distance from a vertex to a segment on the opposite side of the triangle). This establishes correct understandings necessary for the concepts that will find area, surface area, and volume.

To end the activity, have the students answer the following prompt in their math *learning logs* ([view literacy strategy descriptions](#)):

Describe how to determine the distance between a line and a point not on the line. How could you use this information to help you find the distance between a plane and a point that is not contained in that plane?

Activity 6: Ladders and Saws (GLEs: 10, 11)

Materials List: pencil, paper, unlined paper, colored pencils (at least three different colors), protractor, ruler, triangle cutouts, copies of the directions from the website listed in the activity

Instructions for the Ladder and Saws Activity can be found in many texts and on the web. For example, see <http://library.thinkquest.org/28318/ladders.html>. In this activity, students use parallel segments in hands-on activities to discover various relationships between angles, segments, and triangles. The website provides a materials list as well as directions for completing the activity. The last line of the webpage is a link to a pdf file. This file organizes the same information as presented on the webpage into a printable format which will help with the execution of this activity.

Below is a list of some of the possible concepts/principles that students may “discover” by engaging in the Ladders and Saws Activity (as listed on the website <http://library.thinkquest.org/28318/ladders.html>).

- The sum of the angle measures of any triangle is equal to 180° .
- Vertical angles are congruent.
- Linear pairs are supplementary.
- Alternate interior angles are congruent when lines are parallel.

- Corresponding angles are congruent when lines are parallel.
- Same-side interior angles are supplementary when lines are parallel.
- The measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.
- Two lines parallel to a third line are parallel to each other.
- Opposite sides of a parallelogram are parallel and congruent.
- Opposite angles of a parallelogram are congruent.
- Adjacent angles of a parallelogram are supplementary.
- The segment joining the midpoints of two sides of a triangle is parallel to the third side and has a length equal to half the length of the third side.
- The ratio of the perimeters of two similar triangles is the same as the scale factor of the similar triangles.
- The sum of the exterior angle measures of any convex polygon is 360° .
- The sum of the angle measures of a quadrilateral is 360° .
- The sum of the angle measures of a hexagon is 720° .

Teachers should insure that students understand the relationships among the angles formed by the parallel lines and transversals as discussed in Unit 2 Activity 7. Have students discuss the other concepts mentioned above and retain this listing of relationships so that they may refer to it during discussions in subsequent units that refer to these concepts. The concepts which emerge concerning triangles and quadrilaterals will be very important in Unit 4. While students may have been introduced to much of this information prior to this course, this activity connects their knowledge of parallel lines and the angle relationships formed by those lines, to their prior knowledge of triangles and quadrilaterals.

Activity 7: Parallel Line Facts (GLEs: 10, 11, 19)

Materials List: pencil, paper, Parallel Line Facts BLM, ruler

Teacher Note: The focus of this activity is to apply the properties of parallel lines learned in earlier activities. This is a good application of proof using the parallel line properties. It also introduces information relative to angle relationships in triangles, connecting this unit to Unit 4.

Give students copies of the Parallel Line Facts BLM and have them complete the following steps.

- Have students draw a line through one vertex of a triangle so that the line is parallel to a side of the triangle. Have students write a proof (using parallel line relationships from Unit 2 and earlier activities) to show that the sum of the angles in a triangle is 180° .
- Use the same diagram to write a proof to show that the measure of an exterior angle in a triangle has the same measure as the sum of the two remote interior angles. This is the Exterior Angle Sum theorem.

- Have students investigate the area of different triangles formed between two parallel lines by moving one vertex along one of the parallel lines. Students should recognize that the height of each triangle is always the same since the distance between the parallel lines will not change. Since the base length doesn't change, students should realize that the areas are the same.

Sample Assessments

General Assessments

- The student will create a portfolio containing samples of work completed during activities. For instance, he/she could include the graphs from Activity 2 and explain what happened in the activity and what was learned from the activity.
- The student will respond to journal prompts that include:
 - Describing at least three different ways to prove two lines are parallel.
 - Explaining how to write the equation of a line perpendicular to $y = -\frac{2}{3}x + 5$ through the given point $(-4,6)$.
 - Explaining the relationship between the Pythagorean theorem and the distance formula for distance on the coordinate plane.
- The student will create a “scrapbook” of pictures taken in a real-world setting (i.e. railroad tracks) that depict parallel and perpendicular lines. This scrapbook will include pictures and indicate how the items in the picture demonstrate the term chosen. The student will have a minimum of three pictures for each term. See the Scrapbook Rubric BLM for more information.

Activity-Specific Assessments

- Activity 1: The teacher will provide the student with several sets of graphs of perpendicular lines, intersecting lines which are not perpendicular, and intersecting lines which are almost perpendicular drawn on coordinate graph paper. The student will use slope to determine if the lines are parallel.
- Activity 2: The teacher will give each student a copy of the What's My Line? BLM and one of the five What's My Line? graphs. The five graphs have different slopes so there is a larger probability that the students will have different responses. The student will draw and label the x - and y -axes anywhere on the coordinate plane that he/she chooses. Based on where the x - and y -axes are drawn, the student will then
 - find the slope of the given line and write the equation of the given line.
 - write the equation for a line which passes through the given point and is parallel to the given line.

- write the equation of the line which is perpendicular to the given line and passes through the given point.

A What's My Line? Rubric BLM is provided for this assessment.

- Activity 6: The teacher will provide the student with nets or diagrams formed by intersecting lines (parallel and nonparallel) and a minimal number of angle measures for the diagram. The student will calculate the missing angle measures using either the formula $S = 180(n - 2)$ or the angles created by transversals that intersect parallel lines.

Geometry

Unit 4: Triangles and Quadrilaterals

Time Frame: Approximately five weeks



Unit Description

This unit introduces the various postulates and theorems that outline the study of congruence and similarity. The focus is on similarity and congruence treated as similarity with a ratio of 1 to 1. It also includes the definitions of special segments in triangles, classic theorems that develop the total concept of a triangle, and relationships between triangles and quadrilaterals that underpin measurement relationships. The properties of the special quadrilaterals (parallelograms, trapezoids, and kites) are also developed and discussed.

Student Understandings

Students should know defining properties and basic relationships for all forms of triangles and quadrilaterals. They should also be able to discuss and apply the congruence postulates and theorems and compare and contrast them with their similarity counterparts. Students should be able to apply basic classical theorems, such as the Isosceles Triangle theorem, Triangle Inequality theorem, and others.

Guiding Questions

1. Can students illustrate the basic properties and relationships tied to congruence and similarity?
2. Can students develop and prove conjectures related to congruence and similarity?
3. Can students draw and use figures to justify arguments and conjectures about congruence and similarity?
4. Can students state and apply classic theorems about triangles, based on congruence and similarity patterns?
5. Can students construct the special segments of a triangle and apply their properties?
6. Can students determine the appropriate name of a quadrilateral given specific properties of the figure?
7. Can students apply properties of quadrilaterals to find missing angle and side measures?

Unit 4 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
1.	Simplify and determine the value of radical expressions (N-2-H)(N-7-H)
Algebra	
6.	Write the equation of a line parallel or perpendicular to a given line through a specific point (A-3-H) (G-3-H)
Geometry	
9.	Construct 2- and 3-dimensional figures when given the name, description, or attributes, with and without technology (G-1-H)
10.	Form and test conjectures concerning geometric relationships including lines, angles, and polygons (i.e., triangles, quadrilaterals, and n -gons), with and without technology (G-1-H) (G-4-H) (G-6-H)
16.	Represent and solve problems involving distance on a number line or in the plane (G-3-H)
17.	Compare and contrast inductive and deductive reasoning approaches to justify conjectures and solve problems (G-4-H) (G-6-H)
18.	Determine angle measures and side lengths of right and similar triangles using trigonometric ratios and properties of similarity, including congruence. (G-5-H)(M-4-H)
19.	Develop formal and informal proofs (e.g., Pythagorean theorem, flow charts, paragraphs) (G-6-H)
Data Analysis, Probability, and Discrete Math	
23.	Draw and justify conclusions based on the use of logic (e.g., conditional statements, converse, inverse, contrapositive) (D-8-H) (G-6-H) (N-7-H)

Sample Activities

Teacher Note: Before beginning this unit, the teacher should make sure the students have a good understanding of the Angle Sum theorem and Exterior Angle theorem found in Unit 3 Activity 7. Students should also be able to classify triangles according to their side and angle measures. If necessary, the teacher may take a day or two to review these concepts; however, this should be kept to a minimum.

Activity 1: Analyzing Isosceles Triangles (GLEs: 1, 10, 16,)

Materials List: pencil, paper, patty paper, compass, straightedge, ruler

Teacher Note: Safety compasses can be used in schools where sharp, pointed instruments are prohibited. Techniques for using patty paper to duplicate segments, construct perpendicular lines, angle bisectors, etc., can be found in Patty Paper Geometry by Michael Serra (Key Curriculum Press).

Have students use patty paper or tracing paper to draw an acute, isosceles triangle. Have them start by drawing an acute angle and labeling it C . Then they should mark equal lengths on each side of the angle, label them A and B , and draw \overline{AB} . Have students use patty paper constructions, compass/straight edge constructions, or measurements with a ruler to mark equal lengths. Have students fold the triangle in half so that the equal sides lie on top of each other. Have students make observations about base angles A and B . Repeat the activity with obtuse and right isosceles triangles as well. Have a class discussion in which students form conjectures that lead to the Isosceles Triangle theorem (If two sides of a triangle are congruent, then the angles opposite those sides are congruent.) and its converse.

Have students practice their algebra skills to find the measures of sides and angles of isosceles triangles, when given information about the isosceles triangles.

For example:

- A. In isosceles triangle $\triangle ABC$ with base \overline{BC} , $m\angle ABC = 5x - 4^\circ$, and $m\angle ACB = 7x - 20^\circ$, find the measure of each angle.
Solution: $x = 8$, $m\angle ABC = 36^\circ$, $m\angle ACB = 36^\circ$, and $m\angle BAC = 108^\circ$.
- B. In isosceles triangle $\triangle DEF$, $\angle F$ is the vertex angle. If $DE = 5x$ inches, $EF = 4x - 3$ inches, and $DF = 2x + 7$ inches, find the length of the base.
Solution: $x = 5$, and $DE = 25$ inches.
- C. $\triangle ABC$ has vertices $A(2,5)$, $B(5,2)$, and $C(2,-1)$. Use the distance formula to show that $\triangle ABC$ is an isosceles triangle and name the pair of congruent angles.
Solution: $AB = 3\sqrt{2}$, $BC = 3\sqrt{2}$, $AC = 6$, $\overline{AB} \cong \overline{BC}$, and $\angle A \cong \angle C$. All distances are in linear units.

Activity 2: Congruent Triangles (Using Technology) (GLE: **10**)

Materials List: pencil, paper, Internet access

Have students quiz themselves and track results concerning terminology used with congruent triangles at the website <http://www.quia.com/jq/13397.html>. This activity is an assessment of the students' prior knowledge. Repeat the activity later in the unit to see if gains were made. If the class does not have Internet access so that each student can take the quiz online, then the quiz may be printed from the site so copies can be made.

Activity 3: Corresponding Parts (CPCTC) (GLEs: 10, 18)

Materials List: pencil, paper

Note: While this activity does not ask students to find the measures of segments or angles, it does require them to determine corresponding parts of congruent triangles. This is a skill necessary when determining corresponding sides to write proportions for similarity.

The focus of this lesson is to make students aware of correct ways to name congruent triangles to preserve corresponding parts. Have students work in small groups. Give each group several diagrams of pairs of congruent triangles. Have students measure sides and angles to determine congruent, corresponding parts. Lead a discussion about writing congruence statements and focus on writing letters in the correct order. Provide students with congruence statements such as $\triangle ABC \cong \triangle XYZ$ and ask them to name the corresponding angles and sides. Additionally, instruct students to find equivalent congruence statements for figures (e.g., for the example given above, $\triangle BAC \cong \triangle YXZ$ is an equivalent statement).

Activity 4: More about Congruent Triangles (Using Technology) (GLEs: 10, 19)

Materials List: pencil, paper, geometry drawing software (if the software is unavailable use straws, protractor, ruler, compass, patty paper), computers

Using *Geometer's Sketchpad*[®] or similar geometry software, give students a set of three lengths for line segments that can be used to form a triangle. Have students construct the triangle. Next, have students compare their constructions. They should make a conjecture about the triangles created (e.g., the triangles are congruent).

Next, have students repeat the activity using two side lengths and an included angle. Again, have students conjecture the relationship between the constructed triangles. Repeat the activity using two angles and an included side, and then again, using two angles and a non-included side.

Ask students to use their constructions to help them develop convincing arguments for the postulates discovered in this activity (SSS, SAS, ASA, and AAS). If the class does not have access to geometry software, have students use straws cut to certain lengths, and protractors. Provide alternative ways for students to draw the given triangles by hand if materials are not accessible (compass/straightedge or patty paper).

Activity 5: Are They Congruent? (GLEs: 10)

Materials List: pencil, paper, ruler, protractor, compass, geometry drawing software (optional), computers (optional)

Provide students with the measures of two sides and a non-included angle for a triangle, or the measures of three angles and no sides. Have students construct a triangle using the given measures (either with or without technology). Next, have students compare their constructions. Have students make a conjecture about the relationships of these constructed triangles. Repeat this activity with several sets of SSA or AAA measures. Students should make a conjecture about whether SSA and AAA can be used to justify two triangles being congruent.

Activity 6: Proving Triangles Congruent (GLEs: 17, 19, 23)

Materials List: pencil, paper, Proving Triangles Congruent BLM

Have students work in groups. Provide each pair of students with one of the sheets in the Proving Triangles Congruent BLM that present diagrams involving congruent triangles. Each group should have different diagrams. Develop more diagrams with different given information to accommodate the class. The Proving Triangles Congruent BLM provides some samples of the types of diagrams teachers can use. Since students have not yet learned the properties of parallelograms, provide given information that allows students to prove two (or more) triangles congruent. Ask students to prove two of the triangles in the diagram are congruent or parts of congruent triangles are congruent. Allow students to use various methods of proof: two-column, flow, or paragraph.

Have groups share their proofs with the class. Using a modified *questioning the author (QtA)* ([view literacy strategy descriptions](#)) technique, have students ask questions about the proofs to clarify their own understanding. The goal of *QtA* is to help students construct meaning from text. Instead of students asking the questions during reading, in this activity, students will be asking questions after they have reviewed the proofs of other groups. Some possible questions might be:

- Does the flow of the proof make sense logically?
- What information in the diagram led you to that statement?
- Is the correct reason given for the statement presented?
- Are the statement and reason necessary to complete the proof?
- Is there a step missing that would help the reasoning sound more logical?

To end the activity, have students employ techniques used in class to prove two triangles from a diagram are congruent. This individual work will show that students have mastered the skill.

Activity 7: Altitudes, Angle Bisectors, Medians, and Perpendicular Bisectors of a Triangle (GLE: 10)

Materials List: pencil, paper, patty paper, geometry software, computers

Have students work in groups of three or four. Give each group 12 sheets of patty paper. Instruct students to draw the following types of triangles (one triangle on each sheet of patty paper): four acute scalene triangles, four right scalene triangles, and four obtuse scalene triangles. Have students label one sheet of patty paper from each group of triangles with one of the following: angle bisectors, medians, perpendicular bisectors, or altitudes.

Provide students with the definition of *angle bisector*. Have students construct the angle bisectors for all the angles in each triangle labeled with angle bisector. This may be done via patty paper folding, measuring, or with the use of a drawing program such as the *Geometer's Sketchpad*[®]. The purpose of this activity is to have students learn the definition by having to create angles of equal measure. Ask students to share their work with other class members. If done properly, the angle bisectors will intersect at one point. Have students discuss any differences they observed when constructing the angle bisectors on the different types of triangles (acute, right, and obtuse). For all three types, the angle bisectors should intersect inside the triangle.

Provide students with the definition of *median*. Have students construct the three medians in each triangle labeled with median. This may be done via patty paper folding, measuring, or with the use of a drawing program such as the *Geometer's Sketchpad*[®]. The purpose of this activity is to have students learn the definition by having to first find the midpoint of a side, and then draw a segment from the midpoint to the opposite vertex in the triangle. Ask students to share their work with other class members. If done properly, the medians will intersect in one point. Have students discuss any differences they observed when constructing the medians on the different types of triangles (acute, right, and obtuse). For all three types, the medians should intersect inside of the triangle.

Provide students with the definition of *perpendicular bisector of a segment*. Have students construct the perpendicular bisectors for all sides in each triangle labeled with perpendicular bisector. This may be done via patty paper folding, measuring, or with the use of a drawing program such as the *Geometer's Sketchpad*[®]. The purpose of this activity is to have students learn the definition by having to locate the midpoint of a side and then drawing a line through the midpoint so that the line is perpendicular to the side of the triangle. Ask students to share their work with other class members. If done properly, the perpendicular bisectors of the three sides of the triangle will intersect in one point. Have students discuss any differences they observed when constructing the perpendicular bisectors on the different types of triangles (acute, right, and obtuse). For the acute triangle, the perpendicular bisectors will intersect inside the triangle. For the right triangle, the perpendicular bisectors will intersect on the hypotenuse. For the obtuse triangle, the perpendicular bisectors will intersect outside the triangle.

Provide students with the definition of *altitude* in a triangle. Have students construct the three altitudes in each triangle and label each line with the word altitude. This may be done via patty paper folding, measuring, or with the use of a drawing program such as the *Geometer's Sketchpad*[®]. The purpose of this activity is to have students learn the definition by creating a line that passes through a vertex of the triangle and is perpendicular to the opposite side. Ask students to share their work with other class members. If done properly, the altitudes will intersect in one point. Have students discuss any differences they observed when constructing the altitudes on the different types of triangles (acute, right, and obtuse). For the acute triangle, the altitudes will intersect inside the triangle. For the right triangle, the altitudes will intersect at the vertex of the right angle. For the obtuse triangle, the altitudes will intersect outside the triangle.

Teacher Note: Instructions for using patty paper to fold segments in this activity can be found in Patty Paper Geometry by Michael Serra (Key Curriculum Press). It is not recommended that these constructions be made with a compass/straightedge as students seldom remember the construction steps. If compass/straightedge constructions are used, time must be taken to explain and demonstrate how the constructions relate to congruent triangles (i.e., the construction of a perpendicular bisector of a segment is based on the creation of two triangles by SSS). Patty paper constructions or use of the Geometer's Sketchpad are much more intuitive for students, and their use does not present safety issues.

Have students practice constructing their own altitudes, perpendicular bisectors, medians, and angle bisectors to help internalize the definitions.

Extensions: An acute, scalene triangle works best for these activities. Have students draw the triangle on a sheet of patty paper. The triangle should be as large as possible.

Have students fold or draw all angle bisectors in a triangle. Tell them the name of the common intersection point for the three angle bisectors in a triangle is called the *incenter*. Have students measure the distance from the incenter to each side. This reinforces the concept of distance between a point and a line. These distances should be the same, indicating that the incenter is the center of a circle which can be inscribed in the triangle. Have students use a compass to draw the inscribed circle.

Have students fold or draw all the perpendicular bisectors in a triangle. Tell them the name of the common intersection point for the three perpendicular bisectors in a triangle is called the *circumcenter*. Have students measure the distance from the circumcenter to each vertex of the triangle. This reinforces the concept of distance between two points. These distances should be the same indicating that the circumcenter is the center of a circle which passes through each vertex of the triangle. Have students use a compass to draw the circumscribed circle.

Have students fold or draw the three medians in a triangle. Tell them the name of the common intersection point for the three medians is called the *centroid* and is the center of

gravity for the triangle. Have students transfer the location of the three vertices of the triangle and the centroid to a sheet of cardstock (an old manila file folder works well, too). This can be done by placing the sheet of patty paper on the card stock and making an indentation with a pencil or pen point through the patty paper onto the card stock. Have the students use a straight edge to draw the sides of their triangle on the cardstock and then cut it out with scissors. If done properly, the triangle should balance when the centroid is placed on the lead end of a sharpened pencil. Use the eraser end, if needed. Position the pencil at a location other than the centroid, and the triangle will tilt to one side and fall off. It will not stay balanced.

Some students may want to create art designs using triangles and the inscribed and/or circumscribed circles for portfolio entries.

Activity 8: Altitudes, Medians, and Perpendicular Bisectors on the Coordinate Plane (GLEs: 6, 9)

Materials List: pencil, paper

Provide students with information that allows them to graph triangles on a coordinate plane. Have them draw the medians, perpendicular bisectors, and altitudes of those triangles. Ask students to write equations that represent those segments. Writing equations reinforces skills learned in Algebra.

Examples:

- A. $\triangle ABC$ has vertices $A(-3,10)$, $B(9,20)$, and $C(-2,21)$. Find the coordinates of P such that \overline{CP} is a median of $\triangle ABC$. Determine if \overline{CP} is an altitude of $\triangle ABC$.

Solutions: P is at $(3,15)$; \overline{CP} is an altitude of $\triangle ABC$.

- B. The following equations intersect to form a triangle. Identify the vertices of the triangle.

$$x - 2y = -6 \qquad 3x + 2y = -2 \qquad 9x - 2y = 26$$

Draw one of the perpendicular bisectors in the triangle and identify the slope and point used to draw it. Then write the equation for that perpendicular bisector.

Solutions: Vertices are $(-2,2)$, $(4,5)$, and $(2,-4)$. Students should have one of the following for the point, slope, and equation:

- A. $(1,3.5)$, $m = -2$, $y = -2x + 5.5$;
 B. $(0,-1)$, $m = \frac{2}{3}$, $y = \frac{2}{3}x - 1$
 C. $(3,\frac{1}{2})$, $m = -\frac{2}{9}$, $y = -\frac{2}{9}x + \frac{7}{6}$

Activity 9: More on Angle Bisectors, Medians, and Perpendicular Bisectors of a Triangle (GLEs: 10, 19)

Materials List: pencil, paper, geometry software, computers

Have students complete this activity with a partner. They will need an automatic drawer.

1. Using an automatic drawer, such as that found in *Geometer's Sketchpad*[®], draw scalene triangle ABC and measure the lengths of \overline{AB} and \overline{AC} .
2. Construct m , the angle bisector of $\angle BAC$.
3. Construct the midpoint D and the perpendicular bisector of \overline{BC} .
4. Draw the median from point A to \overline{BC} .
5. Move point A until the angle bisector, perpendicular bisector, and the median coincide. Record the lengths of \overline{AB} and \overline{AC} .
6. Drag point A to find two other positions for point A in which angle bisector, perpendicular bisector, and the median coincide. Again, record the lengths of \overline{AB} and \overline{AC} .

Ask students to make a conjecture about $\triangle ABC$ when the angle bisector of $\angle BAC$, the median from A to \overline{BC} , and the perpendicular bisector of \overline{BC} coincide. Have students write a proof to show that if a segment is a median and an angle bisector in the same triangle, then the triangle is isosceles.

Activity 10: Proving Right Triangles Congruent (GLEs: 10, 19, 23)

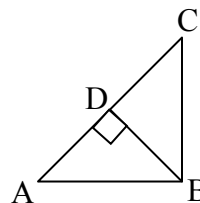
Materials List: pencil, paper

Give students diagrams of pairs of right triangles. Each pair of right triangles should have different sides (either both legs or a leg and the hypotenuse) and corresponding acute angles marked congruent. Help them to determine which pairs of the triangles are congruent and to explain why they are congruent. Then have the students write congruence proofs for these triangles using methods already discussed, like SAS, AAS, and ASA. Connect these proofs and methods to the LL, HA, and LA theorems for right triangles. Also, introduce the HL Postulate. Provide opportunities for students to write proofs for different sets of right triangles formed by perpendicular bisectors and altitudes in triangles as well.

Example:

Given: \overline{BD} is a perpendicular bisector in $\triangle ABC$

Prove: $\triangle ABD \cong \triangle CBD$



Solution: Since \overline{BD} is a perpendicular bisector, $\angle BDA$ and $\angle BDC$ are right angles because perpendicular lines form 4 right angles. That makes $\triangle ABD$ and $\triangle CBD$ right triangles. By the definition of perpendicular bisector, D is the midpoint of \overline{AC} . By the definition of midpoint, $\overline{AD} \cong \overline{CD}$. Also, $\overline{BD} \cong \overline{BD}$ because congruence of segments is reflexive. Therefore, since two pairs of legs of $\triangle ABD$ and $\triangle CBD$ are congruent, $\triangle ABD \cong \triangle CBD$ by LL.

Activity 11: Inequalities for Sides and Angles in a Triangle (GLEs: 1, 10)

Materials List: pencil, paper, Angle and Side Relationships BLM, patty paper, ruler, protractor

Have students work in groups of three. Give each student a copy of the Angle and Side Relationships BLM. In each group, have each student draw one of the following types of triangles on a sheet of patty paper: acute, obtuse, and right (they should all be scalene—if they are isosceles, the activity will not work as well). Each triangle in the group should be labeled differently. For instance, the acute triangle can be labeled $\triangle ABC$, the obtuse triangle can be labeled $\triangle DEF$, and the right triangle can be labeled $\triangle GHI$. Make sure each triangle has the group members' names on it so it can be returned after the activity. Ask groups to exchange triangles so that they do not measure their own triangles. Using a modified *story chain* ([view literacy strategy descriptions](#)), have the students in each group measure each side and each angle, then record the measurements on the Angle and Side Relationships BLM. Have students record which group the triangles came from, and the name of the triangle they measured. A *story chain* typically has a group of students create a story based on content that has already been presented. Each member of the group adds a line to the story until the story is completed. In this modification, students will take turns measuring the angles and sides of the triangles and make observations about the measurements they have taken. They will not be writing a story but they will be working together to form a conjecture/conclusion about the relationships that exist between the sides and angles of a triangle. Have students use the following procedure to use the *story chain* effectively:

- Each student should measure ONE of the angles in a triangle, record the measurement in the correct place on the Angle and Side Relationships BLM, then pass the triangle to the next group member. This step should be repeated with the remaining two angles in the triangles. (If students are in groups of three, then each student will have measured three angles.)
- After the third angle has been measured and recorded in all three triangles, each student in the group should write down all of the measurements from the group. Then each student should take one triangle and check to be sure the angle measures are sensible (i.e., the sum of the measures of the angles is 180, and angles which are acute/obtuse by sight have acute/obtuse measures). Once the measures have been deemed acceptable, the students should pass the triangles again.

- Each student should measure ONE of the sides of the triangle, record the measurement in the correct place on the Angle and Side Relationships BLM, then pass the triangle to the next group member. This step should be repeated with the remaining two sides of the triangles.
- After the third side has been measured and recorded in all three triangles, each student in the group should write down all of the measurements from the group. Then each student should take one triangle and check to be sure the side measures are sensible (check units, be sure the stated measure “looks” close).
- Have the students list the angles in order from largest to smallest to help identify the relationships between the angles and sides.
- Have each student in the group look at one triangle and make observations about the angle opposite the longest side in relation to the other two angles, as well as, the side opposite the smallest angle in relation to the other two sides. Students may need help in understanding that they are to determine whether the angle opposite the longest side is greater than or less than the remaining two angles. The same is true for the sides. Have the students write down their observation of that triangle and pass the triangles. Students should check the group members’ observations after each pass, until all three group members have looked at all three triangles.

Have students share their findings with the whole class. Students should hear that all groups have the same observation: The longest side is opposite the largest angle and the shortest side is opposite the smallest angle, and vice versa. If there are any groups that found other observations, discuss their validity and have the class help those groups realize their mistake(s) if necessary.

Activity 12: Applying Inequalities for Sides and Angles in a Triangle (GLEs: 1, 10, 16)

Materials List: pencil, paper, learning log

After completing Activity 11, have students solve problems that require an understanding of this concept. Provide students with diagrams showing triangles and their angle measures. Ask students to list the sides in order from longest to shortest or shortest to longest. Provide other diagrams showing triangles and the lengths of the sides. Have students list the angles in order from least to greatest or vice versa. Incorporate a review of algebra skills and coordinate geometry as indicated in the examples below.

Examples:

- A. Find the value of x and list the length of the sides of $\triangle ABC$ in order from shortest to longest if $m\angle A = 5x - 5^\circ$, $m\angle B = 4x^\circ$, and $m\angle C = 17x + 3^\circ$.

Solution: $x = 7$, $AC < BC < AB$

- B. $\triangle DEF$ has vertices $D(-2,1)$, $E(5,3)$, and $F(4,-3)$. List the angles in order from greatest to least.

Solution: $\angle F$, $\angle E$, $\angle D$

End this activity with *SPAWN writing* ([view literacy strategy descriptions](#)) from the *What if?* category. Give the students the prompt “What relationships between sides and angles occur if the triangle is isosceles? Do these relationships still fit with the observations you found completing this activity? Explain.” Since students have already studied isosceles triangles in Activity 1, they should realize that if the measures of the base angles are greater than the vertex angle, then the legs of the isosceles triangle are longer than the base and vice versa. This does fit with the conjectures they will make through this discovery activity. They should also understand that when listing the sides of an isosceles triangle in order by its lengths, two of the sides will need to be designated as equal, (e.g., $(AC = BC) > AC$). Students may write this as an entry in their math *learning logs* ([view literacy strategy descriptions](#)) or they may turn it in as a separate assignment.

Activity 13: The Triangle Inequality (GLE: 10)

Materials List: straws, rulers, timer, pencil, paper

Students should work in groups of two or three. Give each group a set of straws which have been cut into different lengths. First, have students measure the length of each straw. Instruct students to make as many different triangles with the segments as possible within a certain time. Have them record all trials including sets that do not work. After the activity, have a whole class discussion about which combinations of triangles will form a triangle as opposed to those that would not form a triangle. Ask students to form conjectures on how they can determine if 3 given segments will form a triangle. Provide students with 4 to 5 real-life problems in which they would need to know which combinations of segments will form a triangle. Give students two side measures and ask them to determine the range of measures for the length of the third side.

Example: Two sides of a triangle measure 4 inches and 7 inches. What is the range of measures of the third side of the triangle? *Solution:* $3 < x < 11$.

Activity 14: Similar or Not? (GLEs: 10, 17, 19, 23)

Materials List: pencil, paper

Begin by using *student questions for purposeful learning (SQPL)* ([view literacy strategy descriptions](#)). To implement this strategy teachers develop a thought-provoking statement related to the topic about to be discussed. The statement does not have to be factually true, but it should generate some level of curiosity for the students. For this activity, pose the statement “All triangles are similar.” This statement can be written on the board, projected on the overhead, or stated orally for the students to write in their notebooks.

Allow the students to ponder the statement for a moment and ask them to think of some questions they might have, related to the statement. After a minute or two, have students pair up and generate two or three questions they would like to have answered that relate to the statement. When all of the pairs have developed their questions, have one member from each pair share their questions with the class. As the questions are read aloud, write them on the board or overhead. Students should also copy these in their notebooks. When questions are repeated or are very similar to others which have already been posed, those questions should be starred or highlighted in some way. Once all of the students' questions have been shared, look over the list and determine if the teacher needs to add his/her own questions. The list should include the following questions:

- What is the definition of similar polygons?
- Can isosceles triangles be similar to scalene or equilateral triangles?
- Can acute triangles be similar to obtuse or right triangles?
- Are congruent triangles similar?
- What must be true about two triangles in order for them to be similar?
- How can you prove two triangles are similar?
- What is the proof that all triangles are similar?

At this point, be sure the students have copied all of the questions in their notebooks and continue with the lesson as follows. Tell the students to pay attention, as the material is presented, to find the answers to the questions posted on the board. Focus on those questions which have been starred or highlighted. Periodically, stop throughout the lesson to allow the student pairs to discuss which questions have been answered from the list. This may be followed with a whole class discussion so all students are sure to have the correct answers to each question.

Review with students the definition of similar figures. Use different activities which allow students to formalize the definition (i.e., corresponding angles are congruent and corresponding sides have the same ratios). For example:

- Have students construct a pair of triangles in which the corresponding angles are congruent, but the corresponding side lengths are not congruent. Have students determine the ratios of the corresponding sides of the two triangles to determine if the triangles are similar.
- Draw a triangle on a transparency and label the diagram with the measures of the angles and sides. Use an overhead projector to display the triangle's image on the chalkboard or whiteboard. Have various students measure the angles and sides of the image, and then determine the ratios of the corresponding sides. (This also works well as a teacher demonstration.)

Reinforce with students that constructing triangles with congruent angles (AA or AAA), creates similar, but not necessarily congruent, triangles. Lead a discussion about why congruent triangles are considered to be similar. Provide activities that allow students to investigate SSS and SAS similarity.

Once students have developed the definition of similar figures, provide students with diagrams that have pairs of similar triangles. Have students prove that these triangles are

similar using the SSS similarity, SAS similarity, and AA similarity. Some pairs of triangles should also be congruent.

To end the lesson, review the questions posed by the students and be sure all students have the answers to the questions for further study.

Activity 15: Conjectures about Quadrilaterals (GLE: 10)

Materials List: automatic drawer, computers (depending on type of automatic drawer), pencil, paper, Quadrilateral Process Guide BLM

Have students work in groups of two (preferred), three or four using an automatic drawer, such as found in *The Geometer's Sketchpad*[®] software. The purpose of this activity is to allow students to investigate the properties of special convex quadrilaterals.

Use a *process guide* ([view literacy strategy descriptions](#)) to help students develop conjectures about quadrilaterals. *Process guides* are used to guide students in processing new information and concepts. They are used to scaffold students' comprehension and are designed to stimulate students' thinking during and after reading. *Process guides* also help students focus on important information and ideas. In this activity, students will be given a process guide that will lead them through the steps to discover the relationships inherent in all quadrilaterals. Create a process guide by reviewing the information to be studied and by deciding how much help students will need to construct and to use meaning. Copy the Quadrilateral Process Guide BLM for each student. Make one copy for each convex quadrilateral that will be studied. The BLM provided is generic as the process will be the same, but the answers will be different for the various quadrilaterals. An example of the process to be used is provided below using a kite as the convex quadrilateral.

- Provide students with an electronic file in which a kite has been drawn. Have students measure the four angles and the four sides, then record the measures.
- Instruct students to resize the quadrilateral by dragging the vertices of the kite. Measure the angles and sides of the resized kite and record the information.
- Have students resize and make measurements until they can form conjectures about the measures of the angles and the lengths of the sides in any kite. For example, a kite has two pairs of congruent and adjacent sides. A kite has one pair of congruent angles which are formed by a pair of non-congruent sides.
- Instruct students to construct the diagonals of the kite and then answer questions relative to the behavior of the diagonals. Are diagonals perpendicular? Do the diagonals bisect the angles of the quadrilateral? Do the diagonals bisect each other?

Repeat the process using trapezoids, isosceles trapezoids, parallelograms, squares, rectangles and rhombi. Have the pairs/groups share their findings with the rest of the class. Students should be told that they are required to be able to support their statements, conjectures, and answers with evidence from their investigation with the process guide.

Have students compare the properties of the various convex quadrilaterals used in the investigation. At this point, students may wish to begin grouping the quadrilaterals based on the properties they have in common. Lead a summary discussion of the conjectures and help students to organize the results by using classifications of quadrilaterals (e.g., any quadrilateral that is a parallelogram has congruent opposite angles and supplementary consecutive angles).

Activity 16: Quadrilaterals on the Coordinate Plane (GLEs: 1, 6, 9, 16)

Materials List: pencil, paper, graph paper

Present students with sets of ordered pairs which form various quadrilaterals on the coordinate plane. For each of these sets, have students determine which quadrilateral is presented by using the distance formula, midpoint formula, and slope formula to determine which properties, if any, apply to the given quadrilateral. Students should be given time to explore this on their own, at first, to see how they would begin to determine whether the given quadrilateral is a parallelogram or not. After some time, lead a discussion about which properties might be most important to show first, in order to determine if the quadrilateral is a square. This would mean it is also a rectangle, rhombus, and parallelogram. Also, discuss which formulas would be used to show congruent segments, perpendicular segments, and that segments are bisected.

Once students have been able to determine the type of quadrilateral, have students write the equations of the lines that will produce the quadrilateral. Depending on the type of quadrilateral, some of the lines will be parallel and perpendicular, while others may not be related at all. At some point, be sure that students write the equations of the lines that form the diagonals, especially, on rectangles, rhombi, and squares.

Activity 17: The Quadrilateral Family (GLEs: 10, 23)

Materials List: pencil, paper, Quadrilateral Family BLM

Directions: Using the *graphic organizer* ([view literacy strategy descriptions](#)) provided in the Quadrilateral Family BLM, fill in the names of the quadrilaterals so that each of the following is used exactly once:

- | | |
|-----------------------|---------------|
| • PARALLELOGRAM | KITE |
| • SQUARE | QUADRILATERAL |
| • TRAPEZOID | RECTANGLE |
| • ISOSCELES TRAPEZOID | RHOMBUS |

Explanation: Following the arrows: The properties of each figure are also properties of the figure that follows it. Reversing the arrows: Every figure is also the one that precedes it.

Have students complete the *graphic organizer* given in the Quadrilateral Family BLMs, and then lead a class discussion to summarize how different quadrilaterals are related to one another. Students should be able to identify a square as being a rectangle, rhombus, parallelogram, and quadrilateral and be able to justify their reasoning.

Activity 18: Median of a Trapezoid (GLEs 10, 16)

Materials List: pencil, paper, graph paper

Discuss with students the definition of the median of a trapezoid. Give students a set of ordered pairs which form a non-isosceles trapezoid and have them graph it on the coordinate plane. Using the definition of the median of a trapezoid, have them find the endpoints of the median. Students should then find the measures of the two bases and the median. Have students verify that the median of a trapezoid is parallel to the two bases at this point, also. Once the students have found the measures of the three segments, ask them to determine if there is any relationship between the measures. Encourage them to use various operations to combine the numbers to find the relationships. In an effort to help the students, give the students more examples of trapezoids on the coordinate plane, or diagrams of trapezoids with the measures of the bases and medians already determined. This will give the students more data to investigate. In time, lead the students to the formula for determining the measure of the median of a trapezoid (half of the sum of the two bases). Relate this formula to the formula for the median of a set of data which students learn when studying measures of central tendency. Have students work problems finding the measure of the median given the measures of the two bases, and finding the measure of one base given the measure of the median and the other base. Some of these examples should include problems that require the use of algebra skills as well.

Sample Assessments

General Assessments

- The student will complete learning log entries for this unit. Grade the learning log. Topics could include:
 - Explain the statement “A square is a rectangle, but a rectangle is not a square.”
 - In an isosceles triangle, is a perpendicular bisector drawn from any vertex always the same segment as altitude and median? Explain your reasoning.
 - Suppose you have three different positive numbers arranged in order from greatest to least. Which sum is it most crucial to test to see if the numbers could be the lengths of the sides of a triangle? Explain your answer and use examples if necessary.

- Provide the student with a net which gives some of the angle measures, and a net in which other angles are labeled with variables representing the measurements of the angles. The student will find all the missing angle measures using the angle relationships learned in the unit, and will defend his/her answers by identifying the property or properties used to determine each missing value. The nets should have special quadrilaterals and other polygons embedded within the diagram so that the properties learned must be used to find some of the angle measures.
- The student will write proofs of congruent or similar triangles using information provided by the teacher. Evaluate proofs for accuracy (use of correct postulates and theorems) and completeness (not missing any steps in the reasoning process), allowing the student to use any method of proof desired.

Activity-Specific Assessments

- Activity 6: The student will complete a product assessment in which he/she designs a 5 by 5 inch tile using various types of triangles. The triangle will be correctly marked to show an understanding of methods used to determine triangle congruency. See Activity 6 Specific Assessment BLM and the Activity 6 Specific Assessment Rubric BLM.
- Activity 7: Provide the student with different triangles and have him/her draw the angle bisectors, perpendicular bisectors, medians, and altitudes for the triangles. Provide one isosceles, one obtuse, one right, and one scalene triangle and have the student draw different special segments on each. The student will draw the altitude on the right and obtuse triangles, all three special segments on the isosceles triangle (one segment should satisfy this), and the angle bisectors on any type triangle. The student will also explain the processes used to make the drawings.
- Activity 15: The student will complete a Venn diagram to demonstrate understanding of the properties of the *parallelograms* discussed in class. See Activity 15 Specific Assessment BLM. This example is only a guide and may be expanded to include trapezoids, isosceles trapezoids, and kites by drawing a larger rectangle around the Venn Diagram shown, thereby eliminating the requirement that properties numbers be shown. Isosceles trapezoids have congruent diagonals and are not parallelograms, so they would have to be drawn within a quadrilateral set and outside the parallelogram set. As a result, the congruent diagonal characteristic would need to be repeated.

Geometry

Unit 5: Similarity and Trigonometry

Time Frame: Approximately four weeks



Unit Description

This unit addresses the measurement side of the similarity relationship which is extended to the Pythagorean theorem, its converse, and their applications. The three basic trigonometric relationships are defined and applied to right triangle situations.

Student Understandings

Students apply their knowledge of similar triangles to finding the missing measures of sides of similar triangles, and to using the Pythagorean theorem to find the length of missing sides in a right triangle. The converse of the Pythagorean theorem is used to determine whether a given triangle is a right, acute, or obtuse triangle. Students can use *sine*, *cosine*, and *tangent* to find lengths of sides or measures of angles in right triangles and the relationship to similarity.

Guiding Questions

1. Can students use proportions to find the lengths of missing sides of similar triangles?
2. Can students use similar triangles and other properties to prove and apply the Pythagorean theorem and its converse?
3. Can students relate trigonometric ratio use to knowledge of similar triangles?
4. Can students use *sine*, *cosine*, and *tangent* to find the measures of missing sides or angle measures in a right triangle?

Unit 5 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Number and Number Relations	
1.	Simplify and determine the value of radical expressions (N-2-H) (N-7-H)
2.	Predict the effect of operations on real numbers (e.g., the quotient of a positive number divided by a positive number less than 1 is greater than the original dividend) (N-3-H)
3.	Define <i>sine</i> , <i>cosine</i> , and <i>tangent</i> in ratio form and calculate them using technology (N-6-H)
4.	Use ratios and proportional reasoning to solve a variety of real-life problems including similar figures and scale drawings (N-6-H) (M-4-H)
Measurement	
8.	Model and use trigonometric ratios to solve problems involving right triangles (M-4-H) (N-6-H)
Geometry	
10.	Form and test conjectures concerning geometric relationships including lines, angles, and polygons (i.e., triangles, quadrilaterals, and n -gons), with and without technology (G-1-H) (G-4-H) (G-6-H)
12.	Apply the Pythagorean theorem in both abstract and real-life settings (G-2-H)
18.	Determine angle measures and side lengths of right and similar triangles using trigonometric ratios and properties of similarity, including congruence (G-5-H) (M-4-H)
19.	Develop formal and informal proofs (e.g., Pythagorean theorem, flow charts, paragraphs) (G-6-H)
Data Analysis, Probability, and Discrete Math	
23.	Draw and justify conclusions based on the use of logic (e.g., conditional statements, converse, inverse, contrapositive) (D-8-H) (G-6-H) (N-7-H)

Sample Activities**Activity 1: Striking Similarity (GLEs: 4, 10)**

Materials List: pencil, paper, grid paper, ruler, protractor, Striking Similarity BLM

Have students work in pairs. Each pair should be given a copy of the Striking Similarity BLM. Students should also be given a piece of grid paper which has squares sized differently than the BLM (larger or smaller) but that has the same number of squares. For instance, on the BLM, the grid is 8 x 10 and each square is 1 centimeter. The students should be given a section of grid paper that also has 8 squares by 10 squares but the squares are a different size. Students will then reproduce the shapes on the blank grid by drawing the segments in the corresponding squares on the blank grid. Once students have enlarged or reduced the figures (be careful not to have the students reduce the figures too much), have students measure the segments and angles of both the original drawing on

the BLM and the new drawing. Have students participate in a discussion that describes the relationship between pairs of corresponding angles and segments in the original and enlarged/reduced figures. Remind students about the information obtained in the previous unit on corresponding sides and angles of similar triangles and have the students develop a definition for similar figures.

Have students complete a *RAFT* writing ([view literacy strategy descriptions](#)) to apply their knowledge of similar figures. This form of writing gives students the freedom to project themselves into unique roles and look at content from unique perspectives. From these roles and perspectives, *RAFT* writing is used to explain processes, describe a point of view, envision a potential job or assignment, or solve a problem. This kind of writing should be creative *and* informative.

Students should write the following *RAFT*:

R – Role—the role of the writer is a regular polygon like an equilateral triangle, a square, or some other regular polygon (the polygon can be assigned to each student or chosen by the student).

A – Audience—the regular polygon will be writing to other polygons in their family. For instance, equilateral triangles should write to scalene triangles or non-equilateral isosceles triangles; squares should write to non-square rectangles, non-square parallelograms, trapezoids, etc.; other polygons should write to non-regular polygons in their same family.

F – Form—the form of this writing is a letter

T – Topic—the focus of this writing is to explain why the regular polygon cannot be the non-regular polygon's partner because they are not similar.

In their *RAFTed* letters, students should include the definition of similar figures and an explanation of why the two figures are not similar. They can include drawings to help their explanation if they choose. Students should include these writings in their portfolios.

Activity 2: Similarity and Ratios (GLE: 4)

Materials List: pencil, paper, pattern blocks, Similarity and Ratios BLM, centimeter cubes or sugar cubes, learning logs

In groups of three or four, instruct students to use equilateral triangle pattern blocks and cubes to make generalizations about the ratios of sides, areas, and volumes in similar figures, using an activity like the one below. Give each student a copy of the Similarity and Ratios BLM so he/she can follow the directions and answer the questions that follow.

- Given an equilateral triangle, use pattern blocks to create a similar triangle so the ratio of side lengths is 2:1. If there are not enough pattern blocks for each group to create the correct triangle, have students trace the pattern blocks to create the similar triangles. Ask: “What is the ratio of areas of the two similar

triangles?” Next, have students use pattern blocks to create a triangle similar to the original triangle so the ratio of side lengths is 3:1. Ask: “What is the ratio of the areas of these two similar triangles?”

The sketches below are not included in the BLM but are provided here to illustrate what the students should be creating at their desks as they are working through the BLM.

Sample sketches:



- Have students use other pattern block shapes to investigate other similar polygons in the same manner as described above and record their findings in the tables provided on the Similarity and Ratios BLM. An example of the table is provided below for easy reference. The easiest pattern blocks to use would be parallelograms and rhombi. This can be accomplished using hexagons as well. Students can use triangles and rhombi to fill in “empty” space and to know how the area of the rectangles and rhombi relate to the area of the hexagon. Prior to completing this activity in class, “experiment” with other types of polygons to determine what obstacles students may encounter.

description of similar shapes	ratio of sides	ratio of areas
.	.	.
.	.	.
.	.	.

- Have students form generalizations based on their investigations in the two activities and have them answer the following question: If the ratio of sides of two similar polygons is $n:1$, what would the ratio of areas be?
- Given a cube, have students create a similar cube with ratio of edges 2:1 using cm or sugar cubes. What is the ratio of volumes? Then have students create a similar cube with ratio of edges 3:1. What is the ratio of volumes? Have students record their observations in the tables on the Similarity and Ratios BLM and use their observations to answer the question, “If the edges of two cubes were in a ratio of $n:1$, what would the ratio of volumes be?” An example of the table is provided on the next page.

description of similar 3-D shapes	ratio of edges	ratio of volumes
.	.	.
.	.	.
.	.	.
.	.	.

At the completion of this activity, have students answer the following prompt in their math *learning logs* ([view literacy strategy descriptions](#)):

Using what you have learned about the relationships between the ratio of the sides and the ratio of the areas of similar figures, determine the relationship between the ratio of the sides and the ratio of the perimeters of those same similar figures. Be sure to explain your reasoning and provide examples/calculations to aid your explanation.

A *learning log* is a notebook students keep in order to record ideas, questions, reactions, and new understandings. Students should use their math *learning log* other times in class in addition to those listed throughout the curriculum. This will provide opportunities to demonstrate understanding.

Activity 3: Exploring Similarity Using Scale Drawings (GLEs: 2, 4, 10)

Materials List: Internet access or printed copy of instructions, boxes (full or empty—enough to have one for each group), rulers, pencil, graph paper, scissors, tape, calculator

Visit the website

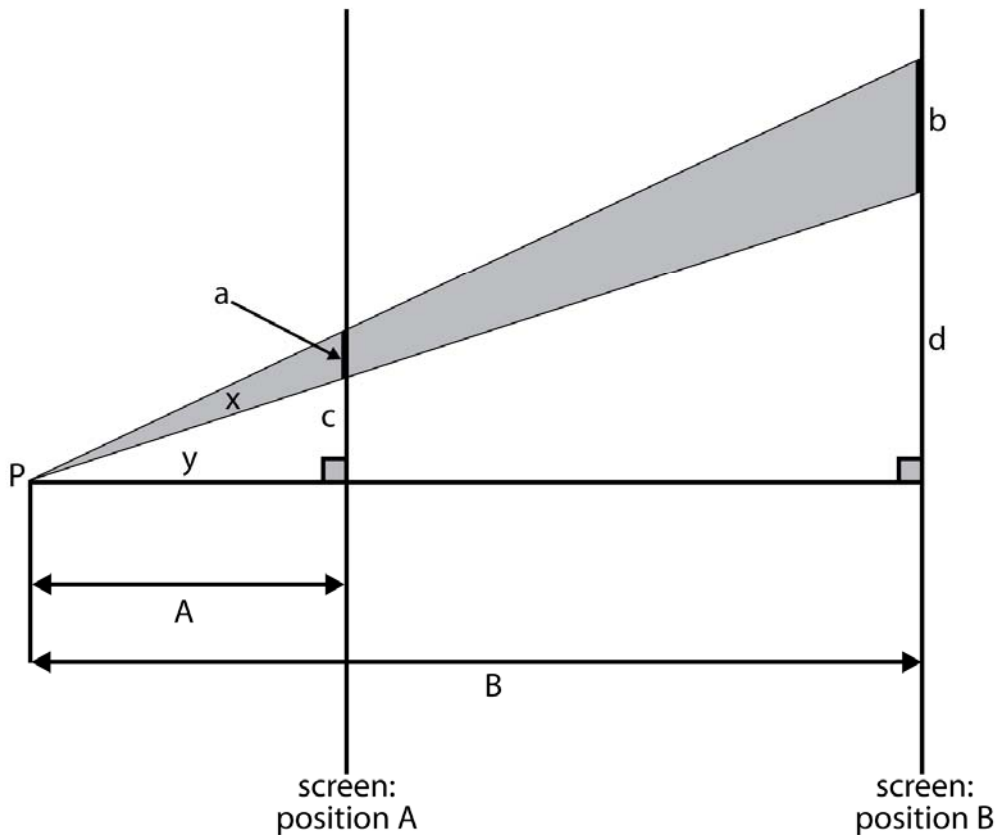
<http://ericir.syr.edu/Virtual/Lessons/Mathematics/Geometry/GEO0003.html> to access the information for this activity which allows students to use their knowledge of similar figures. There should be no need for students to access the website since the material can be printed for the class. This website provides instructions for students to create scale models applying their knowledge of similar figures. When the students are creating their scale models, they will have to decide on a different scale so that the model is not the same size as the box they were given. They will calculate the scale factor in the last step of the activity. Before the students calculate the surface area and volume of the original and scale model, have students predict what they think the surface area and volume should be based on the measurements of the two figures. This will assist students in determining if their solutions are reasonable and allows them to apply the information learned in Activity 2. Then have the students calculate the surface area and volume of the boxes, and the scale factors for the length, surface area, and volume. Since students are measuring the items themselves, help them to understand why their results may not be exactly what theory says they should be.

Activity 4: Spotlight on Similarity (GLEs: 2, 4, 19, 23)

Materials List: pencil, paper, Spotlight on Similarity BLM, overhead projector

Use the Spotlight on Similarity BLM to make a transparency for the overhead (or a copy can be made for each student) to help students investigate the following problem:

- A spotlight at point P throws out a beam of light.
- The light shines on a screen that can be moved closer to or farther from the light. The screen at position A is a horizontal distance A from the light and at position B is a horizontal distance B.
- The lengths a and b indicate the lengths of the light patch on the screen.



- Show that the ratio of the length b to the length a depends only on the distances A and B, and not on the measure of angle y of the beam to the perpendicular, nor on the measure of angle x of the beam itself.

Use similar triangle relationships, $\frac{a+c}{A} = \frac{b+d}{B}$. This means $\frac{a}{A} + \frac{c}{A} = \frac{b}{B} + \frac{d}{B}$ and $\frac{c}{A} = \frac{d}{B}$ because the triangles are similar. Hence, $\frac{a}{A} = \frac{b}{B}$, which is equivalent to $\frac{b}{a} = \frac{B}{A}$. The ratio $\frac{B}{A}$

is independent of the angles x and y and is the *scale factor* relating the distances of the two screens and the sizes of the images on the two screens.

Once students develop an understanding of the term scale factor, give students the measurements of certain figures and a scale factor. Have them predict whether the new figure is going to be larger or smaller than the given measures, and then check themselves by finding the actual measures using proportions.

Activity 5: Applying Similar Figures (GLEs: 2, 4, 18)

Materials List: pencil, paper, calculator

Give students various real-life situations in which similar triangles are used to find missing measures (i.e. shadow problems, distance across a river, width of a lake). The types of triangles should vary. Have students discuss why the triangles are similar before finding the requested missing measures. Provide students with practice in determining the missing sides of other pairs of similar figures in real-life settings.

Example:

Alex is having a snapshot of his grandparents enlarged. The original snapshot is 4 inches by 6 inches. He needs the enlarged photo to be at least 13 inches on the shortest side. What must the minimum length be of the longer side?

Solution: 19.5 in.

Activity 6: Parts of Similar Triangles (GLEs: 2, 4, 10)

Materials List: pencil, paper, automatic drawing program (optional), ruler, protractor

Students should investigate how the lengths of the special segments (altitude, median, angle bisector) in similar triangles relate to the measures of the sides of the similar triangles. Have them construct similar triangles, either with a drawing program or by hand, and draw the altitudes, angle bisectors, and medians. Instruct students to determine the scale factors of the sides and compare them to the ratios of the special segments. Have students refer to their math *learning log* entries relating to the ratio of the perimeters of the similar triangles from Activity 2. Lead a class discussion to summarize that the ratios of the sides, altitudes, medians, angle bisectors, medians, and perimeters in similar figures are equal.

Activity 7: Midsegment Theorem for Triangles (GLEs: 4, 10, 18, 19)

Materials List: pencil, tracing or patty paper, scissors, ruler, protractors

Separate the class into groups of four. Give each group a sheet of tracing paper (or patty paper) and have each draw a triangle of any type and cut it out. Have students:

1. Find the midpoints of any two sides of the triangle by folding.
2. Fold (or draw) the segment that connects the two midpoints. Tell students that this is the midsegment and have them define the term based on what they have done so far.
3. Unfold the triangle and make any observations that seem to be true about the triangle and the midsegment. Students should use rulers and protractors to verify the observations they make concerning the midsegment of the triangle.
4. Fold and unfold the remaining two midsegments of the triangle.
5. Have students make observations regarding the three midsegments of the triangle. Ask them to look for a geometrical relationship between the midsegments and the sides, and to determine the numerical relationship between the lengths of the midsegments and the lengths of the sides. Each midsegment is parallel to one side of the triangle, and each midsegment has a length that is one-half the length of the parallel side.

If there is access to a computer drawing program such as *Geometer's Sketchpad*[®], use it to construct the midsegments of several other triangles to determine if the observations made in the part above hold true.

Lead the class in a discussion which includes the use of similar triangles to prove the Midsegment theorem (the segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half its length). Ask students to discuss the relationship of the triangle formed by the three midsegments to the original triangle (i.e., the inner triangle is similar to the original and its perimeter is half the perimeter of the original triangle).

Activity 8: Math Masters (GLEs: 4, 10)

Materials List: expert attire (optional), pencil, paper

Use a modified *professor know-it-all* ([view literacy strategy descriptions](#)) strategy to review all of the concepts taught concerning similar figures. The *professor know-it-all* strategy allows students to question “experts” concerning a topic that has been studied through reading from a text, a lecture, a field trip, or any other information source. The only modification to the strategy is that students could be called “Math Masters” rather than *professor know-it-alls* as high school students might be more receptive to the name. To implement the strategy, divide the class into groups of three or four. Give the students time to review the material covered in the last seven activities concerning similar figures. Tell the students, groups will be called on randomly to come to the front of the room and

provide “expert” answers to questions from their peers about similar figures. Ask the groups to generate 3 – 5 questions about similar figures they think they might be asked or that they would like to ask other experts. Provide novelty items like ties, graduation caps, lab coats, clipboards, etc., to don when the students are the Math Masters.

After giving the students time to review material and create their questions, call a group to the front of the room and ask its members to face the class standing shoulder to shoulder. The Math Masters invite questions from the other groups. With the first question, model how the Math Masters should answer their peers’ questions. Students should huddle together after each question to discuss and decide upon the answer, then have the spokesperson give the answer.

Direct the students to think carefully about the answer they receive and to challenge or correct the Math Masters if the answers are not correct or need additional information. After 5 minutes or so, have a new group take its place as Math Masters and continue the process.

Some questions that might be asked are:

- What is the definition of similar figures?
- What information must be provided to prove that two triangles are similar?
- Are all triangles similar?
- Are all quadrilaterals similar?
- Are squares similar to rectangles?
- How do the areas of similar figures relate to the scale factor of the figures?
- How do the perimeters of similar figures relate to the scale factor of the figures?
- How do the volumes of similar solids relate to the scale factor of the given solids?
- How does the measure of the midsegment of a triangle relate to the measure of the side?

The activity is complete when all groups have had a chance to be Math Masters or the peers have no new questions to ask the experts.

Activity 9: Pythagorean Theorem (GLEs: 12, 19)

Materials List: pencil, paper

Provide students with a pair of similar right triangles whose leg measures are known. Ask students to determine if the triangles are similar and, if so, to provide a proof (i.e., the right angles are congruent and the legs in the two triangles are proportional) to support their ideas. Have students calculate the length of the hypotenuse of each triangle. Ask: “What is the scale factor between the two similar triangles? Is the hypotenuse of one triangle a multiple of the hypotenuse of the second triangle? What is the multiple?” Students should recognize and use common Pythagorean triples (e.g., 3-4-5, 5-12-13, 7-24-25, 8-15-17) and their multiples as shortcuts to solving problems. For example, if a

right triangle has lengths of 15, _____, 39, the missing side is 36 since 15, 36, 39 is three times 5-12-13.

Activity 10: Application of the Converse of the Pythagorean Theorem (GLEs: 10, 12)

Materials List: pencil, paper, automatic drawing program, protractors (if drawing program is unavailable)

In this activity, students should apply the converse of the Pythagorean theorem to determine if a triangle is right, acute, or obtuse. Begin by reviewing the converse of the Pythagorean theorem as a method of determining whether three segment measures could represent the measures of a right triangle. Then, give the students several different sets of measures that form a triangle (be sure that most of them are NOT right triangles). Have students apply the converse of the Pythagorean theorem to determine which of the trios forms a right triangle. Using a computer drawing program like *Geometer's Sketchpad*, have students construct triangles using side lengths that do not form right triangles, to determine that some triangles are acute while others are obtuse. Have students make a conjecture about the sum of the squares of the smaller sides in relation to the square of the largest side, in both acute and obtuse triangles. Students should explain that if $a^2 + b^2 < c^2$ then the triangle is obtuse and if $a^2 + b^2 > c^2$, then the triangle is acute. Ask students to classify other triangles based only on the lengths of their sides.

If a drawing program is not available, provide students with diagrams in which the triangles have been drawn to scale and the lengths of sides are labeled. Have students apply the Pythagorean theorem (or the rules concerning Pythagorean triples) to determine which triangles are right triangles. For the remaining triangles, have students use a protractor to measure angles, classify each triangle as acute or obtuse, and then determine the relationship between $a^2 + b^2$ and c^2 in the two types of triangles.

Activity 12: Discovering Trigonometry (Using Technology) (GLEs: 3, 8, 12)

Materials List: Internet access for each student (or pairs of students), pencil, paper

The website, <http://catcode.com/trig/index.html>, provides a series of activities that define and help explain the uses of trigonometry. The activities help students to expand their understandings of similar figures, as they apply to the study of trigonometry. Only the first five activities and the activity titled "A Quick Review" should be viewed. Be sure that the computers students will be working on have Java enabled, so students can use the interactive activities. This cannot be printed because the interactivity with the figures will be lost. If necessary, students may be paired depending on class size and the number of available computers. If students do not have access to the Internet, present this information using presentation equipment. Create notes from the information presented on the website which students will be able to use for the remainder of the activity.

Employ a *directed reading-thinking activity (DRTA)* ([view literacy strategy descriptions](#)) to aid students in reading and processing the information presented in the website. The *DRTA* approach invites students to make predictions and check their predictions through the reading. It requires students to pause as they read the information to ask/answer questions. Take the students through the following steps:

- *Introduce background knowledge.* Begin the lesson with a discussion about trigonometry. Elicit information students may already know about trigonometry. Many students may have limited prior knowledge of trigonometry, and that is okay. Discuss the title of the activity. Record students' ideas on the board or chart paper.
- *Make predictions.* Ask questions that invite predictions, such as: "What do you expect to learn from this activity? Based on what we have learned already, what information do you think the author will include?" Have students write their predictions in their notebooks.
- *Read a section of text, stopping at predetermined places to check and revise predictions.* The first stopping point may be after students read the Frequently Asked Questions About Trigonometry. Students should reread their predictions and change them if they feel it is necessary. If they decide to change their predictions, they should cite the new evidence for doing so. Repeat this cycle as the students read through the information on trigonometry. Other recommended "stopping" points are after Shadows and Triangle (students should click on "See the difference here" to understand the effect of the moving sun on shadows), Measuring the Sides, sine and cosine, and A Quick Review (instruct the students to go back to the Index and click on A Quick Review to skip the other information at this time—the remaining information is too much to include at this point). Have students consider the following key questions: "What have you learned so far from the text?" (summarize) "Can you support your summary with evidence from the text?" "What do you expect to read next?"
- *Once the reading is completed, use student predictions as a discussion tool.* Ask students to reflect on their original predictions and to track their changes in thinking and understanding trigonometry, as they confirmed or revised their predictions. Students should write their statements of overall understanding in their notebooks.

Activity 13: Special Right Triangles (GLEs: 1, 3, 10, 12, 18)

Materials List: pencil, paper, construction materials (compass and straight edge or patty paper), scientific calculator

Have students explore special right triangles by starting with an equilateral triangle with side lengths of 2 units. Have students construct an altitude to create two 30° - 60° - 90° triangles. Identify the parts of the 30° - 60° - 90° triangle as short leg, long leg, and hypotenuse. The resulting right triangles have a short leg of 1 unit (remind students that the altitude of an equilateral triangle is also a median which is why the right triangles

have a short leg of 1 unit). A 30° - 60° - 90° triangle whose short leg is 1 is called the unit triangle. Have students use the Pythagorean theorem to calculate the length of the long leg (side opposite 60°) in simplified radical form. A review of simplifying radicals may be necessary.

Next, have students create a unit triangle for 45° - 45° - 90° , using 1 unit as the length of each of the two legs. Using the Pythagorean theorem, students will calculate the length of the hypotenuse in simplified radical form.

Repeat the activity several times, but use different measures for the sides of the equilateral triangle. Start with equilateral triangles whose sides are 4 units, and then 6 units and also use the isosceles right triangle. Do this several times until students see a pattern in the numbers. The goal is to have students write these as formulas:

short leg = $\frac{1}{2} \times$ *hypotenuse* and *long leg* = $\sqrt{3} \times$ *short leg* in 30° - 60° - 90° triangles. For 45° - 45° - 90° triangles, the relationship is *hypotenuse* = $\sqrt{2} \times$ *leg*. Additionally, show students how proportions are an alternative way of calculating the same values.

To help students become familiar with the definition of *sine* and *cosine*, have them calculate the ratios using the side lengths of special right triangles. Have students use the examples from the above activity and the definition of *sine* to determine that $\sin(30) = \frac{1}{2}$. Allow them to use the calculator's *sin* function to verify this. This step may require instruction on the use of the calculator. Help students to understand that formulas, proportions, and trig functions are related to each other, and that each is a different way to write the ratios that exist. See that students become familiar with the idea that trigonometric functions represent ratios of sides in a right triangle.

Activity 14: Trigonometry (GLEs: 2, 3, 8, 12, 18)

Materials List: pencil, paper, trig tables, scientific calculators (minimum)

Extend Activity 13 to define the *cosine* and *tangent* ratios in right triangles. Be sure to include information which shows students how to find the measures of the acute angles in a right triangle if the side lengths are known. Assist students in learning to use the calculator to find trig ratios and to use the ratios to solve problems. In order to facilitate understanding, have students read information from a standard trig table. For example, in order to solve $\tan x = \frac{12}{17}$, students need to understand that there is one angle which has the same decimal ratio as 12 divided by 17. Looking through the list of tangent ratios to find this number helps students understand that the calculator has these ratios stored in its memory. When the student requests $\tan^{-1}\left(\frac{12}{17}\right)$, he/she is requesting the calculator to search for the angle whose ratio is the same as 12 divided by 17.

Have students practice finding the measures of missing sides and angles by applying the trigonometric ratios to right triangles. Once an understanding of the process is mastered, have students apply the trigonometric ratios to real-life problems. These problems can be finding the differences between the heights of two buildings, distance two boats are apart from each other, construction of airplanes, angles of elevation or depression, etc.

Sample Assessments

General Assessments

- The student will create a portfolio containing samples of his/her activities. For instance, the student could choose a particular drawing from class and enlarge it using a given scale. In this entry he/she would also explain the process and how to prove that the new drawing is similar to the given drawing.
- The student will complete math *learning log* entries for this unit. For example:
 - Discuss the proof for the special right triangles: 30° - 60° - 90° and 45° - 45° - 90° . In your discussion, explain why this information can be generalized to all triangles that have these angle measures.
 - Explain how the Pythagorean theorem can be used to determine if a triangle is a right, obtuse, or acute triangle.
- The student will find pictures of similar figures in magazines, newspapers, or other publications and will explain how he/she knows that the figures are similar. The teacher will challenge the student to find pairs of similar figures that are not congruent.

Activity-Specific Assessments

- Activity 1: Give each student a floor plan for a house. The floor plan should not have any measurements on it. The student will enlarge the floor plan to the size of a poster using a given scale. The student will find the actual dimensions of the rooms and the dimensions of the entire house from the scale used to create the floor plan.
- Activity 5: Provide instructions for making and using a hypsometer given on the Making a Hypsometer BLM. The student will write a rationale for the proportion that is given in the instructions. The student will determine the height of various objects throughout school showing all calculations necessary to indirectly find the height of the chosen object.
- Activity 12: The student will use a clinometer to determine the height of something on the grounds of the school (e.g., flag pole, light post, goal post) using the trigonometric functions. The student will produce a scaled diagram of the measurements made and show all calculations used to indirectly

calculate the height of the chosen object using trig functions. Instructions for making a clinometer can be found in most geometry textbooks and on numerous websites, such as:

<http://web4j1.lane.edu/partners/eweb/ttr/mckenzie/resources/ideabank/clin.html>.

Geometry

Unit 6: Area, Polyhedra, Surface Area, and Volume

Time Frame: Approximately five weeks



Unit Description

This unit provides an examination of properties of measurement in geometry. While students are familiar with the area, surface area, and volume formulas from previous work, this unit provides justifications and extensions of students' previous work. Significant emphasis is given to 3-dimensional figures and their decomposition for surface area and volume considerations.

Student Understandings

Students understand that measurement is a choice of unit, an application of that unit to the object to be measured, a counting of the units, and a reporting of the measurement. Students should have a solid understanding of polygons and polyhedra, the meaning of regular, the meaning of parallel and perpendicular in 3-dimensional space, and the reason pyramids and cones have a factor of $\frac{1}{3}$ in their formulas.

Guiding Questions

1. Can students find the perimeters and areas of triangles, standard quadrilaterals, and regular polygons, as well as irregular figures for which sufficient information is provided?
2. Can students provide arguments for the validity of the standard planar area formulas?
3. Can students define and provide justifications for polygonal and polyhedral relationships involving parallel bases and perpendicular altitudes and the overall general $V = Bh$ formula, where B is the area of the base?
4. Can students use the surface area and volume formulas for rectangular solids, prisms, pyramids, and cones?
5. Can students find distances in 3-dimensional space for rectangular solids using generalizations of the Pythagorean theorem?
6. Can students use area models to substantiate the calculations for conditional/geometric probability arguments?

Unit 6 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Measurement	
7.	Find the volume and surface area of pyramids, spheres, and cones (M-3-H) (M-4-H)
Geometry	
9.	Construct 2- and 3-dimensional figures when given the name, description, or attributes, with and without technology (G-1-H)
10.	Form and test conjectures concerning geometric relationships including lines, angles, and polygons (i.e., triangles, quadrilaterals, and n -gons), with and without technology (G-1-H) (G-4-H) (G-6-H)
12.	Apply the Pythagorean theorem in both abstract and real-life settings (G-2-H)
18.	Determine angle measures and side lengths of right and similar triangles using trigonometric ratios and properties of similarity, including congruence (G-5-H) (M-4-H)
21.	Determine the probability of conditional and multiple events, including mutually and non-mutually exclusive events (D-4-H) (D-5-H)

Sample Activities**Activity 1: Why Does That Formula Work? (GLE: 10)**

Materials List: grid paper, pencil

Teacher Note: Students should be able to find the area of basic figures such as rectangles, triangles, and trapezoids through work in previous grades. This activity explores how the formulas were derived and introduces a formula for the area of a rhombus.

Have students investigate why the rectangle area formula is base times height. The easiest way to have students see this is to use grid paper and count the squares. Have them investigate a parallelogram that is not a rectangle or a square, recognizing that the formula for the area of a rectangle is the same formula as the area of a parallelogram. Have students review the definition of distance between two lines to understand why the height of a general parallelogram is different than the measure of the sides. Emphasize that although students have always learned that the area of a rectangle is length times width, the better interpretation is base times height since a rectangle is a special type of parallelogram. This makes the formula applicable to any quadrilateral that is a parallelogram. For most students this will be only a matter of changing the variables from prior knowledge and recognition that height is a measurement taken along a segment which is perpendicular to the base.

After investigating the parallelogram, have students examine the formulas for other plane figures – triangles, trapezoids, and rhombi. Have students explain how each formula is derived from other formulas, as well, as how to apply them to various problem situations.

Review with students the fact that the diagonals of a rhombus are perpendicular and bisect each other. Have students generate the formula for finding the area of a rhombus using the lengths of the diagonals ($A = \frac{1}{2}d_1d_2$). Lead a discussion in which students have the opportunity to consider other quadrilaterals whose areas could be found using the same formula (i.e., squares and kites). This discussion should be based upon the characteristics of the diagonals that were investigated in the unit on quadrilaterals.

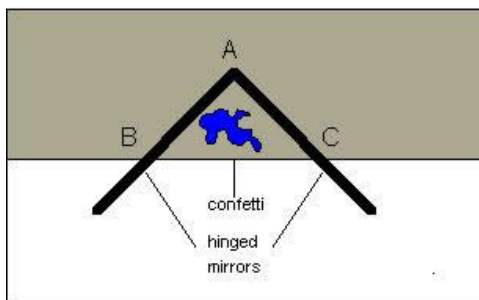
Activity 2: Area of Regular Polygons (GLEs: 7, 9, 10, 12, 18)

Materials List: hinged mirrors, protractors, rulers, pencil, paper, compass

Teacher Note: While GLE 7 refers to the volume and surface area, the area of regular polygons should be reviewed to assist students in finding the volume and surface area of polyhedra with bases that are regular polygons.

In this activity students use hinged mirrors, protractors, and rulers to draw regular polygons and to investigate the measures of their central angles. This will be pivotal to helping find the lengths of apothems that are not given when finding the area of regular polygons. A similar activity with activity sheets is available at http://illuminations.nctm.org/index_d.aspx?id=379.

1. Have students draw a line on a plain sheet of paper. Position the hinged mirror so that the sides of the mirror intersect the line at two points that are equal distances from the hinge of the mirror.



2. Remind students, to sketch what they see and then place the protractor on top of the mirror to determine the angle of the mirror. Record this measurement; this is the measure of a central angle of the polygon that they see in the mirror.
3. Have the students open the mirror wider; students must make sure that the mirror intersects the line at equal distances from the hinge. They should observe what happens to the figure, sketch the new figure, and record the new angle measurement.

4. Have students open or close the mirror until a regular hexagon is formed and record the angle of the mirror. Then students should answer the following questions about the six-sided figure:
 - a. What is the sum of the angle measures for those angles whose vertex is the mirror's hinge?
 - b. How many angles are there in the image for each number of sides, and what is the measure of each of those angles?
 - c. How does the measure of the hinged mirror compare to the measure you calculated in b above?
5. Have students use the hinged mirror to find the central angle for the following regular polygons: triangle, square, pentagon, octagon, decagon, and dodecagon.

Direct students to construct a circle using a compass and to inscribe a regular hexagon in the circle by making 6 congruent arcs along the circle. Have students divide the hexagon into six congruent triangles and write the formula for the area of the hexagon. Lead a class discussion to determine that students were able to generate the formula as $A = 6\left(\frac{1}{2}sa\right)$, where s is the measure of one side of the hexagon, and a is the height of the triangle, or the apothem of the hexagon. Ask students to consider the meaning of $6s$ as this expression relates to the hexagon. Once it has been established that $6s$ is the perimeter of the hexagon, rewrite the formula for the area of a hexagon as $A = \frac{1}{2}Pa$, where A is the area of the polygon, P is the perimeter of the polygon, and a is the apothem of the polygon. This formula can be used for all regular polygons.

Have students practice calculating the area of other regular polygons. Provide diagrams with labeled measurements or provide real objects in which students can measure the parts needed for the formula. Provide instances in which students must use special right triangles or trigonometric ratios to find the apothem or the side measure of the polygon. Students can find the length of the apothem of a regular polygon by using trigonometry. The formula for the length of the apothem of a regular polygon is

$a = \frac{1}{2}s \tan\left(\frac{90^\circ(n-2)}{n}\right)$, where s is the length of the side of the polygon and n is the number of sides of the polygon.

Activity 3: Experiment with Volume (GLEs: 9, 10)

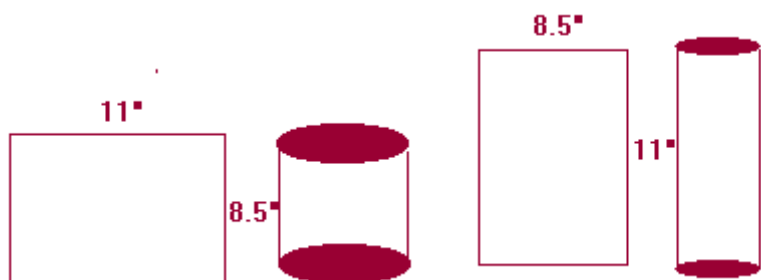
Materials List: paper, pencil, stiff paper, cardstock, transparency sheets, shallow box, rice or other filler

Teacher Note: This activity relates to the construction of 3-D figures as stated in GLE 9; however, it is also included as a review of the volume concepts mastered in grade 8.

Begin this activity by having students complete modified SPAWN writing ([view literacy strategy descriptions](#)). SPAWN is an acronym that stands for five categories of writing options, *Special Powers*, *Problem Solving*, *Alternative Viewpoints*, *What If*, and *Next*.

Using these categories, create thought-provoking prompts related to the topic. In this case, students should be given a prompt which will cause them to predict the relationship between two cylinders with different bases and heights. This is modified because it does not ask students to predict what will happen next, but having them make the prediction does fall under the *Next* category.

To present the writing topic take two sheets of paper and create two baseless cylinders like the ones shown below.



Present the cylinders to the class and have them write a few sentences to answer the following topic from the *Next* category (post the topic on the board or the overhead):

Next: Based on your prior knowledge of volume, predict whether you believe the volumes of the two cylinders are equal or whether the shorter or taller cylinder has the greater volume. Explain why you predicted as you did.

Do not have the students calculate the volume at this step—their predictions should be based solely on their prior knowledge and their observations.

After giving the students time to write their responses, have students share their predictions. Have students pair up and discuss their predictions, then have the pairs join another pair to form a group of four. In these groups of four, the students should discuss their predictions and their reasons for their predictions. After a few minutes of discussion, have groups report their predictions to the class. Then, perform a demonstration for the students (or have the students perform the demonstration by giving them the necessary materials) by taping two sheets of paper to form the two cylinders, one short and one tall. (Stiff paper is helpful. Transparency sheets may be used). Hold the tall cylinder upright in a shallow box and fill with rice. Now fill the shorter cylinder and compare the two amounts of rice. Have the students determine whether their predictions were correct.

Now, have the students calculate the volumes of both cylinders. Lead the students in a discussion as to why the volumes are not the same even though the numbers are the same. Include in the discussion, a study of the areas of the bases and a determination of the effects of squaring the radius when calculating the areas of those bases.

To conclude the lesson, present other *SPAWN* prompts which challenge the students to reflect on or think more critically about what they have just learned:

What If? Given a cylinder which has a base with a diameter of 10 inches and a height of 12 inches, what would happen to the height of the cylinder if the volume was to remain the same and the base was increased by 50%? What would happen to the base of the cylinder if the volume was to remain the same and the height was increased by 75%?

Once students complete their writings, have students share their ideas with a partner, then a group, and then the entire class as they did after the *Next* prompt. These discussions should include their calculations of volume and the new measures for the height and base.

Activity 4: Cube Coloring Problem (GLEs: 9, 10)

Materials List: a large quantity of unit cubes (sugar cubes can be used), graph paper, colored pencils or markers, pencils, learning log

Teacher Note: This activity provides a review of surface area and volume of prisms mastered in grade 8.

In this activity, students will investigate what happens when different sized cubes are constructed from unit cubes, the surface areas are painted, and the large cubes are taken apart.

- Hold up a unit cube. Tell students this is a cube on its first birthday. Ask students to describe the cube (eight corners, six faces, twelve edges). Find the cube's total surface area and volume.
- Ask student groups to build a "cube" on its second birthday, that is, to double the length of each side of the cube. Ask the students to describe it in writing. Find the surface area and volume.
- Ask students how many unit cubes it will take to build a cube on its third birthday, fourth, fifth, etc through the cube's tenth birthday. Find the surface area and volume on each of the cube's birthdays. (*On the cube's third birthday, the cube would be $3 \times 3 \times 3$, on the fourth $4 \times 4 \times 4$, etc.*)
- Pose this coloring problem: The cube is ten years old. It is dipped into a bucket of paint. After it dries, the ten-year-old cube is taken apart into the unit cubes. How many cubes are painted on three faces, two faces, one face, no face? (If using sugar cubes, students can make a model and place a dot on each exposed side with a marker.)
- Have students chart their findings, including surface area and volume, for each age cube up to ten and look for patterns.
- Have students use exponents to write the number of unit cubes needed to make a larger cube. Expand this to the number of cubes painted on three faces, two faces, one face, or no face.
- Have students graph the findings for each dimension of cube up to ten and look for the graph patterns.

Students will note that the three painted faces are always the corners—eight on a cube. The cubes with two faces painted occur on the edges between the corner and increase by twelve each time. The cubes with one painted face occur as squares on the six faces of the original cube. The cubes with no painted faces are the cubes within the cube. As an entry in the students' math *learning logs* ([view literacy strategy descriptions](#)), have the students explain the patterns they have observed. Also, have the students explain how they believe the surface area and volume formulas for a cube were developed based on their findings in this activity. A *learning log* is typically a notebook a student keeps in order to record ideas, questions, and new understandings. Students should keep this as a separate notebook or as a separate section in their binders (if binders are used). Students should use their math *learning logs* other times in class in addition to those listed throughout the curriculum to provide opportunities to assess understanding.

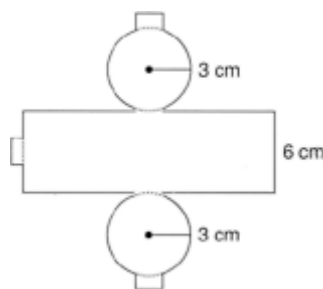
To end the activity, have students complete a *SPAWN writing* ([view literacy strategy descriptions](#)) from the *What If?* category. Post the following prompt by writing it on the board or on the overhead: “How would this activity change if rectangular solids which are NOT cubes were created with the sugar cubes? For instance, how would the information change if the width and height were the same but the length was one more than the width ($1 \times 1 \times 2$, $2 \times 2 \times 3$, $3 \times 3 \times 4$, etc.).” Have students discuss their ideas in groups of four concentrating on their calculations and any patterns they have observed. Then have the students report as a whole class and discuss the patterns they found and how they differ from the cube problem posed in the activity.

Activity 5: Cylinder in 3-D (GLEs: 9, 10)

Materials List: compass, scissors, tape, metric ruler, pencil, paper, thin cardboard (optional)

Teacher Note: This activity also provides a review of a surface area formula mastered in grade 8.

Using groups of two, have students draw the figure shown below on a sheet of paper or thin cardboard, using the indicated measurements. Cut out this net and then fold and tape the edges to form a cylinder.



Have students go back to Activity 3 and use the formula for the volume of a cylinder to find the volume of the cylinder they have constructed. Have students generalize the

measurements (identify the radius and height) to generate the general formula for finding the volume. Have students relate the formula for the volume of the cylinder to the volume formula for various polyhedra $V = Bh$ where V is the volume of the polyhedron, B is the area of the base, and h is the perpendicular height of the polyhedron. Also, students should generate the general formula for finding the surface area of a cylinder. Students should remember that the surface area of any three-dimensional object is the sum of the area of its bases and lateral faces. While a cylinder does not fit the definition of a polyhedron, the general understandings for volume and surface area still apply.

Activity 6: Building a Pyramid (GLEs: 9, 10, 12)

Materials List: two 8.5" x 11" sheets of paper, two pairs of scissors, two rulers, tape for each pair of students; for each student—pencil, calculator

Have students work in pairs and use the materials provided to construct a square pyramid which has a height of 7 cm and a base length of 4 cm. Each of the students is to make a pyramid, but the use of pairs allows them to think through the process together. Remind students that they should use the Pythagorean theorem to figure the slant height (height of the triangle for one of the sides) in order to construct the faces of the pyramid.

The most expedient way to build the pyramid is to make a net consisting of a square with an isosceles triangle drawn on each side. The isosceles triangle should be constructed so that the height of the triangle is the calculated length of the pyramid's slant height. Students who have trouble visualizing the net may use other methods, such as drawing each side individually and taping them all together. It may be a good idea to make two paper models in advance of the activity and then dismantle one of the pyramids to show what the net looks like. If necessary, lead a discussion of the location of each measurement on the net in comparison to the 3-dimensional pyramid.

Have students discuss ways they might find the volume of the pyramid. This discussion should include the understanding that the volume of a polyhedron, in general, is $V = Bh$. However, students should realize that this will not apply to a square pyramid because the pyramid is only a part of a cube or rectangular solid (which is the volume that would be found if the general formula is used). The students may not be able to find the volume at this point—it will be discussed more in Activity 8. Also lead the students in a discussion about the information that would be necessary to find the unknown height of a pyramid (or possibly an unknown base length of the pyramid if the height were given). Ask students to apply this knowledge by creating another pyramid with a different regular polygon as the base.

Activity 7: Surface Area (GLE: 7)

Materials List: models of pyramids from Activity 6, pencil, paper

Using their models from Activity 6, have students determine the total surface area of their constructed pyramids and describe the process of finding the surface area of the pyramids. Students should find the surface area of the pyramids with square bases and other regular polygon bases as well.

Activity 8: Volumes of Pyramids and Cones (GLE: 7)

Materials List: volume model kit(s), rice or other filler, pencil, paper

Compare the volumes of a pyramid and prism with the same base and height as a demonstration using a volume model kit. If enough model kits are available, have students work in groups of 2 or 3 when completing the activity. Students could also make their own models using old manila file folders and then complete the activity.

Fill the pyramid from the kit with rice or unpopped popcorn. Ask students to estimate how many times the pyramid must be filled in order for the prism to be filled. Do the same thing with a cone and cylinder. Develop the concept that the volume of a cone (pyramid) is one-third the volume of a cylinder (prism) if the two solids have the congruent heights and bases. As an extension, ask students to estimate the relationship between the cone and the sphere which are also a part of the kit. Since the cone must be filled twice before the sphere is filled, the sphere is twice as large as the cone's volume or $\frac{2}{3}$ the volume of the cylinder. Provide real-life applications in which students must find the volumes of cones, pyramids, prisms, and spheres. Students will have to be given the actual formula for the volume of a sphere in order to accurately find the volume of spheres.

Activity 9: More with Volume and Surface Area (GLE: 7)

Materials List: pencil, paper

Have students review the processes for finding the surface area of prisms and pyramids. They should generalize that to find the total surface area of a prism or pyramid, they need to find the sum of all the areas of the lateral faces and the base(s). The volume of the prisms can be generalized as the product of the area of the base and the height of the prism ($V = Bh$). The volume of the pyramids should be generalized as finding $\frac{1}{3}$ of the product of the area of the base and the height of the pyramid.

After these generalizations are made, have students practice finding the surface area and volume of prisms and pyramids with regular polygons as their bases.

Activity 10: Volume of Irregular Objects (GLE: 7)

Materials List: cylindrical container, water, irregular objects (like an egg-shaped paperweight), tube of toothpaste (with the box), pencil, paper, calculator

Teacher Note: Although GLE 7 refers to only the volume of pyramids, cones, and spheres, this activity gives students another opportunity to determine the volume of an object for which no formula exists.

Provide students with a cylindrical object whose volume can be calculated and with markings to measure a predetermined amount of water (a beaker from a science class would do well). Ask them to place water in the beaker but not to fill it to the top. Discuss with the students the volume of water in the beaker. Ask them to place an irregular object, like an egg-shaped paperweight, into the water, being careful not to spill any water. Note the displacement of the water and determine the volume of the paperweight.

Next, repeat the activity with a tube of toothpaste. Have the box for the tube on hand as well. After students determine the volume of the tube of toothpaste, have them determine the volume of the box. Instruct students to determine what percent of the box is used by the toothpaste and what percent is empty space. Use this activity for a discussion about packaging efficiency.

Activity 11: Geometric Probability (GLE 21)

Materials List: pencil, paper, calculator

Students should be given problems that require them to find the area of a variety of shapes. This should include basic figures as well as figures within other figures, or combinations of figures. Ask students to find the probability of randomly selecting a point in a shaded region of the given figure.

Example:

Mark created a game consisting of 32 squares on a rectangular game board. The board measures 1-foot by 2-feet. 16 of the squares are 3-inches by 3-inches while the other 16 squares are 2-inches by 2-inches. He earns 3 points for hitting the board and not hitting a square, 5 points for hitting one of the larger squares, 10 points for hitting one of the smaller squares. What is the probability that he will earn 10 points with one throw of a dart? *Solution:* $\frac{64}{288} = \frac{2}{9} = 22\%$.

Ask students to apply geometric probability using the Length Probability Postulate—the probability of a point lying on a smaller portion of a segment is equal to the length of the smaller portion divided by the length of the entire segment.

Example:

A radio station will play the song of the day once during each hour. The 101st caller will win \$100. If you turn on the radio at 2:35 p.m., what is the probability that you have missed the start of the song during the 2:00 p.m. to 3:00 p.m. hour?

Solution: $\frac{35}{60} = \frac{7}{12} = 58\%$.

Sample Assessments

Performance and other types of assessments can be used to ascertain student achievement. Examples include:

General Assessments

- The student will complete learning log entries for this unit. Suggested topics include:
 - Explain why pyramids and cones have $\frac{1}{3}$ as a factor in their formulas for volume.
 - Show how to find the volume and surface area of a solid that combines a cylinder with a cone, or a prism with a pyramid. Be specific.
- Provide the student with three-dimensional models. The student will sketch diagrams and take appropriate measurements from actual objects needed to calculate volume and surface area. The student will label sketches with measurements and then show the process used to calculate volume and surface area. Since this task is time consuming, the student will be given no more than three objects, some type of prism, either a cone or pyramid, and a cylinder.

Activity-Specific Assessments

- Activity 1: The student will determine the total living area and total area of a given floor plan. This floor plan should have odd-shaped rooms that would require using most of the formulas discussed in this activity.
- Activity 6: The student will build a pyramid with a minimum surface area and minimum volume. The student will show the measurements of the bases, height of the faces, and the height of the pyramid based on the given minimum surface area and volume.
- Activity 10: The student will design a container to hold a specific volume of a specified product. Assign each student a different volume and specific shape or assign the volume and allow the student to choose the shape. The student will design a label for the container and determine the area of that label. The student will create a newspaper advertisement about the product to fit within a specified area.

Geometry

Unit 7: Circles and Spheres

Time Frame: Approximately five weeks



Unit Description

This unit focuses on justifications for circular measurement relationships in two and three dimensions, as well as the relationships dealing with measures of arcs, chords, secants, and tangents related to a circle. It also provides a review of formulas for determining the circumference and area of circles.

Student Understandings

Students can find the surface area and volume of spheres. Students can apply the relationship of the measures of minor and major arcs to the measures of central angles and inscribed angles, and to the circumference in various situations. They can also explain the relevance of tangents in real-life situations and the power of a point relationship for intersecting chords.

Guiding Questions

1. Can students provide an argument for the value of π and the way in which it can be approximated by polygons?
2. Can students provide convincing arguments for the surface area and volume formulas for spheres?
3. Can students apply the circumference, surface area, and volume formulas for circles, cylinders, cones, and spheres?
4. Can students apply geometric probability concepts using circular area models and using area of a sector?
5. Can students find the measures of inscribed and central angles in circles, as well as measures of sectors, chords, and tangents to a circle from an external point?
6. Can students use the power of a point theorem (intersecting chords and intersecting secants) to determine measures of intersecting chords in a circle?

Unit 7 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Measurement	
7.	Find volume and surface area of pyramids, spheres, and cones (M-3-H) (M-4-H)
Geometry	
10.	Form and test conjectures concerning geometric relationships including lines, angles, and polygons (i.e., triangles, quadrilaterals, and n -gons), with and without technology (G-1-H)(G-4-H)(G-6-H)
13.	Solve problems and determine measurements involving chords, radii, arcs, angles, secants, and tangents of a circle (G-2-H)
17.	Compare and contrast inductive and deductive reasoning approaches to justify conjectures and solve problems (G-4-H) (G-6-H)
19.	Develop formal and informal proofs (e.g., Pythagorean theorem, flow charts, paragraphs) (G-6-H)
Data Analysis, Probability, and Discrete Math	
21.	Determine the probability of conditional and multiple events, including mutually and non-mutually exclusive events (D-4-H) (D-5-H)
22.	Interpret and summarize a set of experimental data presented in a table, bar graph, line graph, scatter plot, matrix, or circle graph (D-7-H)

Sample Activities**Activity 1: Vocabulary Self-Awareness (GLE: 13)**

Materials List: pencil, Vocabulary Self-Awareness BLM

Give each student a copy of the Vocabulary Self-Awareness BLM. *Vocabulary self-awareness* ([view literacy strategy descriptions](#)) is a literacy strategy which helps to assess students' knowledge of terms before reading text or other tasks. This awareness is helpful for students because it highlights what students already know, as well as what they still need to learn in order to fully understand the concept.

Throughout this unit on circles and spheres, have students maintain the *vocabulary self-awareness* chart on the Vocabulary Self-Awareness BLM. A target vocabulary has been provided in the BLM (additional words may be added if necessary). Have students complete self-assessments of their knowledge of the words using the chart provided on the BLM. Do not give students definitions or examples at this stage. Ask students to rate their understanding of each word with a “+” (understand well), a “✓” (limited understanding or unsure), or a “–” (don't know). Tell students to provide a definition and example for any term they feel they understand well (they would put a “+” for these terms). Over the course of the unit, students should be told to return often to the chart to add new information to it. Students will add definitions, examples, and other words, as

well as revise definitions and examples from their initial self-assessment. The goal is to replace all the check marks and minus signs with a plus sign. Because students continually revisit their vocabulary charts to revise their entries, they have multiple opportunities to practice and extend their understanding of key terms related to circles and spheres.

Activity 2: Derivation of the Area of a Circle Formula (GLEs: 13, 17)

Materials List: pencil, paper, paper circles, scissors, automatic drawing program

Lead students in an exercise to show how the formula for the area of a circle can be developed. Have students cut a circle into 8 or 16 sectors, rearrange the sectors to form a parallelogram, and then use algebra to generate the area formula from the formula for the area of the parallelogram. Using this process allows students to review the circumference formula and area formula of a parallelogram.

Another way to show the derivation is to increase the number of sides of a regular polygon inscribed in a circle. Using an automatic drawing program such as *The Geometer's Sketchpad*®, have students draw a circle and inscribe an equilateral triangle. Have them find the area of the triangle and compare it to the area of the circle. Then have the students inscribe a square, regular pentagon, regular hexagon, regular octagon, regular decagon, and a regular dodecagon. Students should notice that the length of the apothem of the regular polygon approaches the length of the radius of a circle as the number of sides in the polygon increases. Have students derive the area of a circle formula by looking at the process for calculating the area of a regular polygon (i.e., area of one triangle times number of triangles or by using the formula discussed in Unit 6 Activity 2), and generalize this formula as the length of the polygon's sides gets smaller and smaller (as the number of sides increases). Using this technique allows students to review the process and/or formula for finding the area of a regular polygon.

Have a class discussion about the use of deductive logic in this process. What assumptions must be accepted as truth? How is the process different from induction?

Activity 3: Throw That Dart! (GLEs: 13, 21)

Materials List: paper, pencil, teacher-created “dart” boards, calculators, learning logs

Provide students with several “dart” boards made of circles. For example, use circles on squares with the circles cut up into quarter pieces and placed in the corners of a square, or use several concentric circles. Have students shade in some of the circular regions, determine the areas of these regions, and then figure the probability of a dart which is thrown randomly at a dartboard landing in a shaded region (assume each dart hits the board). Have students create dartboards that possess specific probabilities for a randomly thrown dart landing in a shaded region.

As an entry in the students' math *learning logs* ([view literacy strategy descriptions](#)), have the students explain the process they should employ to find the probability of a dart hitting a certain region of the board. The prompt could be:

Create a dart board that includes at least one circle and one other polygon. Shade at least one region of the dart board. Calculate the probability of a dart hitting the shaded region(s). Explain the process you used to find the probability for your dart board.

A *learning log* is typically a notebook students keep in order to record their ideas, questions, and new understandings. Students should keep this as a separate notebook or as a separate section in their binders. Students should use their math *learning logs* other times in class, in addition to those listed throughout the curriculum, to provide opportunities to assess understanding.

Activity 4: Central Angles and Arcs (GLE: 13)

Materials List: Sample Notes BLM, Split-Page Notes Model BLM, paper, pencil, circle diagrams, string or tailor's tape,

In this activity, students will use a *split-page notetaking* ([view literacy strategy descriptions](#)) format to take notes on central angles and arcs. *Split-page notetaking* is simply a different way for students to organize their notes to help them use their notes more effectively for study. Model the approach by placing on the board or overhead sample split-page notes from the topic of circles. Explain the value of taking notes in this format by saying it organizes information and ideas from various sources. It helps separate big ideas from supporting details, and it promotes active reading and listening.

Next, ask students to use *split-page notetaking* while listening to a brief presentation on central angles and arcs. This presentation should define the terms central angles, arcs, major arcs, minor arcs, and semicircles. Tell students to draw a line from the top to bottom of their pages about one-third from the left side of the page. On the left side they will write the terms or concepts and on the right side they will write the definition or explanation of the term. They can also draw examples on the right side of the page. After the presentation, have students compare notes with a partner, then answer questions and provide clarification using the Split-Page Notes Model BLM as a guide. Show students how they can prompt recall by bending the sheet of notes so that the information in the right or left column is covered. Then proceed with the remainder of the activity reminding students to add information to the concepts they have recorded already (like formulas and other examples).

Provide pairs of students with a diagram containing a circle with a given radius length and with central angles labeled 1, 2, and 3. The measures of the central angles should be in the ratio of 2:3:4. Have students determine the measures of the central angles and

measures of the respective arcs. Repeat this activity for other circles and other ratios. In addition, have students identify major and minor arcs given a central angle. Students should realize that the measure of the major arc is equal to $360 -$ the measure of the minor arc. To help students visualize major and minor arcs, ask them to determine the type of arc associated with various times of day displayed on an analog clock.

Review with students the fact that the sum of the central angles of a circle is 360° . If needed, have students use string or a tailor's tape measure to find the circumference of the circle and to determine the length of one of its arcs. Help students to internalize the concept that the ratio of the arc's measure to 360 degrees is the same ratio as the arc's length to the circle's circumference by repeating this activity many times.

Provide opportunities for students to find both arc measure and arc length of major and minor arcs using the formula $\frac{\text{arc measure}}{360^\circ} = \frac{\text{arc length}}{\text{circumference}}$.

Following this activity, have students revisit both their split-page notes and their *vocabulary self-awareness* charts to update and revise their entries.

Activity 5: Concentric Circles (GLE: 13)

Materials List: Concentric Circles BLM, protractor, ruler, learning log

Provide students with copies of the Concentric Circles BLM. Have students find the measure of several of the central angles using a protractor. Make certain that students understand that the measure of an arc is the same as the measure of its central angle. Have students measure the radii of several of the concentric circles and calculate the length of each arc intercepted by the central angles. Have students discuss in pairs their observations about the arcs they have measured. During this discussion students should begin to see that arcs which have the same degree measure do not have the same arc length if they are parts of circles whose radii are different lengths.

Have students draw two circles of different sizes. Have them draw a central angle of 75 degrees in each circle, and then calculate the length of each arc intercepted by the 75 degree angles. Have students determine if their calculations follow their observations from their work with the concentric circles.

As an entry in the students' math *learning logs* ([view literacy strategy descriptions](#)), have students explain how arc measure and arc length differ, and under what conditions two arcs can have the same measure, but different lengths. Ask for students to volunteer their explanations and lead a class discussion on the topic.

Activity 6: Graph It! (GLEs: 13, 22)

Materials List: results from group surveys, protractors, calculators, pencil, paper, color pencils or colors, teacher-created data sets and matching circle graphs, Internet access, magazines, newspapers (The last three items are optional and for teacher use)

Have students work in groups to conduct surveys about favorite TV shows, foods, colors, etc. The surveys should be completed outside of class. Students should ask a variety of people the questions, not just the people in their class. Each group can ask a different question: “What is your favorite TV show? What is your favorite food? What is your favorite color?” Each group should compile its results and compute the percent of people in each category for the survey question (for example, what percent of people liked CSI, Hannah Montana, Spongebob, etc). Have groups of students create circle graphs to represent their data. Provide the students with the definition of the term *sector*. When creating their graphs, allow students to use colors or color pencils to shade the sectors of their graphs. Make sure that students use protractors to calculate the correct angle measures based on their data. Once groups create their circle graphs, have them swap graphs and check each other’s work.

Prior to the start of this activity, search the Internet or magazines and newspapers for data which can be presented as a circle graph. Another option is to survey your classes on various topics like their favorite colors, movies, foods, hobbies, etc., months of the students’ birthdays, ages of the students (if there is a variety among all of the classes you teach). This survey can be accomplished as an interest survey.

Using the data collected either from the Internet or the survey, provide the students with the data in table form (using percents) and matching circle graphs that are already drawn. Make sure some of the circle graphs are constructed incorrectly. Have students discuss the data that is presented to them in the graph and compare to the data provided in the table. Using their knowledge of central angles, instruct students to determine if the circle graph has been constructed correctly. If any graph is incorrectly constructed, indicate that students should develop a new graph based on the given data. Have students determine the kind of arc associated with each category of data (e.g., major, minor, semicircle). Instruct students to determine what each of the sectors represents. For instance, if one of the circle diagrams is a survey about favorite television programs and 24% of the 250 people surveyed like *The Frugal Millionaire*, have students determine the number of people who like that show.

Activity 7: Geometric Probability (GLE: 21)

Materials List: paper, pencil, spinners, calculators

Have students practice finding the area of a sector using the formula $A = \frac{N}{360} \pi r^2$, where N is the measure of a central angle. Give students circle spinners divided into unequal

sectors. Have students find the probability of spinning and landing on a certain sector. Allow students to play simulated games to see if there is a way to always win the game if there are points allotted to certain sectors.

Activity 8: Arcs and Chords (GLE: 13)

Materials List: geometry software program (or compass and straightedge), pencil, paper, ruler, protractor, Diameters and Chords BLM, scissors, patty paper

Have students use a geometry software program (or compass and straightedge) to inscribe a variety of polygons in circles. Next, have students determine the measure and length of each arc of the circle subtended by a chord. For example, inscribe a stop sign in a circle and then determine the two measures of each of the 8 arcs.

Use a *process guide* ([view literacy strategy descriptions](#)) to help students examine the relationship between a chord and its arc when a diameter is perpendicular to the chord, and the relationship between two chords that are equidistant from the center. *Process guides* are used to guide students in processing new information and concepts. They are used to scaffold students' comprehension and are designed to stimulate students' thinking during and after reading. *Process guides* also help students focus on important information and ideas. In this activity, students will be given process guides that will lead them through the steps to discover the relationships inherent in all quadrilaterals. To create a process guide, review the information to be studied and decide how much help students will need to construct and use meaning.

Provide each student with a copy of the Diameters and Chords BLM. Have students work as pairs to complete the investigation. Have the pairs share their findings with the rest of the class. Students should be told that they are required to support their statements, conjectures, and answers, with evidence from their investigations with the *process guide*. Lead a summary discussion of the conjectures formed through the investigation. Also encourage students to add these conjectures to their split-page notes to help them organize their new information.

At this point, students should be given the opportunity to apply their understanding of the conjectures by having the students find measures of segments in circles. These problems should include algebra skills as well.

Activity 9: Finding the Center (GLE: 13)

Materials List: paper, pencil, ruler, protractor

Pose the problem of finding the center of a circular picnic table in order to cut a hole for an umbrella. Challenge students to use their knowledge of chords, lines perpendicular to a chord at its midpoint, and the intersection of these lines to find the center of the circle.

Activity 10: Inscribed angles (GLEs: 13, 17, 19)

Materials List: paper, pencil, ruler, protractor

Have each student draw a circle and then inscribe a regular hexagon in the circle. Students should label the hexagon $ABCDEF$ and the center of the circle P . Draw radii \overline{PA} , \overline{PF} , and \overline{PB} . Label the following angles:

$\angle FPA$ as $\angle 1$, $\angle APB$ as $\angle 2$, $\angle FAP$ as $\angle 3$, and $\angle BAP$ as $\angle 4$. Have students measure each numbered angle with a protractor, then find $m\widehat{FA}$ and $m\widehat{AB}$, and then explain their reasoning. Next, instruct students to find $m\angle FAB$ and $m\widehat{BDF}$ and make a conjecture about the relationship between $m\angle FAB$ and $m\widehat{BDF}$. Have students prove this conjecture and consider all three cases: the center of the circle lies on the side of the angle, the center of the circle is in the interior of the angle, and the center of the circle is in the exterior of the angle. Ask students to investigate the measure of an angle inscribed in a semicircle, and the measures of the angles in a quadrilateral inscribed in a circle.

The theorems students should develop in this activity are:

- If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc (or the measure of the intercepted arc is twice the measure of the inscribed angle).
- If an inscribed angle intercepts a semicircle, the angle is a right angle.
- If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

Activity 11: Tangents and Secants (GLE: 13)

Materials List: Tangents and Secants BLM, compass, pencil, paper, straightedge, information on satellites

Provide students with copies of the Tangents and Secants BLM. Use the BLM to discuss the three theorems listed below:

1. If a tangent and a secant intersect at a point on a circle, then the measure of each angle formed is half of the measure of its intercepted arc.
2. If two secants intersect in the interior of a circle, then the measure of each angle is half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
3. If a tangent and a secant, two tangents, or two secants intersect in the exterior of a circle, then the measure of the angle formed is half the difference of the measures of the intercepted arcs.

Throughout the discussion, have students write down the relationships formed by each pair of intersecting lines.

The first theorem is based solely on the inscribed angles discussed in Activity 10. To help illustrate the second theorem, have students use a compass to draw a circle of any radius and choose any point on the interior of the circle (not the center) and label it V . Then, using a straightedge, have students draw two secants that intersect at V . Label one secant as \overline{WY} and the other secant as \overline{XZ} . Label $\angle ZVY$ as $\angle 1$. Next have them draw \overline{XY} and label $\angle VYX$ as $\angle 2$ and $\angle VXY$ as $\angle 3$. Students should see that $\angle 2$ and $\angle 3$ are inscribed angles so $m\angle 2 = \frac{1}{2}m\widehat{WX}$ and $m\angle 3 = \frac{1}{2}m\widehat{ZY}$. Lead the students in a discussion about why $m\angle 1 = m\angle 2 + m\angle 3$ and help them write an equation based on that understanding. Students should be able to understand the second theorem with this illustration.

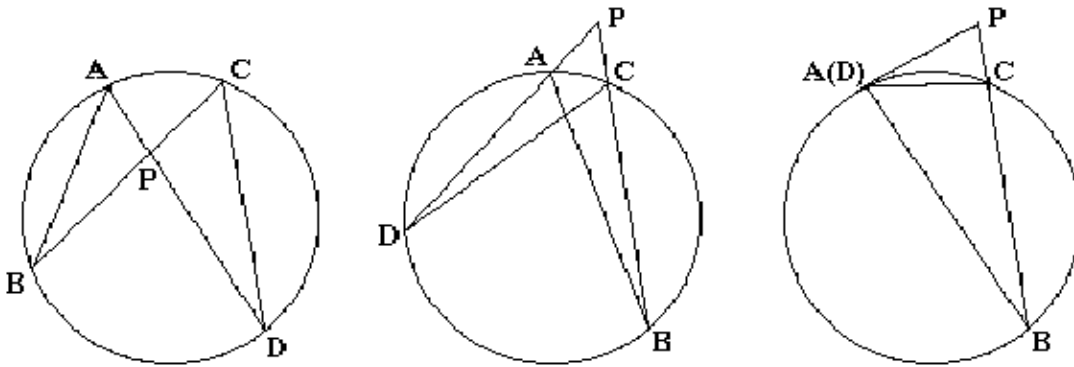
Provide students with information about geostationary satellites and their orbits. Geostationary satellites move in a circular orbit about 26,000 miles above the Earth's center. For a satellite whose orbit is directly over the equator, have students determine the measure of the arc along the equator that is "visible" to the satellite when given the measure of the exterior angle formed by the two tangent lines drawn from the satellite to the earth. For a satellite that is 26,000 miles from Earth's center, the angle formed by two tangent lines measures approximately 17.7° . Ask students to apply the third theorem to find the measure of the required arc. Next, provide students with an arc measure along Earth's equator that is less than $\frac{1}{2}$ of Earth's circumference (so that they have two secant lines intersecting at the satellite's location). Have students determine the measure of an angle of view of the satellite.

Activity 12: Intersecting Chords and Secants (GLE: 13)

Materials List: drawing program, pencil, paper

Have students use a drawing program such as *The Geometer's Sketchpad*® to construct two intersecting chords, two intersecting secants, or a tangent and secant for a circle. These three cases are the basis of the Power of a Point theorem. When point P lies inside the circle, the theorem is called the Intersecting Chords theorem; when point P lies outside the circle, the theorem is called the Intersecting Secants theorem.

Students should first prove that there are two similar triangles created if additional segments are added. $\triangle APB \sim \triangle CPD$, in each case in the diagrams provided below, are the similar triangles. Once the similarity of the triangles is established, then students should be able to write the proportion $\frac{AP}{CP} = \frac{BP}{DP}$, which is equivalent to the statement of the theorem: $AP \cdot DP = BP \cdot CP$. *Note that in the case of the tangent, points A and D coincide and are the same point.*

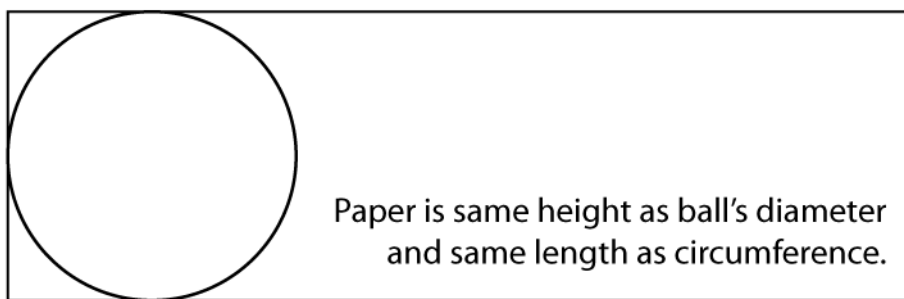


Activity 13: Surface Area of a Sphere (GLE: 7)

Materials List: Surface Area of a Sphere BLM, small spheres, wrapping paper, pencil, paper, scissors, tape

Here is a concrete way to show students why the surface area of a sphere formula is $4\pi r^2$.

Students should already understand that the surface area of an object can be represented by how much wrapping paper it would take to cover it. Ask them to picture a sphere (a balloon or ball) and a piece of paper that is cut as wide as its diameter and as long as its circumference.



If the ball were wrapped with the paper, the paper would cover the entire sphere except for all the overlaps (which would fit into the gaps if they were cut out). If possible, provide each student with a small sphere such as a tennis ball, softball, or golf ball. Have each student cut a rectangle from wrapping paper that will match the specifications listed for his/her ball and then test the concept.

Lead students through the algebraic development of the formula for the surface area of a sphere using this model as a starting point. The formula for the surface area of the paper is

Length \times width which can be written as circumference \times diameter

This is easily understood by looking at the picture above. Now substitute formulas we know:

$$C = 2\pi \times r \text{ and } d = 2r$$

$$C \times d$$

$2\pi r \times 2r$ is equal to the surface area

$$4\pi r^2$$

Provide practical applications problems which require the use of the surface area of a sphere for students to work.

Activity 14: Surface Area and Volume of Spheres (GLEs: 7, 10)

Materials List: Internet access for research, paper, pencil, scissors, tape, centimeter or inch cubes, several types of balls

Spherical balls are used in many sports (e.g., golf, soccer, baseball, basketball). Have students research the various dimensions for selected balls either by searching the Internet or checking the sizes of various balls at the store. Students will then create a circle on paper that represents a great circle associated with the sphere (ball). Using the circle pattern, have students cut the circle into fractional sectors, each of which represents $\frac{1}{8}$ of the circle. Next, have students “cover” a quarter of the sphere (ball) with these sectors. Students will then make a conjecture about the surface area formula for a sphere. To conceptualize the volume formula, have students use centimeter or inch cubes and create a large cube that approximates the size of the ball.

Following the development of the formulas, provide students with several types of balls (e.g., baseball, golf ball, basketball, soccer ball, tennis ball, etc.). Working in groups, students will determine the surface areas and volumes for each type of ball by first making appropriate measurements and then using those measurements in the correct formula.

Sample Assessments

General Assessments

- The student will complete entries in their *learning logs* for this unit. Topics could include:
 - Explain why the formula for the surface area of a sphere is $4\pi r^2$ based on the activity performed in class.
 - Explain the differences between the secant of a circle and the tangent of a circle.
 - $\triangle ABC$ is inscribed in a circle so that \overline{BC} is the diameter. What type of triangle is $\triangle ABC$? Explain your reasoning.
- The student will find pictures in magazines or newspapers of diagrams that show tangents, secants, and chords. The student will explain what the picture is and why it represents the term they are defining. These pictures could be included in a portfolio.
- The student will construct various circles with different areas. The student will construct specified inscribed angles, arcs of given measures, secants, and tangents.

Activity-Specific Assessments

- Activity 3: The student will create a dartboard game using area properties of circles or other figures. The student will determine the probabilities involved with the game and then play the games to determine the experimental probability.
- Activity 5: The student will find examples of circle graphs in magazines or newspapers. He/she will write a paragraph that describes the information presented in the graph and find the measures of the central angles based on the information provided in the graph. He/she will determine if the graph has been constructed correctly.
- Activity 14: The student will:
 1. Find the volume of a tennis ball can. The student will either take measurements from an actual can or the information will be provided in a diagram.
 2. Assuming that the tennis balls are tightly packed, find the total volume of the three tennis balls.
 3. Determine the percentage of the volume of the can taken up by the tennis balls.
 4. Determine the volume of sand needed if sand is poured to fill the remaining air space in the can.
 5. Calculate the percentage of the can's volume taken up by the sand if one of the tennis balls is removed and sand is poured in to replace it.

Geometry

Unit 8: Transformations

Time Frame: Approximately two weeks



Unit Description

This unit provides a deeper mathematical understanding and justifications for transformations that students have seen in previous grades. The focus is providing justifications for the congruence and similarity relationships associated with translations, reflections, rotations, and dilations (centered at the origin).

Student Understandings

Students determine what transformations have been performed on a figure and can determine a composition of transformations that can be performed to mimic other transformations like rotations. They are also able to find new coordinates for transformations without actually performing the indicated transformation.

Guiding Questions

1. Can students find transformations and mappings that relate one congruent figure in the plane to another?
2. Can students provide an argument for the preservation of measures of figures under reflections, translations, and rotations?
3. Can students find the dilation (enlargement or reduction), centered at the origin, of a specified figure in the plane and relate it to a similarity mapping?
4. Can students perform a composition of transformations and explain its relationship to single transformations or other compositions that produce the same image?

Unit 8 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
Geometry	
14.	Develop and apply coordinate rules for translations and reflections of geometric figures (G-3-H)
15.	Draw or use other methods, including technology, to illustrate dilations of geometric figures (G-3-H)
17.	Compare and contrast inductive and deductive reasoning approaches to justify conjectures and solve problems (G-4-H) (G-6-H)
19.	Develop formal and informal proofs (e.g., Pythagorean theorem, flow charts, paragraphs) (G-6-H)

Sample Activities**Activity 1: Understanding Congruence, Similarity, and Symmetry Using Transformations and Interactive Figures: Visualizing Transformations (Using Technology) (GLE: 14)**

Materials List: pencil, paper, Internet access

Begin by using *student questions for purposeful learning (SQPL)* ([view literacy strategy descriptions](#)). To implement this strategy, develop a thought provoking statement related to the topic about to be discussed. The statement does not have to be factually true, but it should generate some level of curiosity for the students. For this activity, pose the statement, “All transformations are congruence transformations.” This statement can be written on the board, projected on the overhead, or stated orally for the students to write in their notebooks. Allow the students to ponder the statement for a moment and ask them to think of some questions they might have, related to the statement. After a minute or two, have students pair up and generate two or three questions they would like to have answered that relate to the statement. When all of the pairs have developed their questions, have one member from each pair share their questions with the class. As the questions are read aloud write them on the board or overhead. Students should also copy these in their notebooks. When questions are repeated or are very similar to others which have already been posed, those questions should be starred or highlighted in some way. Once all of the students’ questions have been shared, look over the list and determine if additional questions should be added. The list should include the following questions:

- What is a transformation?
- What types of transformations are there?
- What does the phrase “congruence transformations” mean?
- How do we know if two figures are congruent?

At this point, be sure the students have copied all of the questions in their notebooks and continue with the lesson as follows. Tell the students to pay attention as the material is

presented to find the answers to the questions posted on the board focusing on those questions which have been starred or highlighted. Students should refer back to these questions throughout the entire unit as all questions will not be completely answered until the end of the unit.

The website, <http://standards.nctm.org/document/eexamples/chap6/6.4/>, allows students to visualize transformations and compositions of transformations, while working interactively with various geometric figures. Students explore the effects of applying reflections, translations, and rotations to any one of three shapes. Periodically stop throughout the lesson to allow the student pairs to discuss which questions have been answered from the list. Ask questions, such as “How does the original shape compare to the shape after the transformation?” and “What is the effect of the transformation on the side lengths and angle measures of the original shape?” Students should begin to realize that reflections, translations, and rotations do not change the size or shape of the figures they use which means they are congruence transformations. This exploration may be followed with a whole class discussion so all students are sure to have the correct answers to each question. This concept will be reinforced in the following activities.

Activity 2: A Basic Look at Transformations (GLE: 14)

Materials List: graph paper, pencil, paper, ruler, protractor

Provide students with a sheet of graph paper with the four quadrants marked. In Quadrant 1, have students construct a polygon by providing a set of coordinates for the vertices. Next, instruct students to perform various translations and reflections of the shape. Have students develop a series of translations or reflections that combine to produce the original shape in its original location. After each transformation, have students determine the vertices of the transformed polygon. To conclude the activity, have students pair up and look for patterns in the coordinates for each type of transformation. For example, each time the polygon is reflected over the x -axis, each vertex will have the same x -coordinate as the original, but the y -coordinates will be the opposites of the original. For a polygon that has been reflected over the y -axis, students should be able to identify that the x -coordinate of each vertex of the transformed figure is the opposite of the original while the y -coordinate is the same as the original. For any translations, students should begin to see that the number of units a figure is moved left or right can be added/subtracted from the original x -coordinate, and the number of units a figure is moved up or down can be added/subtracted from the original y -coordinate. Lead a whole class discussion to have students make conjectures about the “rules” for the transformations. Use these conjectures as background for Activities 3 and 5.

Activity 3: Understanding Reflections (GLE: 14)

Materials List: pictures of reflections, pencil, paper, straightedge

Give students pictures of various reflections using the x -axis, y -axis, the origin, and the line $y = x$. Have students develop a chart that names the type of reflection, the change of the original to the image, a statement about how to find the coordinates of the image, and a numerical example in the coordinate plane. For example, a chart might look like the following:

Reflection	Over x -axis	Over y -axis	Around Origin	Over $y = x$
Original to image	$(a, b) \rightarrow (a, -b)$	$(a, b) \rightarrow (-a, b)$	$(a, b) \rightarrow (-a, -b)$	$(a, b) \rightarrow (b, a)$
Statement	Multiply y -coordinate by -1			
Example				

Provide students with multiple examples for various different figures with different properties to complete this chart. Next, provide students with coordinates of a figure and have them determine the coordinates of the image points for a given reflection without graphing any of the points. Allow students to use the chart to determine what the coordinates of the image points should be. After they have identified the new coordinates for each reflection, have them graph their original images and the reflected images to check their work. Have students compare the lengths of the sides of the polygons as well as the measures of the angles of the polygons. Lead a brief discussion to determine if a reflection represents a congruence transformation. Be sure to have students refer to the questions posed in Activity 1 to be sure all answers are complete. Have students refer back to their conjectures about x -axis and y -axis reflections in Activity 2 to verify or change their conjectures.

Activity 4: Understanding Rotations (GLE: 14)

Materials List: image on a coordinate plane, pencil, paper, straightedge

Teacher Note: While GLE 14 does not refer to rotations, rotations are tested on the GEE21. This activity will also serve as a precursor to the GLEs for grades 11 and 12.

Give students a pre-image on the coordinate plane with the vertices labeled. Have students make a chart like the one below and rotate the pre-image 90° , 180° , 270° , and 360° using the origin as the center of rotation. They should record the new coordinates each time and then graph the new images.

Original	90°	180°	270°	360°
(a, b)				
(c, d)				

(e,f)				
(g,h)				
(x,y)				

After drawing the new figures, have students analyze the coordinates to determine a pattern of changes in coordinates. Provide opportunities to practice performing rotations both with and without this chart on the coordinate plane. Have students compare the lengths of the sides of the polygon as well as the measures of the angles of the polygon. Lead a brief discussion to determine whether a rotation represents a congruence transformation. Be sure to have students refer to the questions posed in Activity 1 to be sure all answers are complete.

Activity 5: Slide It! (GLE: 14)

Materials List: polygons on a coordinate plane, pencil, paper, straightedge, learning log

Provide students with diagrams showing various polygons on a coordinate plane. Give specific instructions about the directions in which to move the figure (3 units up and 4 units right, 2 units down and 4 units left, etc). Have students record the original and new coordinates of the vertices and analyze the results. Lead a discussion to summarize how the x -coordinate is affected when a point is translated left/right, and how it is affected if the point is moved up/down. Have similar discussion about the change in the y -coordinate when the point is moved left/right and up/down. Assuming that a represents a horizontal translation and b represents a vertical translation, the effect of the translation is point $(x,y) \rightarrow (x+a, y+b)$.

Provide the opportunity for students to draw a translation that moves in the same direction as a given vector. The vector may be already drawn on the coordinate plane so that students can identify the number of units to move. Again, they should analyze their ordered pairs and determine if the pattern is still true.

Have students compare the lengths of the sides of the polygon as well as the measures of the angles of the polygon. Lead a brief discussion to determine whether a translation represents a congruence transformation. Be sure to have students refer to the questions posed in Activity 1 to be sure all answers are complete. Have students refer back to their conjectures about translations in Activity 2 to verify or change their conjecture.

To end this activity, have students create an entry in their math *learning logs* ([view literacy strategy descriptions](#)). Present the following prompt:

Explain how you could determine the new coordinates of the vertices of a polygon if it were reflected, rotated, or translated without graphing the transformations on the polygon. Be sure to explain your reasoning for each transformation to be performed.

The students' responses should reflect an ability to apply the coordinate rules developed in Activities 3, 4, and 5.

A *learning log* is typically a notebook students keep in order to record their ideas, questions, and new understandings. Students should keep this as a separate notebook or as a separate section in their binders. Students should use their math *learning log* other times in class, in addition to those listed throughout the curriculum to provide opportunities to assess understanding.

Activity 6: Magnify It! (GLE: 15)

Materials List: figure graphed in a coordinate plane, pencil, paper, drawing program

Have students work in groups of two to develop a specific dilation of a figure that has been graphed in the plane. First, have them create a dilation that is 1.5 times the size of the original figure. Next, instruct students to create a dilation that is .75 times the size of the original figure. Be sure to instruct students to specify the coordinates of the dilated figures. If available, use *The Geometer's Sketchpad*[®] or other drawing program to perform dilations. Have students make statements about the similarity of the original figure and its dilation, and determine the attributes of the original figure that remain unchanged after the dilation is performed. Students should recognize that dilation is not a congruence transformation; however, the figures are similar. Have students refer to the questions from Activity 1 to be sure all answers are complete.

Activity 7: Make a Conjecture and Prove It! (GLEs: 14, 15, 17, 19)

Materials List: drawing program, pencil, paper, diagrams of transformed images

Using a geometry software package such as *The Geometer's Sketchpad*[®], have students create several translations, reflections, rotations, or dilations and combinations of these, and then examine the properties of the transformed figures compared to original figures. Using these inspections, have students make conjectures about the effects of these transformations including conjectures concerning congruence and similarity. Instruct students to prove their conjectures.

For example, "Are any of the combinations of transformations the same as a single type of transformation? Are transformations commutative, that is, can the order of two transformations be changed and get the same result?" Allow students to use inductive and deductive reasoning while comparing conjectures and accompanying proofs. To enhance student understanding, have students begin by reflecting their figures over two parallel lines and compare this with a translation. Next, have students reflect their figures over a pair of intersecting lines (e.g., the x - and y -axes) and compare this with a rotation. Be sure to have students perform this activity through several iterations. Each iteration should focus on a specific set of combined transformations. As an alternative, provide students

with a pre-image and an image in the coordinate plane with vertices labeled and require students to determine the transformation or set of transformations that produced the image.

Sample Assessments

General Assessments

- Provide the student with a polygon in the coordinate plane and instruct the student to perform various transformations on it.
- The student will investigate the transformations that are used in a board game, such as “checkers” or “chess.”
- The student will create a portfolio containing samples of work from the activities. Portfolio entries will include copies of the transformations performed in class with explanations about the procedures used to complete the transformation.
- The student will write entries in his/her *learning log* that are graded. Topics might include:
 - Which transformations produce congruent figures? Why are these figures congruent to their originals?
 - How would you define the words reflection, rotation, translation, and dilation? Are your definitions of these words different from your previous definitions since the completion of this unit? (This *learning log* would be posed twice: the first time, only the first question would be asked, and would be posed before the unit; the second time would be after the unit and both questions would be asked.)
- The student will create a “Transformation Album.” He/she will create a figure and perform at least one of each of the different transformations on the figure in the coordinate plane. Assess the work based on the accuracy of the transformations.

Activity-Specific Assessments

- Activity 3: Ask the student to demonstrate his/her ability to do the following:
 1. Given $A(-2,3)$, $B(-5,7)$, and $C(-1, 10)$, graph triangle ABC .
 2. Reflect triangle ABC over the x -axis. Label the image as $A'B'C'$.
 3. Find the area of triangle $A'B'C'$.
 4. Explain how the area of triangle $A'B'C'$ compares to the area of triangle ABC .
- Activity 4: Provide the student with the coordinates of an image that has already been rotated 90° , 180° , or 270° . The student will find the coordinates of the pre-image using either the coordinate plane or the chart developed in this activity.

- Activity 8: The student will design a tessellation using reflections, rotations, or translations. If materials are available, the student will transfer the design to a cloth square in order to make a class quilt. This can be done using special crayons which can be found in some school and/or art supply stores.