Comprehensive Curriculum

Grade 7 Mathematics

Advanced Course

Cecil J. Picard
State Superintendent of Education

Louisiana Department of EDUCATION
Reaching For Results

Grade 7
Advanced Mathematics
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This course covers both Grade 7 and Grade 8 GLEs. It is designed for Grade 7 students intending to take Algebra I in Grade 8.

Grade 7
Advanced Mathematics
Unit 1: Understanding Relationships

Time Frame: Approximately four weeks

Unit Description:

This unit begins with the use of different problem-solving strategies. The relationship of fractions, decimals and percent will be explored in this unit. A deeper understanding of the relative size of numbers is emphasized. Place values are associated with powers of 10, and scientific notation is introduced as a way to rewrite large values. The unit also introduces operations with integers through the use of manipulatives.

Student Understandings

Students will review the different problems solving strategies as they begin this unit. Students will demonstrate their understanding of fraction, decimal, integer, and ratio/percent representations through comparing, ordering, contrasting and connecting these representations of rational numbers in problem-solving contexts. The students will use order of operations and proportions while exploring rational numbers. The students will demonstrate an understanding of integer operations by observing patterns and conjecturing about rules.

Guiding Questions

1. Can students determine an appropriate problem-solving strategy?
2. Can students represent in equivalent forms and evaluate fractions, percents, decimals, integers and ratios?
3. Can students compare rational numbers using symbolic notation as well as use position on a number line?
4. Can students recognize, interpret, and evaluate problem-solving contexts with fractions, decimals, integers, and ratios?
5. Can students demonstrate the equality of ratios in a proportion?
6. Can students set up and solve simple percent proportions?
7. Can students determine and justify the reasonableness of answers to problems involving rational numbers?
8. Can students explain and use integer operation rules?

Unit 1 Grade-Level Expectations (GLEs)

GLE#	GLE Text and Benchmarks
7th grade	
Number and Number Relations	
1.	Recognize and compute equivalent representations of fractions, decimals, and percents (*i.e.,* halves, thirds, fourths, fifths, eighths, tenths, hundredths) (N-1-M)
2.	Compare positive fractions, decimals, percents, and integers using symbols (*i.e.,* <, ≤, =, ≥, >) and position on a number line (N-2-M)
3.	Solve order of operations problems involving grouping symbols and multiple operations (N-4-M)
6.	Set up and solve simple percent problems using various strategies, including mental math (N-5-M) (N-6-M) (N-8-M)
7.	Select and discuss appropriate operations and solve single- and multi-step, real-life problems involving positive fractions, percents, mixed numbers, decimals, and positive and negative integers (N-5-M) (N-3-M) (N-4-M)
8.	Determine the reasonableness of answers involving positive fractions and decimals by comparing them to estimates (N-6-M) (N-7-M)
9.	Determine when an estimate is sufficient and when an exact answer is needed in real-life problems using decimals and percents (N-7-M) (N-5-M)
10.	Determine and apply rates and ratios (N-8-M)
11.	Use proportions involving whole numbers to solve real-life problems (N-8-M)
12.	Evaluate algebraic expressions containing exponents (especially 2 and 3) and square roots, using substitution (A-1-M)
Data Analysis, Discrete Math and Probability	
35.	Use informal thinking procedures of elementary logic involving *if/then* statements (D-3-M)
8th grade	
Number and Number Relations	
1.	Compare rational numbers using symbols (*i.e.,* <, ≤ , =, ≥, >) and position on a number line (N-1-M) (N-2-M)
2.	Use whole number exponents (0-3) in problem-solving contexts (N-1-M) (N-5-M)
3.	Estimate the answer to an operation involving rational numbers based on the original numbers (N-2-M) (N-6-M)
4.	Read and write numbers in scientific notation with positive exponents (N-3-M)
5.	Simplify expressions involving operations on integers, grouping symbols, and whole number exponents using order of operations (N-4-M)
6.	Identify missing information or suggest a strategy for solving a real-life, rational-number problem (N-5-M)
7.	Use proportional reasoning to model and solve real-life problems (N-8-M)
8.	Solve real-life problems involving percentages, including percentages less than 1 or greater than 100 (N-8-M) (N-5-M)
9.	Find unit/cost rates and apply them in real-life problems (N-8-M) (N-5-M) (A-5-M)

Sample Activities

Activity 1: Missing! (GLEs: 7th – 7; 8th – 6)

It is important for the students to establish problem-solving strategies early in the year. This activity was written to help students recognize the steps in problem solving. Lead a class discussion about problem-solving strategies that the students have used. Have students make a list of basic steps involved in problem solving: a) understand the problem; b) make a plan, sketch or diagram of the problem; c) carry out the plan (do the computation); and d) determine that the solution makes sense. Discuss the different problem-solving strategies such as: a) working backwards; b) sketching models or diagrams; c) making tables, charts or graphs; or, d) setting up and solving a simpler problem first.

Put a situation like one of those listed below on the overhead and have students write a plan for solving the problem. Give the students time to solve it and then have students discuss solution strategies. Ask, Does anyone have a different method that was used to solve the problem? Discuss methods and make sure students verbally explain why their method worked for them.

- Samantha is floating on a raft 75 feet from the shore at 10 a.m. Every hour she moves herself forward 15 feet with her arms, but she stops when she gets tired and the current pulls her backward 6 feet. At this rate, what time will Samantha reach the shore? Explain your solution.

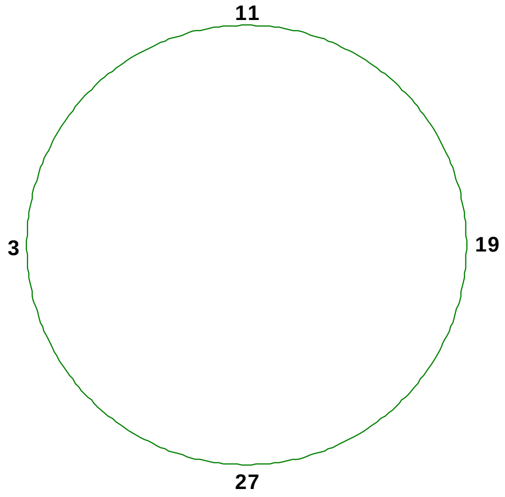
  Solution: 6:20 p.m. Since she gets about 9 feet every hour, 75 ÷ 9 = 8 hours, but she will have three feet left. If there are 3 feet left and she moves 9 ft in an hour, then she will need 1/3 of an hour to get to shore. This is 20 minutes.

- There are six boys in a race. Carl is ahead of Bill, who is two places behind Frank. Allen is two places ahead of Dwight, who is two places ahead of Evan. Evan is last. Which of the boys is closest to the finish line? Explain your solution.

  Solution: Carl is closest to the finish line. Carl is first, Allen second (two places ahead of Dwight), Frank is third (two places ahead of Bill), Dwight is fourth, Bill is fifth (two people behind Frank), and Evan last (two places behind Dwight).

- A group of students has gathered around the center circle of the basketball court. The students are evenly spaced around the circle. Student #11 is directly across from student #27. How many students have gathered around the circle? Explain your solution.

  Solution: 32 students. Sketch a model to get an idea as to positioning. Making a table of the opposites as they draw a diagram will give the students the answer. Example: We know 11 is across from 27, which means that students 12 through 26 must sit between them. Number 19 is in the middle position in a listing of these fifteen numbers (median). This

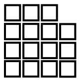


means that the number opposite 19 would have to be the 8th number away from 11, which is 3. 3 would also have to be the 8th number away from 27. The numbers between 3 and 27 would be 1, 2, 28, 29, 30, 31 and 32. The answer is 32 people at the table.

Students also have to be able to determine when there is not enough information available to solve problems and locate the additional information needed to solve a problem. They also need practice in devising problems. Have the students examine problems like the ones below and discuss how these are different from the earlier problems. (*They are missing information*.) Have them work in pairs or small groups to determine the missing information and find the solution.

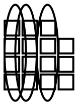
- The world record high dive is 176 feet 10 inches. What is the difference between Jack's highest dive and the world record?
- Mary wants to find the amount of carpet needed to carpet her bedroom. She measures the length of the room. How much carpet does she need to carpet the bedroom?
- Greg Louganis holds 17 U. S. national diving records. How many of these did he earn before the 1988 Olympics?

Activity 2: Grouping Dilemma! (GLEs: 7th – 3; 8th – 5)

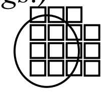


Display the tile pattern at the left on the overhead either with tiles or a sketch. Give the students directions to find the total number of tiles without counting each one. Have the students sketch the pattern and draw circles or "loops" around groups of tiles to help them determine the total number of tiles. It is important that the students explain how their grouping method matches their mathematical expression or equation.

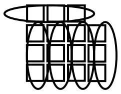
Examples: (There are many more groupings.)



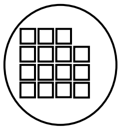
$3 \times 4 + 3$.



$3^2 + (3 \times 2)$

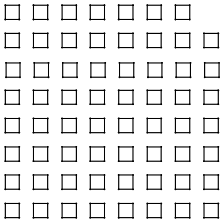


$3+3+3+3+3$ or 3×5



some students even see $(4 \times 4) - 1$

Allow time for as many different groupings as the students develop, asking questions that help them justify how their groupings matches their mathematical expression. Next, ask students to use information from the classroom discussions to determine how many square tiles would be needed if tiles were arranged in this same manner with the top right tile missing, but there were 8 rows and 8 columns. Lead discussion as the students determine which of the methods used earlier make it possible to find the number of tiles. (*63)* Extend the problem by asking students how many rows and columns would be in an arrangement using the pattern of the missing corner tile that has a total of 120 tiles. *(11 columns, 11 rows, with the corner missing)*



Activity 3: Decimal Comparisons - Where's the Best Place? (GLEs: 7th – 2; 8th –1, 2)

Discuss place value and what the decimal part of a number signifies. Make sure the students understand that the part of the number to the right of the decimal point shows the fractional part of a whole. Start with an understanding that the first place to the left of the decimal point is the ones place and the value of that number in that position is determined by multiplying the digit by 1. Ask students for the value of the number in the next position to the left and continue asking questions that lead the students to see that each place to the left of the ones place is simply the next power of 10. Similarly, lead the students in a discussion by looking at the value in the tenths place as some number multiplied by $\frac{1}{10}$ or $\frac{1}{10^1}$. Give students time to establish the powers of ten for each place from the thousandths to the thousands place. Once the students have established the power of ten that represents each place value, introduce the idea of 10^o =1 so they can see that every place value is some power of 10.

Place the students in groups of 4 to play the game, *Where's the Best Place?* The object of the game is to build the greatest number possible. These are the rules for the game:

- Give each player a number card sheet which has several game cards and have groups play the game several times. There is a number card sheet sample after the activity pages.

Example of Game Card 1

0	.				

- Have students shuffle ten cards numbered 0 through 9 and place them face down in a pile.
- One player draws a digit card from the pile. Each player must decide privately where he/she wants to write that digit on his/her game card.
- After the player writes a digit on the card, he/she cannot erase it and place it somewhere else. Once a digit is drawn, it cannot be used again in that game.
- The game is over when all places on each game card are filled.
- Write an inequality using the four numbers generated by the group.
- The player with the greatest number wins.

After the class has played the game, have the students discuss some of the strategies they used as when trying to form the largest number possible. This might be easier for the students if they are asked where they would place a 7 if they got that number on the first draw and have them justify their reasoning.

Activity 4: Fraction Comparisons (GLEs: 7th – 1, 2; 8th – 1)

To check the depth of understanding in dealing with equivalent fractions, have the students complete the pizza problem below while working with a partner or in a group of 4. Circulate around the room and ask questions to find what strategies the students are using to find equivalent fractions.

Provide chart paper for the students to complete their work. Students should be prepared to share their thinking with other groups or the class in about 20 minutes.

Dana, Susan and Bill ordered a pizza. Each would receive the same amount, but Dana wanted her portion divided into 3 pieces, Susan wanted her portion divided in 2 pieces and Bill wanted his portion to be one large piece.

1. Sketch the pizza and label each student's pieces with his/her name and the fraction that represents the size of each piece as related to the whole. Use mathematics to show how the number of pieces each student gets forms a fraction that is equivalent to the other students' portions. Example: If one student asked for his part of the pizza to be 4 parts, this would be $\frac{4}{12}$ of the whole and would equal $\frac{6}{18}$ if another student wanted his part divided into 6 parts.
2. Look at the fractions used in number 1. Write two other fractions that are equivalent to each one.
3. Are the five fractions equivalent to each other? Why or why not?

Ask students to draw and shade in parts of three rectangular pizza pans to represent fractions that are not equivalent. Have students:

1. List the fractions that are not equivalent.
2. Write an inequality statement (using >, < symbols) to compare the fractions.
3. Explain how they know the inequality statement is correct.

Activity 5: Compare and Order! (GLEs: 7th – 1, 2; 8th – 1)

Have students work in pairs. Provide each pair a number line showing only the integers –10 to 10. Give each student a "deck" of index cards containing rational numbers, including some negative rational numbers. Student 1 should get a set of rational numbers in fraction form, and student 2 should get a set of rational numbers in decimal form. Have each student select a card from his/her deck and compare their numbers. The comparison can be done using a calculator, mental math, or paper/pencil. Ask students to correctly place both rational numbers on the number line and then write a correct statement using symbols. For example, if the two rational numbers were $\frac{1}{2}$ and 0.05, they would place a marks at the $\frac{1}{2}$ and the 0.05 points on their number line, with 0.05 being very close to the '0'. They would write a correct statement like "$0.05<\frac{1}{2}$."

Lead a discussion about the placement by asking the students why they are confident about the placement and the inequality. Make sure the students understand all inequality symbols as they write the inequality statements. This activity provides good formative assessment for the teacher to determine what students understand about the relative size of rational numbers.

Continue the process until all cards have been used or time has been called.

Activity 6: Integers! (GLEs: 7th – 7; 8th – 5)

Distribute 10, two-color pieces to each student (*e.g.*, red and black). Students will take all 10 pieces in their hands and drop the pieces on their desk. Have students separate the two colors and count the number of each. For *example, a student might have 3 pieces with black showing and 7 pieces with red showing.* Students should then match each red with one black and determine, emphasizing that they are creating 'zero pairs' as they match reds and blacks. *In the example given, there are 4 reds that don't match up.* Have students record the number of each color and the net value on a chart like the one provided at the end of this unit. Students should repeat this process 10 times and record their values in the chart. Discuss the concept of 'zeros pairs' when using integers. Have students go back to their charts and represent all of the reds as with a negative number and all the blacks as a positive number. Ask students to make observations and/or inferences as to how this relates to math and/or real life situations. (*e.g.,* money and debt, yards gained or lost in a football game, golf scores, altitude and below sea level). Write these observations on the board.

Divide students into pairs and give each pair a deck of playing cards with face cards removed. Instruct each student to turn one card face up. Values on red cards will represent negative integers and values on black cards will be represented by positive integers. Have students write the two integers and place an inequality or equality symbol between them to make a true statement.

Allow students to form groups of 3, and use the playing cards for a different game. Have each student place one card face up and find the sum of the values on the cards. Have students compete to determine who can find the correct sum first. Allow groups of students to challenge each other if they think the first person to answer has given an incorrect sum. They can either collect the cards to count the cards at the end of the game or get a point each time they have answered first correctly. Have students write at least 5 of their integer problems on a sheet of paper to exchange with another group to provide further practice in adding integers.

Activity 7: Multiplying/Dividing Integers (GLEs: 7th – 3, 35; 8th – 5)

Display a number line with positive and negative integers. Use the number line to demonstrate grouping of negative integers.

Figure a

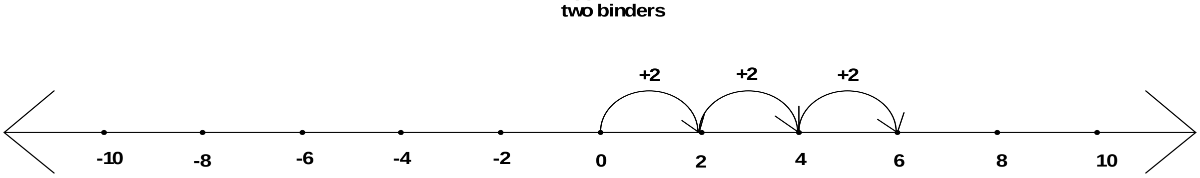


Figure b

3(-2) Susie paid back her three friends $2 each.

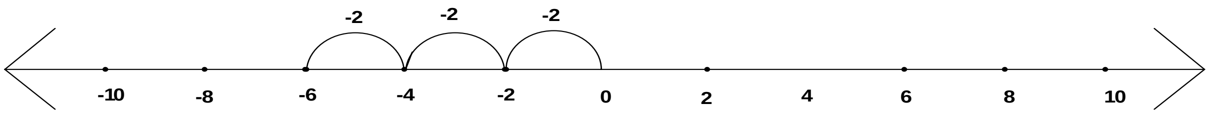
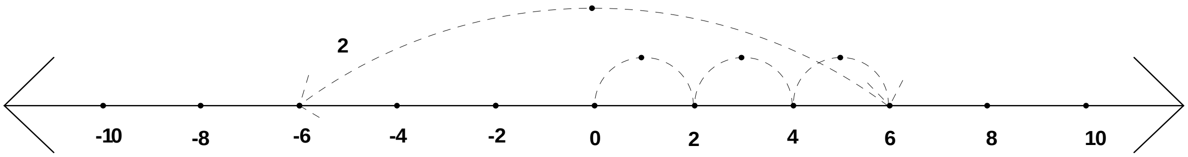


Figure c

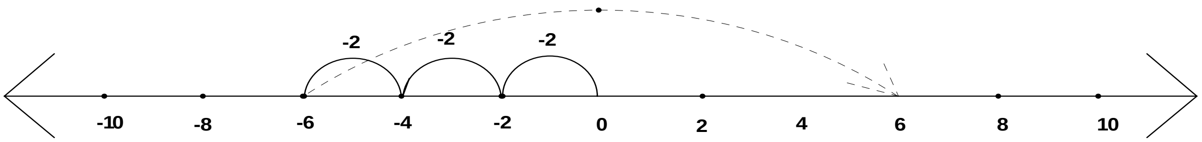
-3(2) This means the opposite of 3 groups of 2; therefore, what would be the opposite of Susie giving 3 friends $2 each? Susie owing three friends #2 each



The opposite of three groups of 2 is negative 6

Figure d

-3(-2) This means the opposite of 3 groups of -2; therefore, what would be the opposite of Susie owing 3 friends $2 each?



The opposite of three groups of -2 is positive 6

Example: 3(2) means three groups of 2 (fig. a); 3(-2) means three groups of -2 or the opposite of three groups of 2 (fig. b); -3(2) means the opposite of three groups of 2 (fig. c); -3(-2) means the opposite of three groups of -2 (fig. d).

Distribute grid paper and have the students sketch number lines to represent 5(-2), -4(3), 6(3), etc.

Have the students develop multiplication rules. Use *if/then* statements to develop rules for dividing integers, such as: If three groups of two equals six, then six divided into what size group will give you three in each group. If 3(2) = 6, then 6 ÷ 2 = 3, and if 3(-2) = -6, then -6 ÷ -2 = 3.

Extend this idea by setting up a situation in which a student has gathered party favors for a swimming party. If she wants to give each friend three favors and she has 81 favors, then to how many friends can she give two favors? Using these if/then statements, have students state the inverse operations.

Activity 8: Target! (GLEs: 7^{th} – 2, 3; 8^{th} – 1, 5)

Review the concepts of equal, greater than, and less than with students who will work in groups of 4. Write two fractions, decimals, integers (a brief review of integers may be needed) or percents on the board and instruct students to write an inequality representing the given numbers. Discuss different inequalities and symbols (<, >, ≤, ≥, or =).

Provide each group of four students a set of playing cards minus the face cards. If students cannot perform integer operations, all cards should represent positive numbers; otherwise black cards represent positive numbers and red cards negative numbers.

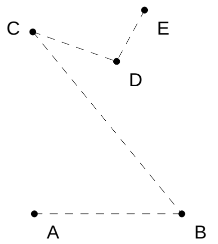
Have Player 1 place the first four cards in the deck face up and identify one of the four numbers as the target number. Allow players 2, 3 and 4 about 45 seconds to build an expression using the three cards that are not the target number and two operation symbols. Have the players compete to be the first to build an expression that results in the target number as the answer. All players should write their expressions using the order of operations and grouping symbols, if needed. If the expression is correct, award the player two points. If no one gets the target number, give the player closest to the target number one point. When the winner of the round has been determined, have group members compare their answers, writing them as a repeated inequality. After each round of play, have the player to the right of the last player turn the cards over and determine the target number.

Example: Suppose the four cards turned up are red 4, red 8, black 3, and red 7. Player 1 selects the black 3 as the target because there are 3 reds and negatives. One student writes 4 x 7 ÷ 8 and gets 3.5, the second student writes 8 ÷ 4 x 7 and gets 14 , and the third student writes 4 ÷ 8 + 7 = and gets 9. The students should write the inequality 14 > 7 ½ > 3.5 > **3**. The player with the answer of 3.5 is closest to 3 and receives 1 point.

Activity 9: Rates (GLEs: 7^{th} – 10, 11; 8^{th} – 7, 9)

Discuss the idea that when finding rates, a person is generally looking for a *unit rate (a ratio with one as the denominator)* to give us a comparison we can understand. Provide the students with a list of items they can purchase along with the prices. The items can be 6-paks of soda, ounces of potato chips, pounds of peanuts, etc. Be sure to use items that can be used to figure unit cost. Have the students figure the unit price of each item. Also, extend this to include rates such as $45.00 for 8 hours of work, driving 297 miles in 5 hours, reading 36 pages in 2 hours, etc. Have the students figure unit rates for these types of problems also. Have students take the unit rates they have found and record these as proportions such as, $\frac{297 miles}{5 hours} = \frac{54.9 miles}{1 hour}$.

Activity 10: Scaling the Trail! (GLEs: 7th – 10, 11; 8th – 7, 9)



Provide students with a sketch of a hiking trail like the one at the left. Label the points A through E. Give students a scale such as 2 inches represents 5 miles and have them find the length of the trail. Challenge the students to add another $1\frac{1}{4}$ miles to the trail by extending the trail from one of the endpoints. This will require them to determine the length of the segment that needs to be added to the diagram. Have students explain which method was used to determine the length of the 1 ¼ mile stretch of the hiking trail.

Activity 11: Is it Reasonable? (GLEs: 7th – 7, 8; 8th – 3)

Provide students with a list of real-life situations involving positive fractions and decimals. Individually, have the students estimate each answer. If students are required to write their estimates in ink, it will help provide an idea as to their number sense when reviewing their papers. As a class, discuss their estimates and methods used for estimating.

Example problem: 24% of the 7th graders at West Middle School are helping tutor 4th graders at West Elementary School. If there are 322 seventh graders at West Middle School, estimate how many seventh grade students are tutoring the 4th graders.

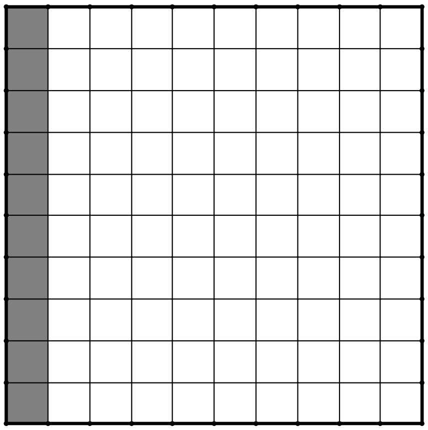
After the students have had time to make estimates, give the students a list of the correct answers, and have them compare their estimates to the correct answers. Discuss the operations needed to solve the problems. Discuss as a class why some estimates may not have been close.

Activity 12: Simple Percent Problems (GLEs: 7th – 6, 8, 9; 8th – 8)

Introduce the idea of shopping when a store is having a sale. Discuss discounts that the students remember having seen in store advertisements. Place a 10 x 10 grid on the overhead. Have students use a 10 x 10 grid at their desk and explain to them how the grid can be used to understand percent problems. Ask students if they can think of a way to represent 10% of the grid (*shade 10 squares*). Practice other percent representations, if necessary.

Tell students that for this activity, the entire 10 x 10 square represents 100% of the original cost of an item. They will have to determine the value of each 1% (one small square) and the value of 10% (one strip) based on the original cost. The diagram below shows a model of a 10% discount on the 10 x 10 grid.

shirt costs $25.00
discount 10% off



entire grid = $25
Shaded area represents 10%

25/100 makes each square is worth $0.25
25/10 makes each strip is worth $2.50

Therefore: 25 - 2.50 = 22.50
Sale price = $22.50

Some students will figure the value of each small square and some will use the value of one strip. Encourage students to represent these percents as a ratio or proportion. Continue asking questions to help students see how to set up and solve simple percent problems with the use of equivalent ratios. There is a practice sheet using 10 x 10 grids after the activity section in this unit.

> *Example:* $\frac{x}{25}=\frac{10}{100}$ *would show the amount of discount; students would then determine the cost of the shirt by subtracting.* $\frac{x}{25}=\frac{90}{100}$, *would show the percent of the whole that was paid and the student would have subtracted the 10% from the 100% initially.*

Using the store sale advertisements from the newspaper, make a transparency of a few items with the prices. Have the students break into groups of four to figure 10%, 20%, 30%, 50%, 75%, $33\frac{1}{3}\%$, and $66\frac{2}{3}\%$ off the cost of items in the advertisements, or figure the sale price using the percent given in the ad. Initiate a discussion about whether an exact cost or an estimate is used when deciding what to purchase from these ads.

Many times items are advertised as $\frac{1}{3}$ or $\frac{2}{3}$ off the original price. Make sure students know that these are the same as $33\frac{1}{3}\%$ and $66\frac{2}{3}\%$ off. Have the students check to see if their answers are reasonable. Have students practice estimating 10%, 20%, 30%, 50%, 75%, $33\frac{1}{3}\%$, and $66\frac{2}{3}\%$ off the items in the ads, and then compare these estimates to the answers they originally figured.

Give each group a budget and assign different discounts. Have students choose items from the sale papers, estimate the percents to determine if they have enough money to make the purchases they want, and then calculate the exact prices. On a quarter/half sheet of poster board, have the students create a display indicating their choices, the method used to calculate each price, and the total cost of their purchases. For example, for a $\frac{1}{4}$-off sale, students could determine how much the discount is and then subtract from the original price, or they could determine that $\frac{3}{4}$ of the original price still has to be paid and thus they would find $\frac{3}{4}$ of the original price. Allow students to cut and paste pictures of the items and require them to show their work. Have students determine the final total price by including the calculation of any taxes.

Discuss strategies for determining the presale price of several "on-sale" amounts and show them in examples. For example, if the sale price of an item is $40 and this reflects a 20% discount, have students determine the original price. This is a good activity for students to justify their problem-solving solution and compare strategies with other students.

Activity 13: Four Is a Winner! (GLEs: 7th – 6; 8th – 8)

Provide students with a sample game card as shown in the Activity 13 game card found after the activity section in this unit. Each pair of students needs 2 different color paper clips, marker chips, and/or two different colored markers.

To play the game,

- Have the tallest student go first by placing one color paper clip on a percent expression and the other color paper clip on a number in the row below the expressions. For example, Student 1 places a blue paper clip on "25% of" and a yellow paper clip on "160" in the bottom row of numbers. Student 1 should then mark his answer for 25% of 160 (*40*) either by placing a chip over the correct answer or by marking with a colored marker.
- Next, instruct Student 2 to move *either* the blue or the yellow paper clip to create a new problem and find the answer on the game board. For example, Student 2 might move the paper clip on "25% of" to "50% decrease" and Student 2 would then place his marker on "80."
- Continue play until one player gets four in a row, horizontally, vertically, or diagonally.
- Have students record the problems and answers on paper so that wins can be justified.

Activity 14: What Is Needed? (GLEs: 7th – 9; 8th – 3, 8)

In groups of four, have students brainstorm to develop a list of scenarios that require exact answers to problems involving decimals and percents and a list of scenarios that do not require exact answers. For example, if a person were shopping for groceries and had $30 in cash, then an estimate of the cost of items in a grocery cart can be used to determine if some of the items should be placed back on the shelf. Have groups write word problems for one scenario that requires an estimate and for one requiring an exact answer. For each scenario have students show the mathematics involved to make a good estimate and share their methods with their groups or the class. Students can exchange their problems with another group and have them solve the problem and check their estimates, then return to the group for checking, or the teacher may want to use some of their scenarios on a quiz.

Sometimes students will see sales tax at ½ % or ¾%. Ask the students how they might determine ½ % of $10.00. Have them determine how to find these small percentages.

Solution: $\frac{\frac{1}{2}}{100} = \frac{x}{10}$

$$5 = 100x$$

$$\$0.05 = x$$

Activity 15: Tipping at a Restaurant (GLEs: 7th – 6, 8, 9; 8th – 7)

Discuss with the class the tip that customers leave at restaurants, noting that customers pay their server a tip for providing good service. A typical tip is 15% to 20% of the cost of the meal. Indicate to students that they need to use estimation skill to figure a tip that will be left for the server because the check will not come out to exact dollars. Discuss with the students how to round in reasonable ways. Discuss the mental math strategies when finding the tip at a restaurant. Present the following situation to the class. Your bill at Logan's Restaurant is $19.45. What is a 10% tip on this bill? Instruct students to round off the amount to something they can reasonably work with. Some may say $19.50 but will this be reasonable for the situation? Would you leave $1.95 or $2.00? So a better process might be to round $19.45 up to $20.00 and then calculate a 10% tip of $2.00.

Activity 16: How Much Did I Start With? (GLEs: 7th – 3; 8th – 5, 6)

Provide the students with the following situation:

> James' mom gave him a large sum of money for his 7th grade field trip. James counted the money left in his wallet on the trip home and he had $12.00. He could not remember how much money his mom had given him, so he started going through what he had spent. He spent 10% of his money on breakfast. He then remembered that he spent $\frac{1}{2}$ of what he had left after breakfast on a watch. The only other money that James had spent was the $6.00 it cost to get into the museum at the end of the day. Determine how much money James' mom gave him for the field trip. Explain the method you used.

Solution: Working backwards, add the $12 and the $6 to get $18. This $18 is $\frac{1}{2}$ *of what he had left after breakfast so he had $36 after breakfast. If the $36 was 90% of what he started with, then 10% is $4 and he therefore started with $40.*

Activity 17: Computing Using Scientific Notation (GLEs: 7th – 3, 12; 8th – 2, 4, 5)

Begin the activity with a discussion including the fact that scientific notation is a method of recording very large and very small numbers using powers of ten. Ask the students to determine a way to rewrite 234 as some number multiplied by a power of ten. Since 234 is not considered a very large number, it is generally written as 234. It may come up that 23.4 x 10 meets the stated criteria. If students don't recognize 2.34×10^2 as another option, lead them to that conclusion. Repeat the process with another number or two, such as 3,506 and 12,000, making sure that students list all the possibilities. Lead students to understand that since there may be more than one way to rewrite a number using a power of 10, a standard format for rewriting numbers is used. Provide students with the rule for writing a number in scientific notation, showing examples of both the number in standard form and the number written in scientific notation. Make sure the students understand that with scientific notation, the largest place value used is the ones place. For example: 1,500,000 miles can be represented using scientific notation as:

$$\underbrace{1.5}_{\textit{A number greater than or equal to one, but less than ten}} \text{ X } \underbrace{10^6}_{\textit{A power of ten}}$$

Other examples:

$7000 \text{ m} = 7 \text{ X } 10^3 \text{ m}$ $360{,}000 \text{ m/s} = 3.6 \text{ X } 10^5 \text{ m/s}$

Students need to understand that using positive exponents in scientific notation allows them to rewrite very large numbers such as those used by scientists at NASA when recording distances in space.

Teacher Note: The GLEs for grades 7 and 8 indicate that only positive exponents are to be addressed. This is because students will probably not have mastered operations with integers and may be confused with the use of negative exponents as a way to indicate division by a power of 10. Teachers should use their discretion in deciding whether to show students how to write very small values in scientific notation; however some calculators give answers in scientific notation. Should this happen, it may be an appropriate time to introduce this concept.

Give students a set of computational problems that involve large values, multiple operations, and grouping symbols. Have students rewrite the problems using scientific notation and then compute. For example:

- The planet Mercury is 58,000,000 kilometers from the sun. The planet Pluto is 10^2 times farther from the sun than the planet Mercury. About how far is the planet Pluto from the sun?
- Samantha's bicycle tire has a diameter of 65 centimeters. She had a counter on her bicycle that counted the number of complete rotations of the front tire of her bicycle. She figured the circumference was about 204 centimeters and that if her counter said 10^6, then she had traveled 204,000,000 cm but her calculator said $2.04 \text{ x } 10^8$. Use your calculators to determine how the calculator representation relates to Samantha's or give an example of how Samantha's calculator represents 204 million centimeters.
- How old is a person who is one billion seconds old? Explain your reasoning. Represent your answer using the number of seconds and scientific notation, next simplify your answer using years, months, weeks, days, minutes, and/or seconds.

Lead a class discussion, having students explain their results and how they represented the numbers in scientific notation. Have students make some comparisons of these large numbers and develop conjectures as to how the power of ten helps determine the size of the number.

Activity 3
Number Card Sheet

Game #1 card		.				
Game #2 card		.				
Game #3 card		.				
Game #4 card		.				
Game #5 card		.				
Game #6 card		.				
Game #7 card		.				
Game #8 card		.				
Game #9 card		.				
Game #10 card		.				

3)

Activity 6
Chart

Integers!

Throw	#red	#yellow	Net value
1	7	3	4 red
2			
3			
4			
5			
6			
7			
8			
9			
10			

Observations from chart:

1)

2)

3)

Activity 13

Four’s a Winner Game Card

320	400	10	250	50	225
90	20	270	100	150	15
150	120	80	30	240	75
180	60	25	200	5	125
40	100	50	135	90	45
75	10	360	20	60	300

Paper clips go on one percent expression and one number in the list below. Solve the problem and place your marker on the game card above.

25% of	25% increase	25% decrease
50% of	50% increase	50% decrease

20	40	60	80	100	120	160	180	200

Sample Assessments

General Assessments:

- The teacher will determine student understanding as the student engages in the various activities.
- Whenever possible, the teacher will create extensions to an activity by increasing the difficulty or by asking "what if" questions.
- The student will be encouraged to create his/her own questions.
- The student will create and demonstrate math problems by acting them out or using manipulatives to provide solutions on the board or overhead.
- The teacher will observe the student's presentations and use a rubric to assess.
- The student will complete journal entries by responding to prompts such as:
    - Explain the meaning of 10%, 20%, 25%, $33\frac{1}{3}\%$, 50%, $66\frac{2}{3}\%$, 75%, and 100% and write their fractional and decimal equivalents. Give examples of their use in real-life situations.
    - Newspapers often have coupons for discounts on many different items. For example, Academy had a coupon for $1.50 off of a pair of socks. The socks regularly sell for $5.00. Shauntell wanted to find out the percent of discount this is. She thought about the problem this way:

      "*I need to find what percent $1.50 is of $5.00. I can think of these amounts as pennies. The fraction I want to represent as a percent is* $\frac{150}{500}$*, which is equivalent to* $\frac{30}{100}$*. As a decimal, this fraction is 0.3 or 0.30. That means the discount is 30%.*"

      Is Shauntell correct in her thinking? Explain your answer.
- The student will create a portfolio and include evidence of knowledge of the content of the unit. Example:

  The 4-H advisor is in charge of buying drinks for the club's landscaping day. She conducted a survey to determine if students liked Dr. Pepper® or Coca Cola®. Here are her results:

	Grade 6	*Grade 7*	*Grade 8*
Dr. Pepper®	*80*	*75*	*85*
Coca Cola®	*70*	*90*	*80*

Tell whether the statements 1-4 are accurate based on the information in the table.

1. 15 more seventh graders prefer Dr. Pepper® to Coca Cola®. Explain your answer.
2. The ratio of seventh graders who prefer Dr. Pepper® to Coca Cola® is 5 to 6. Explain your answer.
3. 50% of the students surveyed prefer Dr. Pepper®. Explain your answer.
4. $\frac{7}{8}$ of the sixth graders prefer Coca Cola®. Explain your answer.

Tell whether an exact answer or an estimate is needed to determine the grade in which 52% preferred Dr. Pepper®. Explain your answer.

The Coca Cola® for the party would cost $168 and the Dr. Pepper® would cost $180. Are the students paying the same price for Coca Cola® and Dr. Pepper®? Justify your answer using unit rates.

- The teacher will give the student a list of about fifteen rational numbers including fractions, decimals and percents, making sure that some of the values are equivalent (*i.e.,* $\frac{1}{4}$ and 25%*)*. The students will make a number line and place all fifteen rational numbers along the number line in the correct position. To complete the assessment, the student will write at least 10 inequality statements using the symbols <, >, ≥, and ≤.

Activity-Specific Assessments:

- Activity 5: Given a list of 8 different representation of numbers (fractions, decimals, percents) and a blank number line, the student will place the numbers in the correct position on the number line and write three inequalities using the given numbers and the symbols <, ≤, ≥, >.

- Activity 6: The student will discuss strategies that could be used by Player 1 when choosing the target number from the four cards turned up. The teacher will ask group members to choose one strategy that they think is best and share with the class.

- Activity 11: The teacher will present the following scenario to the student and evaluate the student's ability to answer the questions asked orally.

  Latoya is at a grocery store near her house. She has $10.00, but no calculator or paper or pencil. At the right is a list of the items she would like to buy. Use mental calculations and estimation to answer the following questions.

Item	Price
Milk	$2.47
Eggs	$1.09
Cheese	$1.95
Bread	$0.68
Honey	$1.19
Cereal	$3.25
Avocado	$0.50

1. Can she buy all the items? Why or why not?
2. What different items could she buy to come as close as possible to spending $5.00?
3. Approximately what percent of the $10.00 did Latoya spend on eggs?

Solutions:

1. No, she cannot buy all the items.
2. Solutions may vary: Sample solutions: milk, avocado, and cheese or eggs, cheese, honey and avocado
3. $1.09 out of $10.00 is about 10%.

- Activity 15: The student will prepare a chart and an explanation to James' problem:

  A local store has a sale rack for clearance merchandise. The sign on the rack says, ***"Marked down an additional 20% each day!"*** James has been thinking about buying a jacket that costs $100. The clerk tells him it will be moved to the sale rack tomorrow. James is happy about this and decides he'll go back to the store in five days when the jacket will be free. When he gets to the store five days later, he sees that the jacket is not actually free. What price is really marked on the jacket? Why did James think the jacket would be free? Explain your thinking.

Solution: Day 1 - $80; Day 2 - $64; Day 3 - $51.20; Day 4 - $40.96; Day 5 $32.77 (this is a rounded answer). On day 5, the jacket is marked $32.77. James thought that 20% of $100 would be subtracted each day, thus leaving a balance of zero on day 5.

Grade 7
Advanced Mathematics
Unit 2: Rational Numbers, Proportional Relationships, and Counting Techniques

Time Frame: Approximately four weeks

Unit Description:

This unit extends the work of the previous unit to include the operational understandings of multiplication and division of fractions and decimals and their connections to real-life situations. Use of rates, ratios, proportions, and percents is presented in real-life settings. An introduction to combinations, permutations, and counting techniques are included.

Student Understandings

Students develop an operational understanding of multiplication and division of fractions and decimals and focus on real-life proportional relationships and solutions involving rates, ratios and percent. At the same time, they will become proficient in rational number computations using the order of operations. Students have a full understanding of percents, including those greater than 100 and less than 1, as well as percent of increase and decrease. They will find rates and apply them in solving real-life problems involving proportions, including those involving fractions and integers. Students will use lists, diagrams and tables to solve problems involving combinations and permutations.

Guiding Questions

1. Can students multiply and divide fractions and decimals with understanding of the operations and accompanying representations?
2. Can students determine when computations are required or just estimates in real-life settings?
3. Can students handle all percentage problems (<1, >100, % increase, % decrease)?
4. Can students set up and solve proportions representing real-life problems, including those with fractions, decimals, and integers?
5. Can students interpret, model, set up, and solve proportions linking the measures of sides of similar triangles?
6. Can students apply concepts of combinations and permutations?

Unit 2 Grade-Level Expectations (GLEs)

GLE#	GLE Text and Benchmarks
7th grade	
Number and Number Relations	
2.	Compare positive fractions, decimals, percents, and integers using symbols (*i.e.,* <, ≤, =, ≥, >) and position on a number line (N-2-M)
3.	Solve order of operations problems involving grouping symbols and multiple operations (N-4-M)
5.	Multiply and divide positive fractions and decimals (N-5-M)
6.	Set up and solve simple percent problems using various strategies, including mental math (N-5-M) (N-6-M) (N-8-M)
7.	Select and discuss appropriate operations and solve single- and multi-step, real-life problems involving positive fractions, percents, mixed numbers, decimals, and positive and negative integers (N-5-M) (N-3-M) (N-4-M)
8.	Determine the reasonableness of answers involving positive fractions and decimals by comparing them to estimates (N-6-M) (N-7-M)
9.	Determine when an estimate is sufficient and when an exact answer is needed in real-life problems using decimals and percents (N-7-M) (N-5-M)
10.	Determine and apply rates and ratios (N-8-M)
11.	Use proportions involving whole numbers to solve real-life problems (N-8-M)
Measurement	
22.	Convert between units of area in U.S. and metric units within the **same** system (M-5-M)
Data, Statistics, Probability and Discrete Math	
34.	Create and use Venn diagrams with three overlapping categories to solve counting logic problems (D-3-M)
35.	Use informal thinking procedures of elementary logic involving *if/then* statements (D-3-M)
36.	Apply the fundamental counting principle in real-life situations (D-4-M)
37.	Determine probability from experiments and from data displayed in tables and graphs (D-5-M)
8th grade	
Number and Number Relations	
1.	Compare rational numbers using symbols (*i.e.,* <, ≤, =, >, ≥) and position on a number line.
3.	Estimate the answer to an operation involving rational numbers based on the original numbers (N-2-M) (N-6-M)
5.	Simplify expressions involving operations on integers, grouping symbols, and whole number exponents using order of operations (N-4-M)
6.	Identify missing information or suggest a strategy for solving a real-life, rational-number problem (N-5-M)
7.	Use proportional reasoning to model and solve real-life problems (N-8-M)
8.	Solve real-life problems involving percentages, including percentages less than 1 or greater than 100 (N-8-M) (N-5-M)

9.	Find unit/cost rates and apply them in real-life problems (N-8-M) (N-5-M) (A-5-M)
Geometry	
29.	Solve problems involving lengths of sides of similar triangles (G-5-M) (A-5-M)
Data Analysis, Probability, and Discrete Math	
36.	Organize and display data using circle graphs (D-1-M)
42.	Use lists, tree diagrams, and tables to apply the concept of permutations to represent an ordering with and without replacement (D-4-M)
43.	Use lists and tables to apply the concept of combinations to represent the number of possible ways a set of objects can be selected from a group (D-4-M)

Sample Activities

Activity 1: The Meaning of Multiplication of Fractions (GLEs: 7^{th} – 5, 8)

Ensure that students get a real sense of multiplying fractions and making the connection to the meaning of multiplication. Ask the students to draw a picture and/or describe in words: 3×4. (*Teacher Note: The students should write in words and model three groups of four and/or four groups of three.*) Have a class discussion of a real-life meaning of this problem (*e.g.*, Sam has three groups of candy bars with four candy bars in each group). Extend this concept to include multiplication of a fraction and a whole number (*e.g.*, $3\times\frac{1}{2}$). Write each problem on the board and ask a student to model it for the class. (*e.g.*, three groups of $\frac{1}{2}$ and/or $\frac{1}{2}$ group of 3, drawing the groups). The advanced seventh grade students should be able to grasp the idea of multiplying fractions as combining fractional groups and move on to practice with situations. Provide many situations such as:

> *Jamie wants to give each of her 8 friends a necklace that requires ¾ yards of ribbon. If she wants to purchase the ribbon in only one color, how many yards should she buy? Explain your thinking.*
>
> *Possible Answer: Jamie should purchase 6 yards of ribbon. She will have ¼ yard left from each of the 6 yards so she will have the equivalent of 6 lengths of ¼ yard each. This is the same amount as 2 units that are each ¾ yard. 6 units plus 2 units will be the needed 8 units of ¾ yards.*

Encourage students to discuss these problems in their groups and justify the reasonableness of their answers. After doing several of these types of problems, have the students make estimates prior to determining a solution. Be sure to ask the students to create a rule for multiplying whole numbers and fractions. Continue practicing and modeling various situations.

Ask the students to think about how they could use a similar process to multiply a fraction by a fraction. First, have them state the problem in context. For example, ½ x ½ , could be “ ½ group of ½ dozen” – which would be ½ of 6 or 3. 3 is ¼ of 12, so ½ x ½ is ¼ of the dozen. It will be important to encourage the attachment of units to these fractions or the students will have difficulty developing the meaning of the situations. Create many possible situations in which the students multiply fractions by fractions. Each time the students model and explain their answers, have them check to see if the answers are reasonable.

Have the students work in groups to create word problems that involve multiplication of fractions, whole numbers, and mixed numbers. Have them show a model of the problem and a mathematical sentence that illustrates the situation. Monitor the groups and ask questions to ensure that their problems are sensible. Have groups exchange their problems with another group for solving and discussion. It is difficult for the students to get started writing word problems, Encourage students to start with whole numbers and substitute fractions into the situation they write, checking to see that the use of the fractions makes sense.

Activity 2: The Meaning of Division of Fractions (GLEs: 7th – 5; 8th – 7, 8)

In multiplication, most students understand that 4 groups of 2 objects gives a total of 8 objects. They need to relate division of fractions to their understanding of the division problem, $8 \div 4$. Students have difficulty in stating the meaning of division—take a total of 8 candy bars and divide the bars among groups of 4 students, which means that each group of 4 students gets 2 candy bars (from 6th grade lessons, this represents 'sharing'). Have the students give the percent of the total number of candy bars that each student now gets. (25%) Have the students think about a situation in which students are filling bags and must put 4 pieces of candy in each bag. If this is the case, they will fill only 2 bags (from 6th grade lessons, this is grouping). Have students give the percent of the total number of candy bars that each student gets.

Provide a situation in which the students have been rewarded by the teacher and will receive 3 candy bars instead of 2 in their bag. Ask, If a student was to get 2 candy bars, but got 3 instead, what is the percent of increase in candy bars? Lead a discussion as to the definition of percent of increase. Since these students received one extra candy bar instead of 2, there is a 50% increase. Have the students determine 50% of 2 and discuss how finding a percent of a number is different than finding a percent of increase.

Change the problem by telling the students that they have 15 candy bars divided into groups of three candy bars. Ask them how a diagram of this would differ from 15 candy bars divided among three people. Have someone put the diagrams on the overhead or board for class discussion. Tell the students that the teacher is going to increase the original number of candy bars to share to 25 candy bars. Have them determine the percent of increase in the number of candy bars.

$$\frac{10\text{ (difference in the number of candy bars)}}{15\text{ (the original number of candy bars)}} = \frac{\text{percent of increase or decrease}}{100} = \frac{1000}{15} = 66\frac{2}{3}\%\text{ increase}$$

Write a problem on the board (*e.g.,* 24 items ÷ 6). Have students write a situation for the problem, and then solve to determine the percent of increase if the total changed to 30 items. Once they determine that the percent of increase is 25%, ask them if a 25% decrease of this new total will give them the 24 items they started with. Repeat the process with these concepts until the students have a good understanding of division and percent of increase and decrease.

Extend student understanding to include division with a fraction: The problem $8 \div \frac{1}{2}$ can be modeled by dividing 8 candy bars into ½ pieces. Have a student model the problem for the class in words and draw a picture. The picture helps students see division (8 candy bars broken in half would result in 16 pieces). Repeat the process using several examples. Record all problems on the board with the intent that one or more of the students will see a pattern or rule after doing a series of problems. The rule must fit all types of problems they will encounter. Take time to discuss students' methods before moving on to dividing fractions by fractions and dividing fractions by mixed numbers. Have the students model the situation – Julie has ½ of a large candy bar and must share equally with 4 friends. What fractional part of the candy bar will each friend receive? One way to analyze the situation is to divide the half into four pieces and say, if ½ has 4 pieces, how many small pieces would be in the whole candy bar? (Thus, each piece of candy is 1/8 of the whole). Do enough of these situations that the students have opportunity to develop a rule for dividing fractions.

Finally, have the students write word problems using fractions. Writing word problems is always difficult and especially with fractions! Just make sure the students attach situations to the problems so that they make sense.

Activity 3: Practice with Order of Operations and Rational Numbers (GLEs: 7th – 3, 5; 8th – 5)

Review the order of operations and steps. Prepare two boxes for each group of 4 students, one with cards containing rational numbers and the other containing operation and grouping symbols. The numbers should include integers -9 through 9; some fractions, and some decimals. Have students, working in groups of four, select three cards from the rational number box and two symbols (operation and grouping). Each group will use the numbers and symbols to create expressions to challenge other students' understanding of the order of operations. Have each group record the expression and its solution on a card. Give students time to form 4 – 5 different expressions and find the solutions. Set a time limit of three minutes for each problem.

When students have completed the task, allow the group that created an expression to present it to the class. Other groups are to find the answer and record it on paper. (Alternative: Have groups write their answers on small white boards using dry erase markers. When time is called, have each group show what is written on their white board.) Give one point to the team presenting a correct solution and give two points to each group that challenges an incorrect solution and has the correct solution. The team with the most points wins.

Teacher Note: Inexpensive 4' x 8' sheets of white tileboard (normally used in bathrooms) are available at home improvement stores and contain the same material used to make more expensive white boards available through school supply stores. Personnel at the store will cut the sheets into 2' x 2' squares for a nominal fee.

Activity 4: Cooperative Problem Solving (GLEs: 7th – 5, 7, 10, 11; 8th – 7, 9)

Have students work in groups of two or three to solve problems involving real-life situations. Separate the facts and the question for the problem and have the parts written on three different cards. Give each student in the group at least one card and instruct the student to keep it in his/her possession. Have students in each group take turns sharing the information on their cards, then work together to find a solution. Make sure that each set of cards involves use of proportions. This is a good tool to get all students involved in the problem-solving process. Even the weakest students have a part, because they must share the information on their cards in order for the problem to be solved. Below is an example of a set of cards that one group might solve:

The seventh graders are planning to sell cups of hot chocolate at the basketball games this winter.

It takes 4 ½ spoonfuls of hot chocolate mix make a cup of hot chocolate.

How many spoonfuls will be needed to make 42 cups of hot chocolate?

(Another example problem: Card 1 - Jared has 4 ¾ pizzas to share. Card 2 - The pizzas are divided into eighths. Card 3 - What fractional part do Jared and each of his 5 friends get if they are divided equally?)

Activity 5: Map It! (GLEs: 7th – 7, 9, 22; 8th – 3, 9)

Divide students into small groups of two or three and give them U.S. road maps or U.S. Atlases to use. (The social studies teacher may be able to provide these.) Write names of cities on paper strips and place them in one bowl and write names of consumer-goods (such as fresh flowers or food products) on other strips and place them in a second bowl. Have each group draw one strip from the consumer-goods bowl and two cities from the city bowl. Perishable items should include items that will spoil quickly if refrigeration should break down or if they are on the road too long. Ask students to use their maps to find the best route for the delivery of their perishable item from one of the cities they have selected to the second city. Have them use their maps or atlases to determine distances between the two cities.

Note: If the atlases are not available, students can go to an online map service for directions and distances written in decimal format. If students do not have access to computers, the teacher may want to go to one of the online map sites and print directions between two cities that the students might recognize; be sure to delete the total distance from the printout. If this method is used, have students use the decimal values provided by the map service to determine the total distance to the nearest tenth of a mile. This also a good time to have students convert distances written in tenths of a mile to miles and feet or yards (to practice the conversions within the same system).

Tell students they are to travel at 50 miles per hour and have students report the time it would take to make their trip. Time should be written as a fractional part of an hour. Have students find the current cost per gallon of gas (or provide it to them) and have them calculate the cost of their gasoline if the car they were driving averages 18 miles per gallon of gasoline. A site which will provide average cost per gallon of gas by zip code can be found at http://autos.msn.com/everyday/GasStations.aspx?m=1&l=1&zip=82070&x=20&y=5 . As an extension, tell them that their gasoline cost for their trip was $50 and have them determine the average cost per gallon for their trip.

Place a third bowl of obstacles/hazards that might occur along the trip (*e.g.,* there was road construction for 10 miles and we traveled at an average speed of 10 mph for this distance, there was an accident with a farm truck and vegetables were spread all over the roadway and we were stopped for 30 minutes). Have the students recalculate their average speed for the trip based time delays created by the obstacle.

Example: You are traveling from Baton Rouge to Mobile at a speed of 65 miles an hour. How long will it take you to get there? There is a wreck on the interstate and you are stopped for 45 minutes. How much time does this trip require? What would the average rate of speed be? If you leave at 6:00 A.M., when will you get there? Your engine light comes on and you slow to 40 miles per hour for the last 50 miles. How long will it take you to get there? How does this affect your average speed for the trip? Hazards can include vehicle break down, road construction, or weather. With each hazard, have students assess the necessity of accuracy in timing depending on the perishable item they carry. Ask students to present their situation and solutions to the class. Have the students decide if it would be appropriate to estimate the answers or if exact answers are needed for each situation and justify their reasoning.

Activity 6: Bull's Eye (GLEs: 7th – 2, 8; 8th – 1, 3)

The Bull's Eye target and the Bull's Eye Chart used for this activity are found after the activity section in this unit. Give students 5 minutes to complete the estimation column of the chart (column 2). Then, ask the students to find the exact answers and fill in column three. Have students calculate the values for column four which provides ratios of the estimates to the exact answers. Have students use the Bull's Eye target to determine their points earned based on how close an estimate was made and write this in column five. Lead a discussion as to why best answers are those closest to one. Have students use > , < , or = to compare their estimates to the exact answers in column six.

Activity 7: How good were my estimates? (GLEs: 7th – 8; 8th – 3, 36)

Using the data from the Bull's Eye Chart, have the students determine the percent of 10 point, 5 point, 2 points and 1 point answers they gave. Instruct students to create a circle graph based on the fraction of their estimates that resulted in each of the point values. Ask students to explain to their partners or group members what the data shows them about their estimated answers.

Activity 8: Which is greater? (GLEs: 7th – 6; 8th – 8)

Have students model percents less than ½ % and greater than 100% by providing students with 10 x 10 grids that represent 100%. This is an extension of the model used in Unit 1 to show a visual representation of percents. Ask students shade in different percents such as 50%, 10%,$12\frac{1}{2}\%$, 150%, 2.5%, 75%, $\frac{1}{2}\%$,$\frac{1}{4}\%$. Check for understanding as they shade these different percents while circulating around the room. Have students share their ideas with the class and discuss different representations. Do not say the percentages aloud; have the students shade them from the written representation as the discussion that surfaces between their understanding of 50% and $\frac{1}{2}\%$ can uncover some misconceptions. After the students have shaded the given percents, have some real-life situations ready for the students to model and solve.

Examples:
1) ½ % of the seventh grade students missed more than 10 days last year. If there are 146 seventh grade students, how many missed less than10 days? *The 10 x 10 grid would equal 146 students or 100% and each 1% represents 1.46 students. So, ½ of 1% would be .73 or 1 student missing more than 10 days, so 145 missed less than 10 days.*

2) The jewelry store marks up gold jewelry 150%. If the store bought a piece of jewelry for $30, show a representation of the price of the jewelry when it is sold by the store? *The grid would represent $30 or 100% of what the jewelry store paid. The student should shade three grids. Two grids should be shaded completely to represent the initial $30 and the second to represent the 100% increase. The third grid should have ½ of the squares shaded to represent $15. The selling price of the jewelry would be $75.*

Activity 9: Order! (GLEs: 7th – 2; 8th – 1, 6)

Provide pairs of students with a number line that starts at 0 and ends at 4, with each unit divided into fourths. Make cards with rational numbers on them, being sure to include fractions, decimals and integers. To model the activity for the class, put masking tape on the floor. Use a strip that is 8 feet long, label one end ‘0’ and the opposite end ‘4’, and mark fourths between each whole number. Ask a student (#1) to select a card which has as distance written on it (*e.g.,* ¼ unit). Model this activity by instructing another student (#2) to start at zero and move ¼ of the distance between 0 and 1. Student 1 selects three more distances from the cards for student 2 to move (*e.g.,* –5/8, +1/2, - ¾) and writes each of these distances in an equation on the board (1/4 + -5/8 + ½ + -3/4 =). Ask student 2 to name the point on which he/she ends. The expression now becomes an equation with a solution (¼ + -5/8 + ½ + - ¾ = -5/8). This can become a game like *Mother, May I?*. Repeat this activity several times.

To move the activity to a more abstract level, have student 1 draw three cards with distances written on them and have student 2 determine the ending point by finding the sum of the values. If needed, students can check their work by modeling the problem on a number line after determining the sum. *For example*, if $\frac{1}{4}$ is the starting point, and the moves are $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$, then the ending point of $1\frac{1}{4}$ would be determined by finding the sum of $\frac{1}{4}+\frac{1}{4}+\frac{1}{2}+\frac{1}{4}$. Encourage students to include reverse moves. For example, if $\frac{1}{2}$ is the starting point and the moves are $\frac{1}{4}$, $-\frac{1}{2}$, $\frac{3}{4}$, then the ending point would be determined by $\frac{1}{2}+\frac{1}{4}-\frac{1}{2}+\frac{3}{4}=1$. Repeat this activity several times.

As an extension to this activity, have student 1 select a starting and ending point on the number line. The second student now has the first number in the expression and the net value or sum. The second student should then give at least 3 moves that will result in the ending point.

Activity 10: Proportional Reasoning in Similar Triangles (GLEs: 7th – 11; 8th – 7, 29)

This activity allows students to apply the concept of similar triangles. Begin by distributing ¼ inch grid paper and have the students sketch right triangles with leg lengths of 5 and 6 units. Once they have sketched the triangle, have them sketch a second triangle with leg lengths twice as long as the original. Discuss the proportions formed: 5 units/10 units = 6 units/12 units. Have students use protractors to measure the angles formed in each of these triangles. Discuss properties of similar triangles.

Have students calculate the height of various objects by measuring the object's shadow and the shadow of a meter stick placed vertically in the ground. Have students use proportions to solve for the unknown heights. Lead students to understand that they can solve the problems by creating a proportion between the corresponding parts of the right triangle formed by the object and its shadow and the right triangle formed by the meter stick and its shadow. Have students sketch and label dimensions of the corresponding parts of the similar triangles formed with these objects.

Distribute another sheet of grid paper and have students draw the x- and y-axes. Have students plot and label the coordinates of three points on a coordinate grid. Students should then connect their points to form a triangle. Ask the students to think about the effect that would be created if each of the coordinates of the vertexes their triangle were multiplied by three? *(An enlargement of the area of the triangle would occur.)* Then ask the students to think about the effect of dividing each of the coordinates by 3. *(A reduction of the area of the triangle would occur.)* Have students test their conjectures.

Activity 11: Refreshing Dance! (GLEs: 7th – 10; 8th – 9)

Have students work in groups of four to prepare a cost-per-student estimate for refreshments at a 7th grade party. Give students a list of items to be served, serving size for each student, and cost of each item. For example, 2 glasses of soda per student is about $\frac{1}{2}$ liter and a 2 liter bottle of soda is about $1.00. Have students determine the total cost of refreshments for each student and the total cost of the dance, if they plan for 150 students. Have the students prepare posters of plans and share these with the class.

Activity 12: The Better Buy? (GLEs: 7th `– 10; 8th – 9)

Provide students with a list of items they can purchase, along with the prices. These items can be liters of soda, ounces of chips, pounds of nuts, etc. Be sure to use items that require them to calculate the unit cost in order to decide which is a better buy. For example, list a 12-pack of soda cans and a 6-pack of the same soda. Extend the activity by including grocery ads from different stores carrying the same items. Have students make projections about savings on groceries by shopping at store A versus store B over a year, etc. Give the students at least one problem where the larger purchase is not the best buy, such as a four-pack of soda costs $2.75 and a twelve-pack costs $8.40. Have students determine which is the better buy.

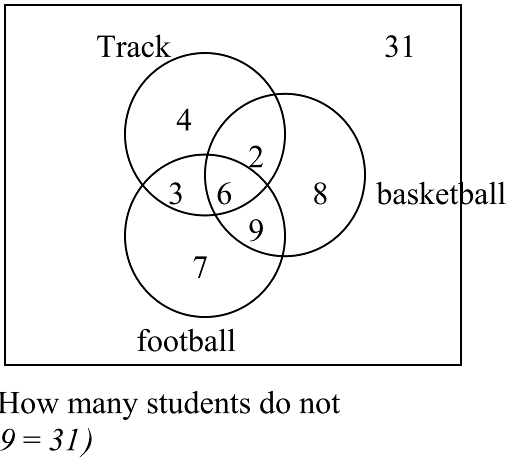
Activity 13: How Many Outfits are on Sale? (GLEs: 7th – 36; 8th – 43)

Provide groups of four students with a copy of one page of a clothing sales brochure that has pants, shoes and shirts. There is a sample sheet provided at the end of this unit. Have the students sketch a diagram to illustrate the different outfits that could be made from the items on the brochure. The outfits should include pants, shirt, and shoes. Ask students to determine which of these outfits would cost the least if they must have shirt, shoes, and shorts. Have students determine a mathematical method of determining the total number of outfits that could be made using all clothing items in the ad. Discuss methods that the students used. Have students write a summary showing their mathematical thinking and give the total number of possible combinations that could be made from the least expensive shirt, shorts, and shoes listed.

Activity 14: Logic Problems with Venn Diagrams (GLEs: 7th – 11, 34, 35; 8th – 7)

Have students use logic to create a Venn diagram. Make this a whole class activity and create the Venn diagram on an overhead transparency. Later pair the students to complete other problems that you provide.

There are 70 students in the 7th grade at Sleepy Hollow Middle School.
15 students participate in track
25 students participate in football
25 students participate in basketball
15 students participate in football and basketball
9 students participate in track and football
8 students participate in track and basketball
6 students participate in all three sports



Ask, What size Venn diagram do we need? *(3 overlapping circles)* How do you know? *(There are three activities listed)* Give students a hint to start with the last statement. Have students discuss and debate where they think the numbers should go based on the information provided. Ask, How many students do not participate in any of these sports? How do you know? (*70 - 39 = 31)*

Ask the students to determine a method to solve *'if/then'* statements such as, If the number of students that participate in all three sports triples and the proportion of students in each of the groups remains the same, what would be the number of 7th grade students at Sleepy Hollow Middle School? If this is the total number of 7th grade students at the school, then how many students would participate in track? If there are 420 students in the 7th grade at another middle school and the proportion of students in each sport remain the same, what would be the total number of students that play all three sports?

> *Solution: There would be 210 students at Sleepy Hollow Middle School, and 45 students participate in track. There would be 36 students that play all three sports in the school with 420 students.*

Activity 15: Combination or Permutation? (GLEs: 7th – 37; 8th – 7, 42, 43)

Have student groups of six write their names on a slip of paper or an index card. Have students determine the total number of combinations of 3 students by making a list or diagram. If students need help, let them use letters of their first names (if all are different) or use A, B, C, D, E, and F to represent the six students. Make sure the students understand that combinations involve an arrangement or listing where order is not important (*i.e.,* ABC is the same as BCA, as these would be the same group of people even though the order in which they are listed is different).

Show them how to make an organized list. After giving students ample time to make the list of combinations, lead a class discussion in which the class agrees on the list of combinations that can be made. Then, have each student determine the ratio of the number of times his/her name appears in a combination compared to the total number of combinations. How would this number change if 4 of the 6 students were selected? Have students discuss the change and any conjectures that can be made at this time.

Next, tell students that these same six names are now in a race, which changes the problem to a permutation because in this case, order is important (*e.g.,* ABC means A came in first, but BCA, means B came in first). Ask, How many arrangements are there for 1st, 2nd, or 3rd place? Ask students to determine whether the number of arrangements (120) is the same as in the previous problem (60) and to explain why or why not? Ask students to determine the number of permutations if 4 people were to be recognized for finishing 1st, 2nd, 3rd or 4th.

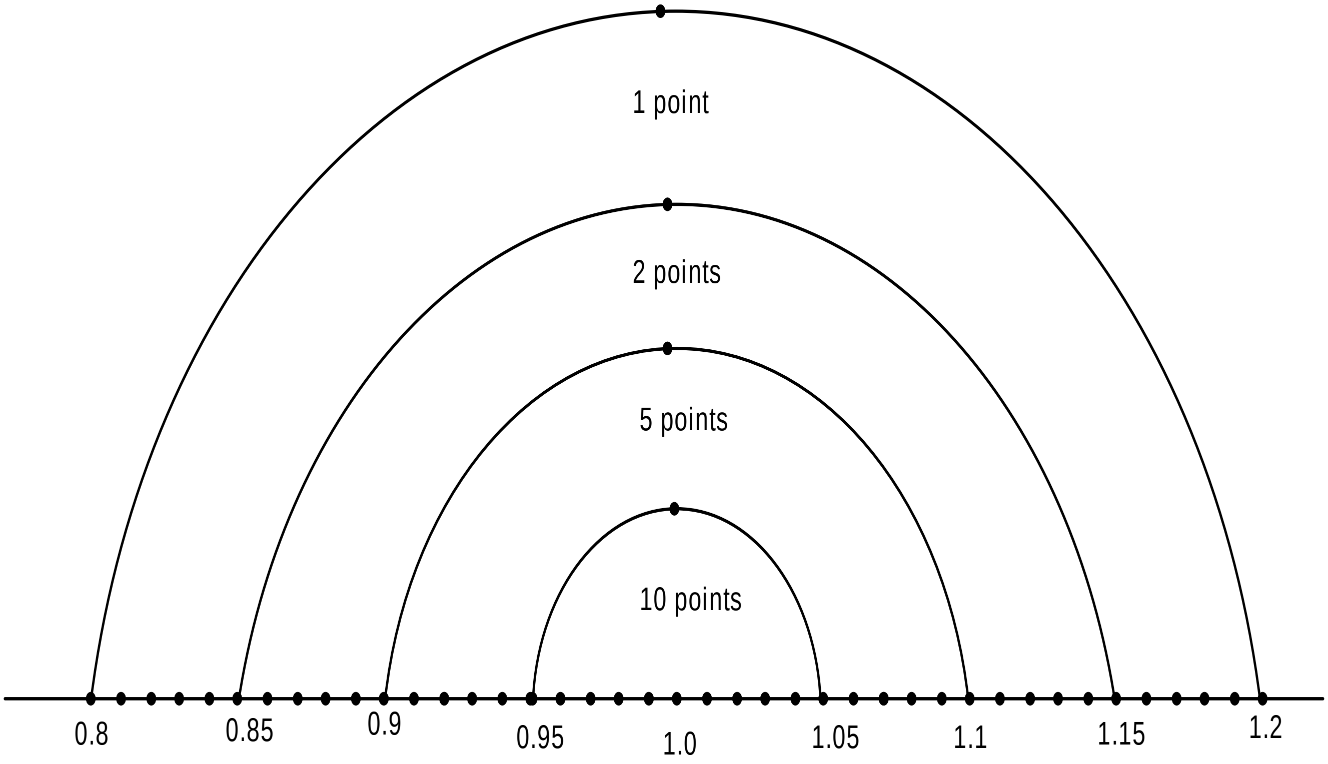
Have students discuss the difference in the two concepts and discuss when order is important. Have students determine the probability and the percent of times they would be in first place, second place, and third place out of the total number of possible outcomes. Have students determine if and how these probabilities would change if 10 students were in the competition.

Activity 6
Bull's Eye Chart

Problems	**Estimated Answer**	**Exact Answer**	**Estimate ÷Exact Answer**	**Points earned**	**Inequality (estimate/exact)**
4.872×3.127					
25.2×20.02					
0.62×0.57					
$19.8 \div 1.52$					
$0.91 \div 12.13$					
$54.45 \div 14.79$					
$\frac{2}{5} \times 8\frac{1}{4}$					
$3\frac{9}{10} \times \frac{5}{7}$					
$9\frac{1}{5} \times 6\frac{3}{4}$					
$\frac{9}{10} \div \frac{2}{5}$					
$\frac{8}{25} \div \frac{3}{8}$					
$11\frac{3}{5} \div 6\frac{1}{2}$					

Activity 6
Bull's Eye Target

This is the target for use after the first three columns of the chart are complete. The target is used to find the points earned. Compare points earned with those earned by your partner, then go back and write your inequalities.



Activity 13
Sale paper sample

Overdyed Piqué Polo

Sale
34.50

B. Web Flip-Flops
$19.50

D. Twill Cargo Shorts
Outdoor-inspired look, all-around-town comfort. A total of nine pockets defin these relaxed favorites. 100% cotton

Sale
29.99

Poplin Camp Shirt

A. Poplin Camp Shirt
CLASSIC FIT
Understated details and princess seaming enhance the classic femininity of our updated camp shirt. Flattering V-neck; cap sleeves with button trim. Not too slim, not too loose; traditional. 100% cotton. Machine wash.
B08 312 5100 Reg. S-XXL

Sale
29.50

2 or more
22.50 ea

E. O Ring Thongs
$39.50

Sample Assessments

General Assessments

- The teacher will determine student understanding as the student engages in the various activities.
- Whenever possible, the teacher will create extensions to an activity by increasing the difficulty or by asking "what if" questions.
- The student will be encouraged to create his/her own questions.
- The student will create and demonstrate math problems by acting them out or using manipulatives to provide solutions on the board or overhead.
- The teacher will use the website http://www.rubrics.com to create a rubric to assess student work.
- The student will complete journal entries using such prompts as:
  - Explain whether the exact product of (1.40)(0.099) will be greater than or less than the estimate (1.4)(1). How can you tell without multiplying 1.4 and 0.099?
  - When you multiply two whole numbers, the product is larger then the factors. Is the product of two fractions larger then the fraction factors? Explain your reasoning.
  - Using the numbers -6, $\frac{1}{2}$, -2, and 5 and any three operations, what problems can you write for which the answer fall between 2 and 0?
- The student will create a portfolio containing samples of their abilities to work problems such as the following:
- The teacher will give the student a list of about fifteen rational numbers including fractions, decimals and percents, making sure that some of the values are equivalent (*i.e.,* $\frac{1}{4}$ *and 25%).* The students will make a number line and place all fifteen rational numbers along the number line in the correct position. To complete the assessment, the student will write at least 10 inequality statements using the symbols <, >, ≥, and ≤.

Activity-Specific Assessments

- Activity 3: The students will respond to the following situation in his/her math journal: Ms. Fields put the problem $\frac{4}{5}+\left(3^2 \div 3 \times 2\right)-2\frac{3}{10}$ on the board. Erica got an answer of 0 and Sammy got an answer of $4\frac{1}{2}$. Explain which of the students is correct and give justification for your answer using correct mathematical language.

  Solution: Sammy is correct. Erica performed the order of operations within the parentheses incorrectly. She divided nine by six and got one and a half.

- Activity 6: The student will respond to the following prompt: Using your data from the Bull's Eye activity, explain when the estimated answer gave a value greater than one on the Bull's Eye and why.

  Solution: When the estimated answer is greater than the exact answer, the value was greater than one because the estimate was divided by the exact answer.

- Activity 12: While watching the LSU football game, Jerrica became very thirsty. The snack stand sold drinks in 4 sizes.

Kiddie Size	Adult Size	Super Size	Super Duper Size
20 ounces for $0.80	32 ounces for $0.90	44 ounces for $0.99	64 ounces for $1.25

  For which size would Jerrica get the most drink for her money? Explain how you made your decision. If the snack stand offered a Mega Size drink, how much should Jerrica expect to pay? Explain your thinking.

- Activity 13: The students should individually show the price of the items used in the sale paper further reduced 15%. They should show all of their thinking.

Grade 7
Advanced Mathematics
Unit 3: Algebraic Thinking

Time Frame: Approximately four weeks

Unit Description:

This unit ties numerical problem solving to algebraic problem solving. The unit begins with computations using the distributive property; and moves into solving and graphing solutions to equations and inequalities and graphing on a coordinate grid. Relationships among units and conversions between units within the same system will be addressed. Representations of the relationships in patterns are made using tables, graphs and equations. Equation solutions and descriptions of how rates of change in one variable affect the rate of change in the other variable are explored as graphs are analyzed and slopes are discussed.

Student Understandings

Students in this unit show a strong command of working with positive whole number exponents in evaluating expressions. Students should be able to solve one- and two-step equations and also solve and represent solutions to inequalities on a number line. They can discuss rates of change, such as found in the graphs of linear relationships. Students develop an intuitive grasp of slope and will be able to compare and contrast slope in linear settings. They are capable of shifting among representations and discussing the nature of such representations for functions as tables, graphs, equations, and in verbal and written formats. Students determine which type of graph is appropriate for given situations

Guiding Questions

1. Can students link algebraic inequalities with their verbal descriptions?
2. Can students apply the distributive property?
3. Can students graph points on a coordinate grid?
4. Can students apply positive whole number exponents in evaluating expressions?
5. Can students apply the order of operations in evaluating expressions involving fractions, decimals, integers, and real numbers along with parentheses and exponents?
6. Can students shift among written, verbal, numerical, symbolic, and graphical representations of functions?
7. Can students solve and graph solutions of multi-step linear equations and inequalities?
8. Can students explain and form generalizations about how rates of change work in linear settings?
9. Can students construct a table of values for a given equation and graph it on the coordinate plane?

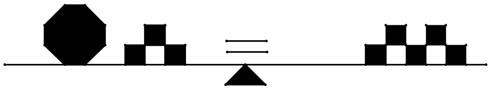
Unit 3 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
7th grade	
Number and Number Relations	
3.	Solve order of operation problems involving grouping symbols and multiple operations. (N-4-M)
4.	Model and apply the distributive property in real-life applications (N-4-M)
5.	Multiply and divide positive fractions and decimals (N-5-M)
7.	Select and discuss appropriate operations and solve single- and multi-step, real-life problems involving positive fractions, percents, mixed numbers, decimals, and positive and negative integers (N-5-M) (N-3-M) (N-4-M)
Algebra	
12.	Evaluate algebraic expressions containing exponents (especially 2 and 3) and square roots, using substitution (A-1-M)
14.	Write a real-life meaning of a simple algebraic equation or inequality, and vice versa (A-1-M) (A-5-M)
15.	Match algebraic inequalities with equivalent verbal statements and vice versa (A-1-M)
16.	Solve one- and two-step equations and inequalities (with one variable) in multiple ways (A-2-M)
17.	Graph solutions sets of one-step equations and inequalities as points, or open and closed rays on a number line (*e.g.,* $x = 5$, $x < 5$, $x \leq 5$, $x > 5$, $x \geq 5$) (A-2-M)
18.	Describe linear, multiplicative, or changing growth relationships (*e.g.,* 1, 3, 6, 10, 15, 21, …) verbally and algebraically (A-3-M) (A-4-M) (P-1-M)
19.	Use *function machines* to determine and describe the rule that generates outputs from given inputs (A-4-M) (P-3-M)
Measurement	
22.	Convert between units of area in U.S. and metric units within the same system (M-5-M)
Patterns, Relations, and Functions	
32.	Describe data in terms of patterns, clustered data, gaps, and outliers (D-2-M)
33.	Analyze discrete and continuous data in real-life applications (D-2-M) (D-6-M)
35.	Use informal thinking procedures of elementary logic involving *if/then* statements (D-3-M)
39.	Analyze and describe simple exponential number patterns (*e.g.,* 3, 9, 27 or 31, 32, 33) (P-1-M)
GLE #	**GLE Text and Benchmarks**
8th grade	
Number and Number Relations	
2.	Use whole number exponents (0-3) in problem-solving contexts (N-1-M) (N-5-M)

5.	Simplify expressions involving operations on integers, grouping symbols, and whole number exponents using order of operations (N-4-M)
Algebra	
10.	Write real-life meanings of expressions and equations involving rational numbers and variables (A-1-M) (A-5-M)
11.	Translate real-life situations that can be modeled by linear or exponential relationships to algebraic expressions, equations, and inequalities (A-1-M) (A-4-M) (A-5-M)
12.	Solve and graph solutions of multi-step linear equations and inequalities (A-2-M)
13.	Switch between functions represented as tables, equations, graphs, and verbal representations, with and without technology (A-3-M) (P-2-M) (A-4-M)
14.	Construct a table of x- and y-values satisfying a linear equation and construct a graph of the line on the coordinate plane (A-3-M) (A-2-M)
15.	Describe and compare situations with constant or varying rates of change (A-4-M)
Measurement	
18.	Apply rate of change in real-life problems, including density, velocity, and international monetary conversions (M-1-M) (N-8-M) (M-6-M)
20.	Identify and select appropriate units for measuring volume (M-3-M)
22.	Convert units of volume/capacity within systems for U. S. and metric units (M-5-M)
Data Analysis, Probability, and Discrete Math	
34.	Determine what kind of data display is appropriate for a given situation (D-1-M)
35.	Match a data set or graph to a described situation, and vice versa (D-1-M)
39.	Analyze and make predictions from discovered data patterns (D-2-M)
Patterns, Relations, and Functions	
46.	Distinguish between and explain when real-life numerical patterns are linear/arithmetic (*i.e.,* grows by addition) or exponential/geometric (*i.e.,* grows by multiplication) (P-1-M) (P-4-M)

Sample Activities

Activity 1: Equations! (GLEs: 7th – 16; 8th – 12)



Distribute 15 - 20 square tiles and 2 - 3 hexagon tiles to each pair of students. Tell the students that the small squares represent one. The hexagon tile represents an unknown value. Sketch a balance on the overhead and put one hexagon tile and 3 square tiles on one side of the balance and 5 square tiles on the other side of a balance, as shown. Begin discussion with the students about how to determine the value of one hexagon tile. Tell the students that the scale is balanced and anything we do to one side, we must do to the opposite side to keep it balanced. Students should see that removing three units from both sides indicates that one hexagon = two squares.

Next place two hexagons and 2 squares to one side of the overhead and set this equal to eight squares. Have students explain methods of determining the value of 'one hexagon'. Translate the pieces to an equation: $2x + 2 = 8$ and have the students record each step using numbers and symbols as they find the value of one hexagon tile. Have students practice other problems using tiles with students recording the steps to their solutions.

Activity 2: Developing the Distributive Property (GLE: 7th – 4)

This activity should be completed without the use of calculators. Ask students to explain a method for multiplying 34×8 or 157×5. Students might show methods such as repeated addition, or multiplication using smaller values. For example, 34×8 might become 34 x 4 resulting in 136, which can be doubled to give 272. Students may use the distributive property and not realize it by stating that 30 x 8 = 240 and 4 x 8 = 32, and then add 240 + 32 to get 272. Provide several of the following types of problems for students to calculate using the distributive property (*e.g.,* $6(24)=6(20+4)=6(20)+6(4)=120+24=144$). Remind students that rewriting 24 as 20 + 4 is writing the number in expanded form, a process they learned in grade school. If students do not mention the distributive process or describe the use of the property for solving such a problem, lead the class to understand that it is possible to work 34×8 by finding 30×8 and adding this answer to 4×8. Ask the students to show how to work 157×5 using the same method. One way to efficiently multiply a two-digit number by a one-digit number is to use the distributive property.

Do not allow students to use pencil, paper, or calculators for the next problem as this is a mental math exercise. Write the problem $(6\times 84)+(6\times 16)$ on the board allowing only 5 - 10 seconds for student think time. Students will not be able to find a solution in the time allowed because they are trying to multiply 6 and 84, then multiply 6 and 16, and add the two. Give students a hint that the distributive property could be used to work this problem quickly. Let students see if they can come up with how the distributive property could be used.
$(6\times 84)+(6\times 16)=6(84+16)=6\times 100=600$.

After practicing several forms of distributive property problems, ask: Will the distributive property work when dealing with subtraction? Have the students justify their answers by making up problems to show proof. *(Yes, it does work:* $(7\times 73)-(7\times 33)=7(73-33)=7\times 40=280$*)* Will the distributive property work with fractions? Try this one $\frac{1}{2}\times 6\frac{2}{3}$. *(Hopefully, this will surface:* $(\frac{1}{2}\times 6)+(\frac{1}{2}\times\frac{2}{3})=3+\frac{1}{3}=3\frac{1}{3}$*. Take half of 6 and half of two-thirds, then, just group the answers together. It might not always be this easy, but it will work.)*

Pair students and have them come up with their own examples using whole numbers and fractions and addition and subtraction distribution. Ask pairs of students to share the examples they wrote with another pair of students to see if they can solve the problems using the distributive property. They should also solve problems provided by the teacher.

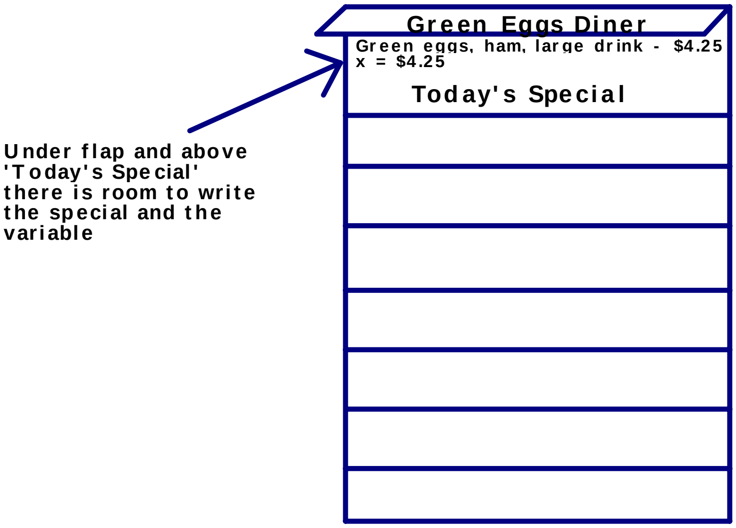
Activity 3: Green Eggs Diner (GLEs: 7th – 4, 7, 14; 8th – 5)

Ask the class if they have ever heard of the story *Green Eggs and Ham,* by Dr. Suess. Tell them that today they will use the green eggs and ham idea in an algebra activity. If available, read the book to the class. The students will make a foldable in this activity that will give experience with substituting values for variables and combining like terms. Give students directions for making the "Green Eggs Diner" menu booklet. Students each need 4 sheets of 8 ½ x 11 paper (use colored paper, if available).

- Fold over one inch at the top of sheet 1 to form a flap. On the flap, write *Green Eggs Diner.*
- Fold over 2 inches at the top of sheet 2. Insert the folded edge of sheet 2 under the flap of sheet 1. The flap of sheet 2 will extend 1 inch below the first flap. Write *Today's Special* on the extension of flap 2.
- Continue with the remaining two sheets of paper. Fold a three inch flap on sheet 3 and a four inch flap on sheet 4. Each of these two sheets should extend 1 inch below the previous flap when inserted.
- When all four sheets have been assembled, staple through all sheets along the fold. There will be 8 flaps showing – four from the folds and four which are extensions at the bottoms of the sheet.
- Write the following on the remaining flaps: flap 3, *Menu*; flap 4, *Sample Order*; flap 5, *Large Orders*; flap 6, *Grease Spot and Ketchup Spills*; flap 7, *Write Your Own Problem*; lap 8, *Daily Totals – Combining Like Terms*.

Directions for foldable activity completion:

- Have students lift flap 1 and write *Green Eggs, Ham, Large Drink* as three menu items above the words, Today's Special. This combination of items is the special. Have them write *"x"* to represent the special and assign it the price of $4.25.
- Make a list of basic items that might be on a breakfast menu in a restaurant, such as: eggs - $2.00, bacon - $1.25, ham - $1.50, large drink - $1.00, small drink - $.75. Have



the students lift flap 2 and write these above the ‘Menu’ label. Make up a situation like “*In a diner, time and efficiency are of the utmost importance, so there is a shorthand used when writing down a customer’s order. It takes too long to write each food item completely on the ticket, so each food item is abbreviated and only the first letter of each item is used. For example, if a customer wanted green eggs, the waiter would write only g.”* The students should determine as a class which variable to use to represent each menu item. Remember that x represents *Today’s Special.*

- Tell the class that they will be assisting the cook by telling him what has been ordered. Assume the waiter has his first order and has written g + h + s. What did the customer order? (*green eggs, ham, small drink).* Have students lift flap 3 and above the label, *Sample Order*, and ask them to make a table of four columns. Label columns: Order, Translation, Cost, and Total. The table below is completed with three sample orders.

Order	Translation	Cost	Total
1) $x + g + s$	Today’s Special, green eggs and small drink	\$4.25 + 2.25 + .75	\$7.25
2) $2g + b$	2 green eggs, bacon	2(\$2.25) + \$1.25	\$5.75
3) $e + 3h + 2l$	Egg, 3 hams, 2 large drinks	\$2.00 + 3(\$1.50) + 2(\$1.00)	\$8.50

- Flap 5 is labeled *Large Orders*. Above this label the students should make another chart like they made for *Sample Orders* and write expressions like the ones below in the first column. The students should write orders that might come from a family to practice order of operations and the use of the distributive property. For example, a table of four might order two specials (2x) , two ham, and two small drinks. This would look like 2(4.25) + 2(1.50) + 2(1.00) or 2 (4.25 + 1.50 + 1.00). This gives them practice with order of operations and using the distributive property. Make sure the orders are different from those used on flap 4.

  $2(g + h)$
  $(g + s) + 2h$
  $x + 3(e + l)$
  $(e + b + 1) + 2x$

  Have students fill in the other columns and make up 2 more orders for their table.

- Flap 6 is labeled *Grease Spots and Ketchup Spills.* Have students copy the following above the label.

  ✺ $(e + l) = \$6.00$
  $x +$ ✺ $+ l = \$6.50$
  ✺$g + h = \$10.50$
  ✺$+ g + b = \$5.50$

  Challenge the students to “substitute” values for the grease spot to be able to read the order. After the students have had time to substitute the correct values from the variables and solve for the missing value in each equation, lead a class discussion about their answers. Note: The grease spot covers some part of the

equation that the waiter wrote. The student's purpose is to determine what the number should be to make the equation true. These grease spots represent missing information, not a variable.

- Flap 7 is labeled *Write Your Own Problem*. Challenge students to write their own order. Then have groups of 4 students write an expression that represents the sum of their orders and set the expression equal to T which represents the total of all orders. This will give good reason to combine like terms. Have students write the simplified equation which represents the total of all their orders. Have them substitute the costs of each item into both the unsimplified and simplified versions of the equation. This should help students understand that simplifying first makes the substitution easier and that the numerical total for each equation should be same when the substitutions are entered.

- Have one or two groups put their equations on the board and explain to the class how they combined the items. Discuss the concept of a term and the idea that the variable (food item in this case) had to be the same before they could combine the terms.

- Have the remaining groups put their equations on the board, and challenge the students to individually simplify each equation and then substitute values to determine the price. This is a good time to have them determine the number of terms in an equation and to make sure that students have a good understanding of the definition of like terms and how to simplify them.

Activity 4: Distribute It With Candy Bars! (GLEs: 7th – 3, 4; 8th – 5)

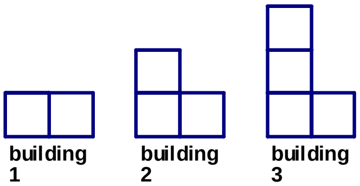
Give students a scenario of selling candy bars. Different types of candy bars are packaged with different numbers of bars in each box. Jolly bars are packaged with 24 bars in each box, Nutty bars with 20, and Cocoa bars with 32. Ask students to write an expression that illustrates buying 5 boxes of each type of candy bar and then use the distributive property to find the total candy bars. Ask, If you sell 3 boxes of Jolly bars, 5 boxes of Nutty bars, and 1 box of Cocoa bars, how many bars did you sell? Make sure that students set up the problem as $3(24)+5(20)+1(32)=$ _____, indicating that distribution cannot be used since the number of boxes for each type is not the same. Give groups of students sales situations for the three types of bars and have students determine the number of bars sold in each situation (*e.g.,* situation a: sell 5 boxes of Jolly bars, 7 boxes of Nutty bars, and 9 boxes of Cocoa bars). Give some problems in which the distributive property can be used and others in which it cannot.

Adding a sales incentive can enhance the activity. For example, if a person sells more than three boxes of Nutty Bars, the company will give an extra bar for each box. Have students write the expression that represents the number of Nutty Bars they will have if they sell 4 boxes. How many candy bars is this? (4 x 20 + 4 extra candy bars = 84 candy bars) Have students show their thinking. Using the price for each bar can extend the activity further. The different bars can cost different amounts. Example: If Cocoa Bars sell for $0.50 each, how much would 5 boxes of Cocoa Bars cost? $5\times(30\times.50)$.

Activity 5: Beaming Buildings! (GLEs: 7th – 16, 18, 19, 32, 35; 8th – 11, 12, 13, 14)

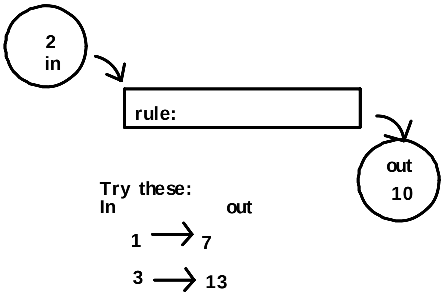
Distribute 15 – 20 toothpicks to each pair of students. Have the students place the toothpicks in the arrangement shown at the right for buildings one through three.

Bldg # x	# beams y
1	7
2	10
3	13



Have them create a table of values showing the building number represented by the x-value and the number of support beams it takes to build the building as the y-value for this pattern of buildings through building #3.

Have students sketch a function machine and determine the rule that would be needed to get from the building number to the number of beams for each building. *One rule that works is that the number of beams needed can be found by multiplying the building number times 3 and adding 4.* *(y=3x +4)*



Have students complete their table of values through building #6. Ask students to use their table values to predict the number of beams needed for building #10. Engage the class in a discussion about their predictions and how they made them.

Ask the class to determine which building would take 61 beams and have the student(s) explain their thinking. This is good practice in working backward. (The students need to subtract the 4 and then divide the 57 by 3 to get the 19th building.)

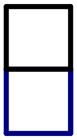
Next, instruct students to plot the ordered pairs of building numbers and number of beams to determine if the relationship is linear. Challenge pairs of students to create a 'what-if' question for another pair of students and be able to justify their answers. Examples of "what-if" questions include: What if the building was number 31, then how many beams would be needed? What if each beam costs $5.25, then how much would building number 25 cost? Give students time to share questions with other pairs of students. Other patterns that can be explored are on an attachment following the Activities section.

Have students begin making the connection between the data in the chart to the linear equation. Students can subtract the consecutive values in the y column and see that each building number increases by 5. The students might be ready to see that when these values are constant, the equation is linear. Lead students to see that the 'y-intercept' as the constant in the pattern. This is a good time to start looking at the slope as the "rise over run." In this graph the students will see that to get from one point to the next point on the graph, they will go 'up three and over one', for a slope of 3. Don't get bogged down at this point, just plant the seed for their understanding.

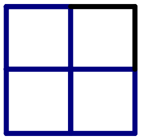
Activity 6: More Patterns! (GLEs: 7th – 18, 19, 32; 8th – 39)

Have the students work in groups to complete the following problem using the concept of a function machine with inputs being the wall number and the output being the number of beams needed for building the wall. Distribute toothpicks that were used in Activity 4 if students need the manipulatives to determine the number of beams.

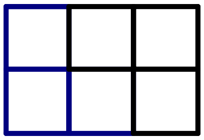
> *An engineer designs the skeleton for the walls of a new stadium from equal lengths of steel beams that are placed in a rectangular pattern as shown below. The engineer knows that one wall has to have a length of 57. How many steel beams are needed for this wall?*



length 1



length 2



length 3

(This is the view from the front.)

<u>The length of the wall is measured by the number of beams along the bottom of the wall.</u>

a. Ask students to model the wall with toothpicks to represent all beams from length 1 to a length of 6.
b. Have them make a table and record the total of beams for each length and share their observations with their group members.
c. Instruct students to predict how many beams are needed for a length of 10 and a length of 17.

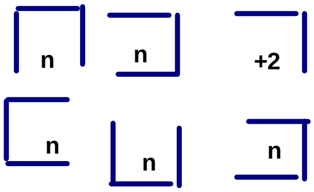
Example of table:

Wall number x	1	2	3	4	5	6	7
Number of beams y	7						

d. Have students write sentences to describe the patterns they see in the table (*i.e.,* write a mathematical statement or rule to describe the rule that relates the number of lengths to the total number of beams). Have each group write their rule on chart paper to share with the class.

   Some example or rules the student may see: a) The total number of beams is equal to a) 5 times the length plus 2. b) The total number of beams is equal two times the length plus one plus three times the length; c) The total number of *beams is equal to 2 times the length of the wall, plus one more than the length of the wall times three, minus 1.*

e. Have the students describe their rules symbolically/mathematically using l to represent the lengths and b to represent the beams.
Example equations that go with the above rules:
a) $b = 5l + 2$; b) $b = 2(l + 1) + 3l$; c) $b = 2l + 3(l + 1) - 1$



The diagram at the right is an example that one student used to determined this rule. Provide students with opportunities to develop an understanding of the equation either by having them sketch how they see the rule or using toothpick manipulatives. Have the students write their rules on the board and make some observations. Ask a question such as, If $b = 5l + 2$ and $b = 2(l + 1) + 3l$ will $5l + 2 = 2(l + 1) + 3l$? Ask them to think back to the *Green Eggs Diner* activity and how they combined terms in that activity? (They combined variables in the menu items that were the same.) Ask students if they think the rules listed are equivalent and ask them to test this by simplifying the expressions. To combine the terms in the *expression* $2(l + 1) + 3l$, the students must first use the distributive property and get $2l + 2 + 3l$ which can be simplified to produce $5l + 2$ as in equation 'a'.

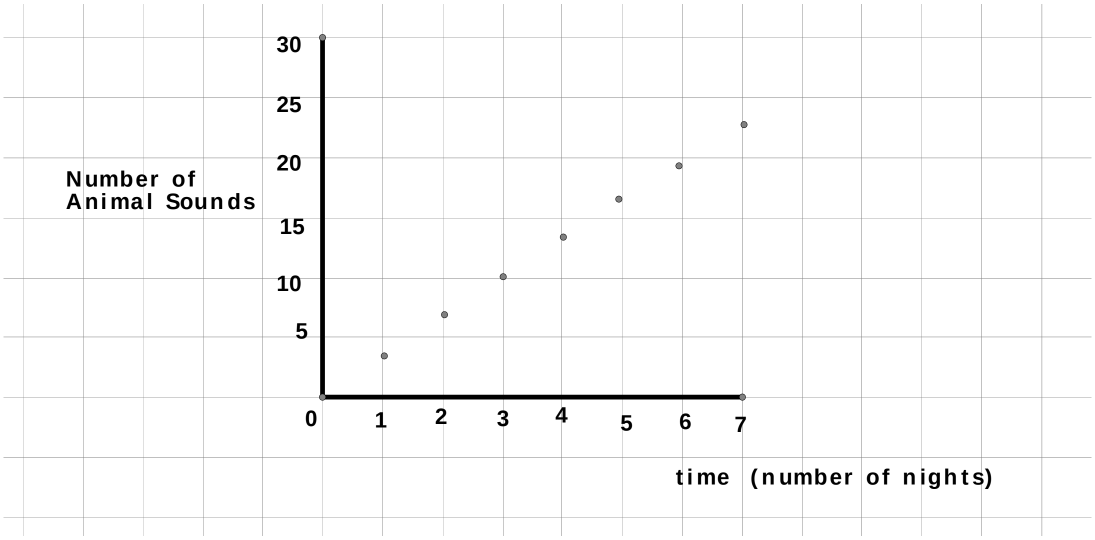
f. Have students graph their table values for lengths 1 – 5. Again, look at the rise over the run (slope – rate of change) in this graph. Ask, How does this line differ from the line we graphed in Activity 4 ($T = 3n + 4$)? The students will probably see that the graph of the line $T = 5n + 2$ is steeper. Relate this to the slope or the '5' in the equation. Refer to the y-intercept and how this equation is different from the equation used in activity 4. Remember, these are concepts that are just beginning to surface.

g. Once the students have combined terms and simplified the equation, have them determine the length of a wall that uses 277 beams. With this the students will again practice working backwards [(277 – 2)/5] to get the length of 55. Challenge them with other equations by giving total numbers of beams and/or wall lengths.

Activity 7: Camping Sounds! (GLEs: 7th – 16, 33; 8th – 12)

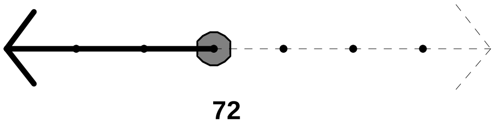
Have the students work in pairs for this activity. Students will begin this activity by looking at the following table of values. Have students write in their math journal a short paragraph describing a situation that the table represents. Ask students to predict the number of animal sounds they would hear if they camped out ten nights. Have students explain their predictions and encourage them to make some rule for the data in the chart ($y = 3x + 1$). Have students plot these four ordered pairs on a coordinate grid and explain the relationship that is shown. The data is discrete, and this is a good time to discuss what is meant by discrete data. The points are not continuous along a line because each animal sound is "one discrete piece of data."

number of nights	number of animal sounds heard
1	4
2	7
3	10
4	13

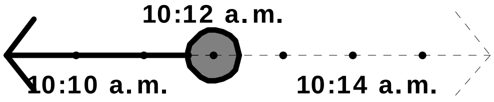


Give groups different situations like the ones below for practice solving and graphing equations or inequalities.

- Joe brought six bags of marshmallows on the camping trip with two friends. Each night they roasted and ate ½ of a bag of marshmallows. If there were 48 marshmallows in each bag and no one ate more than 8 each night, what would be the maximum number *(T)* of marshmallows that were eaten after the 3rd night ?
  $T \leq$ *(3 friends · 8 marshmallows each · 3 nights) ;*
  $T \leq$ *72 marshmallows.*



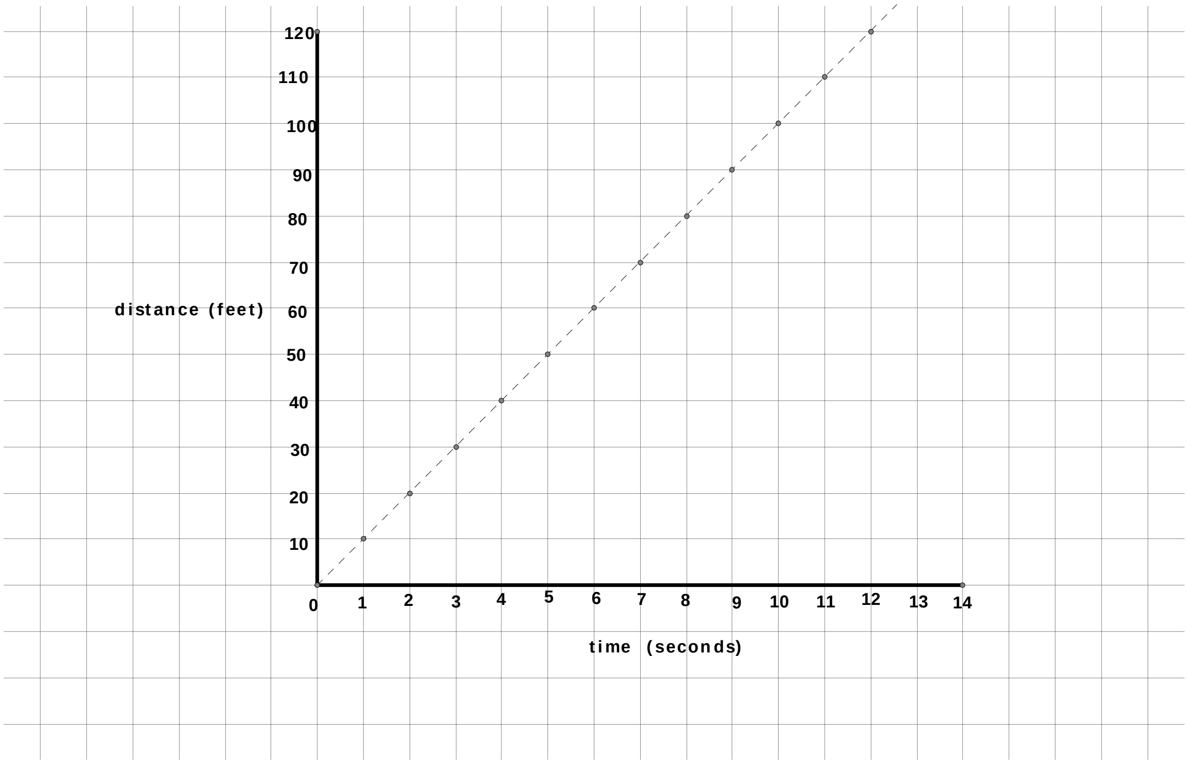
- Joe and his friends hike at least 2 ½ miles each hour on their trip to the lake. The lake was 5.5 miles from their camp site. If they leave the camp at 8:00 a.m., what time will they arrive at the lake? (*at least by 10:12 a.m., maybe earlier*)



- Sam is hiking on a trail that is 280 feet long. He has hiked 20 feet less than half the distance. How far, *d,* has he walked? Write your equation.
  Solution: $\frac{280}{2} - 20 = d; d = 120$ *feet*

  If Sam walks 10 feet per second and completes the trail, make a graph of his hike along the trail.
  Solution graph on next page.

Activity 8: Graphs to Situations! (GLEs: 7th – 15, 18; 8th – 11, 15, 35)



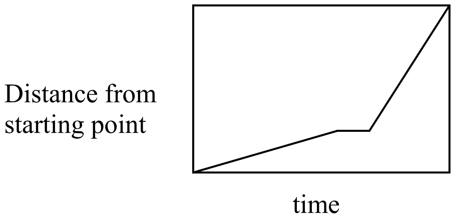
Distribute one different situation similar to the ones listed below to pairs of students. Each of these situations can be represented using the *x*- and *y*-axis and the first quadrant of a coordinate graph:

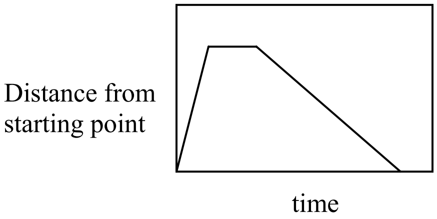
A) *Joe left his room walking slowly, stopped at the refrigerator to get a snack, and then went quickly into the backyard.*
B) *Sally ran quickly to the dressing room after the ball game; she stopped at the door and went back to speak to her parents.*
C) *Stephanie receives $25 a week for allowance, and she spends only $15 a week.*
D) *Jeremy has $200 in his savings account and puts $15 a week in his account but spends $10 a week for snacks after school.*
E) *The rental car company charges $30/day to rent a small car.*
F) *Danny rode his bicycle fast and then stopped for a few minutes to rest before beginning to ride at a slow steady pace.*
G) *The bus was stalled at the intersection for about 10 minutes before the driver started the engine and moved the bus slowly out of the way.*
H) *Jonathan drives slowly until he gets on the interstate, then he speeds up until he gets to an area of construction where he slows down.*
I) *Derrick walks to the store, stops to buy a soda, and then runs back home.*

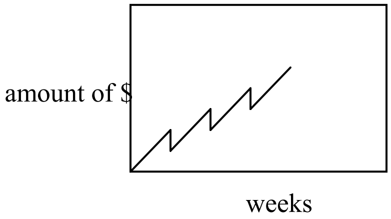
Have each pair of students create a graph that matches the situation they were given, identifying them with the letter of the situation rather than labeling them with their names. See that students are discrete about their situation and the graph they are sketching. Have students post their graphs on the wall and have groups match graphs to the list of situations. Next, provide students with the same list of situations, but replace the letters with blanks in which to write their matches. Have students determine whether the relationship shows a constant or varying rate of change and record that information, too.

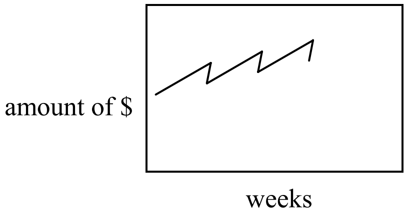
Lead a discussion about student conclusions after they have matched the graphs.

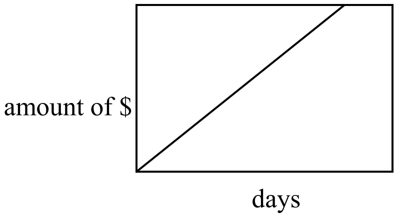
Possible graph representations for the situations are pictured on the next page.

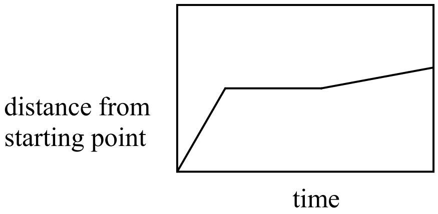


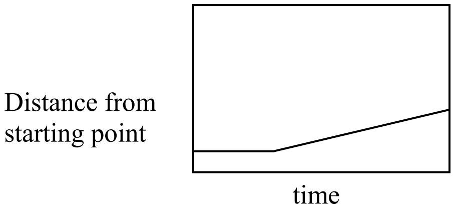


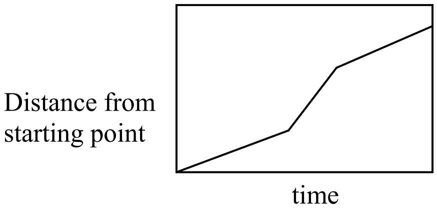


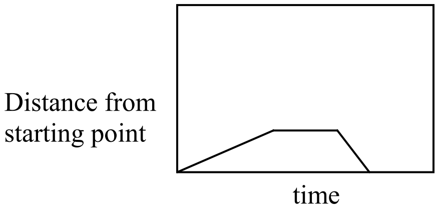










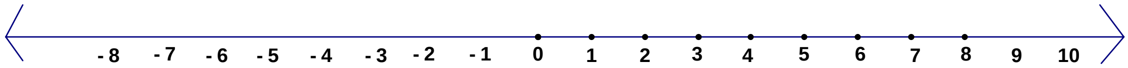


Activity 9: Graphing Solutions to Linear Equations and Inequalities (GLEs: 7th – 5, 16, 17, 33; 8th – 12)

Create sets of cards, consisting of four cards per set and with enough cards in each set to be used by a small group of students.

- One card should contain a multi-step linear equation or inequality (*e.g.,* $6x \ge 25$),
- One card should contain a situation that matches the multi-step linear equation or inequality (*e.g.,* Jacob wants to give his brother at least 25 baseball cards; he knows he can get six cards in one pack. How many packs of cards Jason must buy?).
- One card should contain the answer~~s~~ in terms of the variable (*e.g.,* $x \ge 4\frac{1}{6}$). This card might work in more than one problem, but can only be used once in a game.
- One card should have a number line with the solution graphed.

Have students play a "Go Fish" type card game with all of the cards or, to save time, a matching type of game. The goal is to make a book, which is a set of four cards for each problem. For example, a book might be one card that says "Jacob has no more than 8 baseball cards," one card that says “$y \le 8$ baseball cards,” one card that says “ $y = 5$,” and the fourth card should be the number line below.



(*The number line shows that Jacob may have had 0 cards and may have had up to 8 cards. The data is discrete so the points are not connected with a line.) There is a sample set of cards that can be used or adapted at the end of the activities at the end of the unit.*

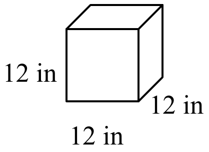
Activity 10: Situations with Equations (GLEs: 7th – 14, 16; 8th – 10, 12)

Have students work in groups of four to make a team. Give each team 2-3 index cards with equations (like the ones they have been studying) written on them. Have each team work together to solve their equations, using symbolic steps and/or diagrams to justify their work. Ask students to write a situation that would represent the equations given to them. After the teams have completed their work, have the students share their work and situations with the class. Be sure that students explain their procedures verbally and that they don’t just give written steps. Instruct one team member to explain the addition/subtraction grouping, one the multiplication/division grouping, one the checking step, and another the situation that represents the equation.

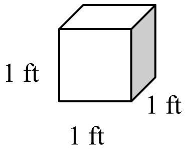
Activity 11: Building a Cube (GLEs: 7th – 22, 39; 8th – 20, 22, 46)

This activity lays a conceptual foundation for understanding square units and cubic units and conversions within the same system. These two concepts are very difficult for students to grasp. Pass out pre-cut 12 inch by 12 inch grid paper to show a square foot. Make sure that students understand that when converting square inches to square feet, they are changing both dimensions, not just one. Have students determine how many square inches are in one square foot. Write the following on the board: ___ square inches = 1 square foot. Discuss the conversion.

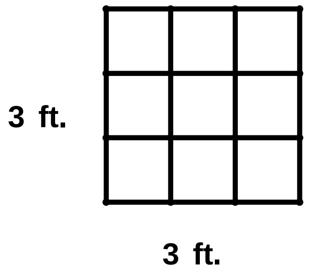
Have students sketch an arrangement of square feet pieces that will fold into a cube with no gaps or overlaps (they may see this as a ‘net’). Discuss the different arrangements and then tape 6 of the student’s square grids together form a cubic foot in the arrangement that the students selected. Ask the students how many cubic inches are inside the cubic foot, encourage the development of a rule, if they are ready. (12 inches x 12 inches x12 inches = 1728 cubic inches)



or



Discuss how students could make a square yard from the square foot grids. Have students that have not had their square foot used already come to the front of the room and tape enough of the square feet on the board to make a square yard. Ask them to determine how many square feet are needed to make one square yard. Discuss the conversion. Make sure students understand that they are changing both dimensions, not just one, when converting square feet to square yards. Write on the board: ____square feet = 1 square yard Have the students make a sketch of the square yard and have them label the dimensions in both inches and feet. There is no need to draw all of the square inches, but it will be a good idea for them to show the sides of the square yard as 12” + 12” + 12” = 36” for the length of the side of a square yard.



9 square feet = 1 square yard

Have the students discuss how to build a cubic yard. Refer to the net that was made with the square feet pieces. Ask, How many cubic feet will be needed to make a cubic yard? (*It takes 27 cubic feet to make the cubic yard.*) If the class is large or all the square foot models are saved from each class, there should be enough square feet to actually build a cubic yard with the students. More discussion will be needed here for student understanding. Discuss the conversion. Make sure students understand that they are changing three of the dimensions, not just one or two, when converting cubic measurements. Be sure to write $3 \times 3 \times 3 = 3^3 = 27$ ft^3 on the board and have the students relate this to the **volume** of the cube in cubic feet.

If possible, hang the models from the ceiling for reference during the year. After completing this activity, discuss and analyze the exponential number pattern that is formed by squaring numbers, and cubing simple numbers such as 2 and 3. Have the students predict the number of units in a 2 by 2 by 2 cubic foot and a 4 by 4 by 4 foot cube in cubic feet and convert the volume to cubic inches. Challenge them to determine the number of cubic feet in three cubic yards. Discuss any patterns that the students discover.

Ask the students to think about what volume unit is used to measure the volume of the following: a) glass of milk (ounces); b) size of a refrigerator (cubic feet); c) amount of cement needed to make a driveway (cubic yards); d) barrel of oil (gallons); and e) pitcher of tea (gallons or liters). Use other models of volume measure in this discussion to help students develop an intuitive sense of the different units of volume.

Activity 12: Formula Madness! (GLEs: 7th – 3, 12, 16; 8th – 2, 5, 12)

Provide students with various types of common formulas. Supply data to be used as replacement values for the variables in the formulas to practice substituting values into equations. Be sure to provide instances where the student must determine a missing value for a variable by supplying the total result. For example, give students the formula for finding the volume of a box of cookies shaped like a rectangular prism, $V = Bh$, and the values of the length of the box, width of the box, and the volume of the box. Then have students determine the missing value or the height of the box. Be sure to use both the formula for converting Fahrenheit to Celsius and Celsius to Fahrenheit in situations, $C = \frac{5}{9}(F - 32); F = \frac{9}{5}C + 32$, such as Jason was going to travel into Canada and looked up the average temperature for the time of year he was traveling and found the temperature to be 25 degrees Celsius. What is the temperature in Fahrenheit degrees and what type of clothing is appropriate for the temperature? *(77 degrees Fahrenheit and spring-type clothing)*

Activity 13: Speed, Time and Distance (GLEs: 7th – 14, 18; 8th – 10, 14, 15, 18, 34)

Instruct students to work in groups of four. One student in each group should have a stop watch or second hand to be used as a timer. If possible, borrow stop watches from the science or P. E. department. Have students mark off a distance of 10 meters and take turns walking the distance and gathering data about the time it takes each student in the group to walk the distance. Ask each student to work independently and create a table of values for the time it takes him/her to walk distances of 15, 20, 25, and 30 meters at the rate that was determined at 10 meters. Have students determine the equations that represent their speeds (unit rates) and make a table of values for the time and distance.

Next, have students plot the coordinates with x representing the time it took and y representing the distance walked for each of the distances on a coordinate grid. Challenge groups of students to develop a conjecture as to the relationship of time and distance shown on the graph. This is a good time to have the students think about the independent and dependent variables and how this is placed on the graph. Discuss graphs from the different students' data and discuss whether the graphs are linear and why or why not. Students should calculate their speed by setting up a proportion and finding the unit rate and relate this to the rate of change on the graph. It is a good time to have students revisit their observations of how equations relate to slopes of the lines from Activities 4 and 5. Lead a discussion to help the students begin to see the connection between their speed and the slope of the line~~.~~ Have students decide as a class how they can best represent the class average walking speed. Assist groups in collecting class data and challenge groups to prepare at least two different graph representations of the class data.

Once the students have completed their graphs of the data, have them post the data from their group for other groups to make observations of the graph representations. Have students predict the rate of change represented by the linear graph if the person was running at a constant rate and crawling at a constant rate. Discuss these changes in rate with the class.

Activity 14: Operations with Inequalities (GLEs: 7^{th} – 16; 8^{th} –12)

Put the inequality $5 > 3$ on the overhead or the board. Ask the class to multiply the values by 2 and rewrite the inequality with the new values (*10 > 6*). Does the statement remain true? Then have the class multiply $5 > 3$ by -2 and rewrite the inequality with these new values.
$-10 > -6$ is not true; the inequality becomes $-10 < -6$. Discuss multiplying an inequality by a negative integer and have them write a conjecture. Test this conjecture with numerous problems.

Next, take the '$-10 < -6$' and have the class divide by -2. Ask them to determine what happens to the inequality symbol (the symbol again is reversed). Discuss their conjecture and test the conjecture. Have students also test adding/subtracting the same number to each side of the inequality.

Have the students summarize the rules for performing each operation on an inequality.

Activity 15: Real-Life Inequalities: (GLEs: 7^{th} – 15, 16, 17; 8^{th} – 11, 12)

Have students write an inequality that represents a situation where a person has an allowance of \$25 a month and must spend no more than 60% of this amount (\$15) on snacks and entertainment (\$15 ≤ the amount spent on snacks and entertainment). Have someone share their inequality and discuss. Have students graph the statement.

Pose the situation:

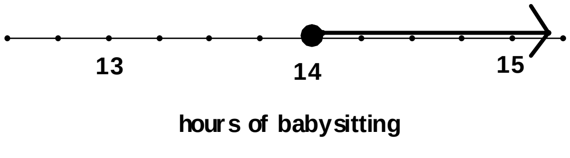
> Jamie puts her Christmas money in a special drawer at home. She wants to have at least \$150 by Christmas. Her grandmother gave her \$35 for her birthday, and she earns \$8.00/hour babysitting. Write an expression to represent how Jamie can determine the least number of hours she needs to baby-sit to have the money she wants by Christmas.

Remind the students that when solving an inequality, they use the same rules as they do for equations, except that when they multiply or divide by a negative, the inequality symbol becomes the opposite.

Have students find solutions to the inequality. (\$$35 + 8x \geq 150$; $x \geq 14\frac{3}{8}$ hours) Sketch a number line on the board and have the students determine a method of plotting their solutions. Lead a discussion about the meaning of the inequality and its solutions.

Have the students work in pairs to complete the following:

> There is a building code in some states that requires at least twenty square feet of space for each person in the classroom. Suppose a classroom is 28 feet long and 18 feet wide. How many people can be in the classroom? Explain your answer and write an inequality to represent the solution. *($20p \leq 504$; $p \leq 25.2$; no more than 25 people)*



Distribute a list of inequality situations to groups of four students. Have the students write the inequality that matches the situation, sketch a graph to represent the solutions, and be ready to justify their solutions to other groups. Some possible situations are:

a) Jamie went to the mall and found a pair of in-line skates that he wanted to buy for $88. He makes $5.50/hour babysitting his little brother. Write and solve an inequality to find how many whole hours he must baby-sit to buy the skates. $(5.50h \geq 88;\ h \geq 16)$

b) A monthly pass for the movie theater is $115. The charge for a movie is $5.50. Write and solve an inequality to find how many times a person should go to the movie so that the pass is less expensive than buying individual tickets$(115 < 5.50m; m > 20)$

c) Coach told the team members that they must each earn $30 this week for a weekend tournament. Tim knows his dad will give him $12 to mow his grandmother's lawn and $8 for each car he washes. If Tim mows his grandmother's lawn, write and solve an inequality to find how many cars he needs to wash to earn at least $30. $(30 \leq 8t + 12; t > 2)$

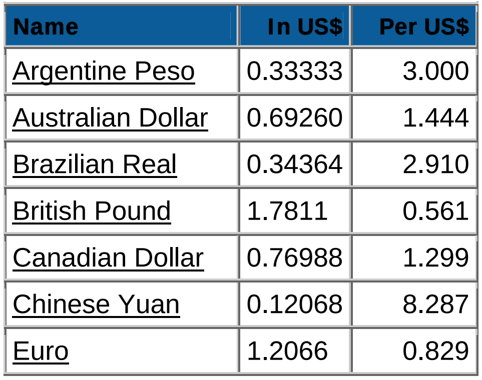
d) Sam wants to go to Washington, D.C. in the spring. The trip will cost him $380 to go with his 8th grade class. Sam has saved $150 and he makes $5.25/hour when he works with his dad after school. Write and solve an inequality to find how many hours Sam must work with his dad to have at least $380. $(380 \geq 5.25h + 150; h > 43)$.

e) Joan was taking a trip to Maine. She checked the temperatures in the newspaper every day. The meteorologist predicted a drop of two degrees Celsius each day for the next month. If the temperature on the day of the prediction is 38° C, how many days will it take for the temperature to drop to at least -4° C? $(21 \leq x)$.

Have students share solutions with other groups and discuss as a class any solutions that they would like to challenge.

Activity 16: How Much Would It Cost? (GLE: 8th – 18)

Prior to the lesson, go to http://moneycentral.msn.com/investor/market/rates.asp. There is an example of the chart to the right. Discuss the chart. The Euro on the chart shows that 1 Euro is $1.2066 and therefore the dollar does not buy as much in Europe. We would spend about $3.62 for something that costs 3 Euros since 3 Euros x 1.2066 dollars is approximately that amount. The chart also shows that 0.829 Euros is equal to $1.00. So $3.00 would be 0.829 Euros x $3 or 2.487 Euros.



Name	In US$	Per US$
Argentine Peso	0.33333	3.000
Australian Dollar	0.69260	1.444
Brazilian Real	0.34364	2.910
British Pound	1.7811	0.561
Canadian Dollar	0.76988	1.299
Chinese Yuan	0.12068	8.287
Euro	1.2066	0.829

Distribute sales papers to the students and a copy of the currency exchange rates. Go over one of the sale items with the students and guide students to determine how to convert the U.S. costs to Euros. Example: A sale paper shows a backpack that costs $5.95. The students should set up a proportion so that they can figure the cost of the item using the Euro. The chart shows the Euro in US$ is 1.2066 and per US$ is 0.829 – the proportion would be $\frac{.829\ Euros}{\$1.00} = \frac{x\ Euros}{\$5.95}$ so the backpack would cost 4.93 Euros.

Give students a budget of $300 in US dollars and use the sale brochure to spend the money. The students should then find the cost of each item in Euros and prepare a poster that explains their procedure and the total amount spent. The poster should show at least 4 different proportions used to calculate the cost in Euros. Students should list their purchases and the cost of each item in Euros.

Activity 17: Equation Match (GLEs: 7th – 12, 16; 8th – 2, 5, 12)

Make up a set of 20 or more equations (at least one set for each pair of students in the class), make copies on colored card stock, and cut them apart. *It is important that some of these equations contain whole number exponents for students to practice.* Duplicate answers on a different colored sheet of card stock and cut them apart. Place a set of equations and a set of answers in separate *Ziplock*® bags. Make enough bags for each pair of students to have one equation bag and one answer bag. Have the students take all the equations and place the equations face up on their desks. At the teacher's prompt, have students match the answers to the equations from the answer bag. Have the first pair of students who completes the matching to raise their hands. At that time, instruct everyone else in the class to stop. Check to see if the answers are matched correctly to the problems and declare a winner or continue the game if a mistake has been discovered. Modify this activity using inequality cards and answers.
Example of cards:

$\boxed{5x+16=91}$ $\boxed{x=15}$

Activity 18: Constant and Varying Rates of Change (GLEs: 7th – 18; 8th – 15)

Have students create a table of functional values that depict a constant rate of change of a specified amount. For example, have students create a table of values (*x, y*) in which the value of *y* is always two-thirds that of the value of *x*. That is, for each increase of 1 unit in the *x*-coordinate, there is an increase of $\frac{2}{3}$ units in the *y*-coordinate. So, the constant rate of change is $\frac{2}{3}$ to 1. Repeat this scenario several times, each time with a different constant rate of change.

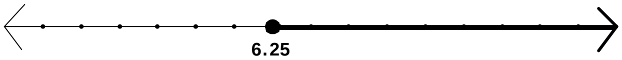
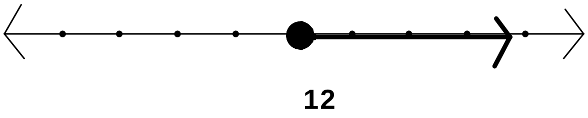
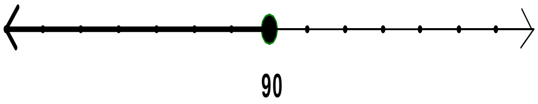
Next, have students create a table of functional values that depict a varying rate of change. For example, have students create a table of values in which the value of the second variable is the square or cube of the first variable. In this case, for each change of 1 unit in the *x*-coordinate, there is a varying change in the *y*-coordinate. Have students compare characteristics of constant and varying rates of change? Have students create a table of values, write equations and identify the rate of change for the following situations. Have student pairs determine whether the rate of change is constant or varying.

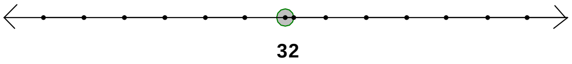
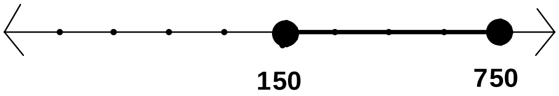
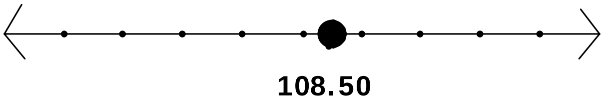
- Sam gets $5.75 an hour for babysitting his baby brother.

- Roderick’s mom gives him $2 for the first hour of babysitting and then doubles his pay each hour he baby-sits.
- Ellen walks every day. It takes her fifteen minutes to walk one mile, 30 minutes to walk 2 miles, 45 minutes to walk 3 miles, etc.
- Denise started a science experiment measuring the growth of a bean plant. The plant grew 2 inches the first week, 9 inches the second week and 16 inches the third week.

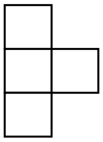
Lead a discussion assuring students they are making the connection between the rate of change and the slope of the line in situations that involve constant rates of change and the constant of the equation that is the y-intercept.

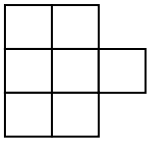
Activity 9
Example cards to use with groups

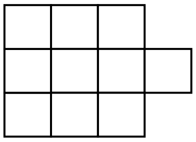
Sam's dad told him to take at least \$50 on the trip with the team. He saves at least \$4/week. If the trip is in 8 weeks, what is the least amount of money he must save each week?	$8x \geq \$50$
	$x \geq \$6.25$
Lucy wanted make 35 football bumper stickers for Charlie. The stickers were on a sheet with 3 stickers per sheet. How many sheets does she need?	$3x \geq 35$
	$x \geq 11\frac{2}{3}$
Joan had \$25 and her mom gave her \$4/week for allowance. Her dad told her that if she saved her money, he would give her no more than double what she saved after 10 weeks.	$x \leq 2(4 \cdot 10 + 25)$
	$x \leq \$90$

Ben saved 4/week. How long will it take for him to save $128?	$4x = 128$
	$x = 7$ weeks
Jeremy delivered newspapers every day. He was paid $25/day. If Jeremy saves at least 20% of his money for a month (30 days), how much money will he have?	$x \geq (\,.20)(25) \cdot 30$
	$750 \geq x \geq 150$
Patty found a jacket, shoes, and a purse on sale for 30% off the price. The jacket regularly sells for $80, the shoes regularly sell for $45, and the purse regularly sells for $30. Patty's father told her to spend no more than $100 on the items. Is the sale good enough for Patty to do as her father said?	$x = (80 + 45 + 30) - [\,.30(80 + 45 + 30)]$
	$x = 108.50$ $\$108.50 > \$100$

Activity 5
Additional Patterns

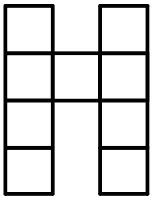

Arrangement 1

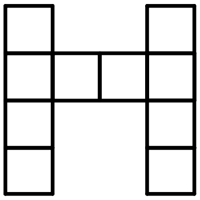

Arrangement 2

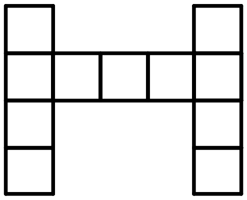

Arrangement 3

Possible questions:

a) Sketch the 4th and 5th arrangement in the pattern.
b) Make a table that shows the arrangement number and the total number of tiles in the pattern.
c) Describe a rule for determining the number of tile in the 25th pattern, 100th pattern. ($y = 3x + 1$)
d) Is this rate of change in this pattern linear? Explain why or why not? (Yes, it is linear because the rate of change is constant and the slope is 3.)


arrangement 1


arrangement 2


arrangement 3

Possible questions:

e) Sketch the 4th and 5th arrangement in the pattern.
f) Make a table that shows the arrangement number and the total number of tile in the pattern.
g) Describe a rule for determining the number of tile in the 25th pattern, 100th pattern. ($y = x + 8$) The 25th arrangement would have 33 tile, and the 100th would have 108 tile. Is this rate of change in this pattern linear? Explain why or why not? (*Yes it is linear, the slope is 1.)*

Sample Assessments

General Assessments

- The students will prepare a brochure comparing mileage of different cars. The student will include graphs of at least three cars and their mileage and explain the relationship of the mileage and the slope of the line. A website that the students can use to find different mileage comparisons is http://www.fueleconomy.gov/feg/FEG2004_GasolineVehicles.pdf
- The student will prepare a presentation using number sequences or pattern sequences and describe when the sequence results in a linear relationship and how they determine this.
- The teacher will provide students with a list of situations that can be represented with an algebraic expression. The student will write the expression that represents the situation.
- The teacher will provide the student with a list of expressions involving variables with whole number exponents up to three. The student will evaluate the expressions using a given set of values for the variables.
- The teacher will provide the student with a table of values that describe a linear situation (*e.g.,* a constant rate of change) and the student will determine the rate of change.
- Whenever possible, the teacher will create extensions to an activity by increasing the difficulty or by asking "what if" questions.
- The student will create portfolios containing samples of experiments and activities.

Activity-Specific Assessments

- Activity 8: The teacher will provide graphs of situations to the student. The student will match a set of graphs with a set of linear equations or inequalities.

- Activity 7: Provide students with a constructed response about cell phones such as "One cell phone plan charges $10/month and $.15/minute. Complete the chart with amounts a person would be charged for each time listed. Graph the data and determine how many minutes a person could talk for a monthly charge of $73."

Time (x) Hours	Cost (y) Per Month
1	$19.00
2	
3	
4	

- Activity 9: The teacher will provide the student with situations in which the student should write the equation or inequality and the solution.

- Activity 18: The student will write a situation with a constant rate of change and create a question that could be answered from the situation. The student will write a situation with a varying rate of change and create a question that could be answered from the situation.

Grade 7
Advanced Mathematics
Unit 4: Patterns in Algebra

Time Frame: Approximately four weeks

Unit Description

This unit connects equation solving to representation of problem situations from an algebraic standpoint. While these problems may often be solved numerically or through mental math methods, this unit develops algorithmic methods of algebra and the related techniques that are used to solve problems involving growth, change, variation, and numerical relationships. There is emphasis on recognizing and differentiating between linear and exponential change and developing the expression for the n^{th} term for a given arithmetic or geometric sequence. Practice is provided in recognizing squares and square roots and estimating the values of square roots that are irrational.

Student Understandings

Students work with equations as they use exponents and square roots. They explore patterns and make conjectures. Students should be able to represent growth (linear and exponential), or alternatively (arithmetic or geometric), in equation or expression form. Students also use function machines' input-output tables and their graphs in making similar analyses of data and relationships. They understand rates of change and can use this to test their generalizations. They understand that a table, a graph, an algebraic expression, or a verbal description can be used as different representations of the same sequence of numbers.

Guiding Questions

1. Can students solve problems involving one- and two-step contexts with fractions, decimals, and integers?
2. Can students recognize and solve problems involving linear or exponential growth, distinguishing between them and their representations?
3. Can students differentiate between additive and geometric growth patterns and discuss each verbally, numerically, graphically, and symbolically?
4. Can students develop and test the n^{th} term for a generalization about a sequence of numbers?
5. Can the students solve problems with squares, cubes, and square roots?

GLE #	GLE Text and Benchmarks
7th grade	
Number and Number Relations	
2.	Compare positive fractions, decimals, percents, and integers using symbols (*i.e.,* <, ≤, =, ≥, >) and position on a number line (N-2-M)
3.	Solve order of operations problems involving grouping symbols and multiple operations (N-4-M)
7.	Select and discuss appropriate operations and solve single- and multi-step, real-life problems involving positive fractions, percents, mixed numbers, decimals, and positive and negative integers (N-5-M) (N-3-M) (N-4-M)
Algebra	
12.	Evaluate algebraic expressions (especially 2 and 3) and square roots, using substitution (A-1-M)
13.	Determine the square root of perfect squares and mentally approximate other square roots by identifying the two whole numbers between which they fall (A-1-M)
14.	Write a real-life meaning of a simple algebraic equation or inequality, and vice versa (A-1-M) (A-5-M)
16.	Solve one- and two-step equations and inequalities (with one variable) in multiple ways (A-2-M)
17.	Graph solutions sets of one-step equations and inequalities as points, or open and closed rays on a number line (*e.g.,* $x = 5, x < 5, x \leq 5, x > 5, x \geq 5$) (A-2-M)
18.	Describe linear, multiplicative, or changing growth relationships (*e.g.,* 1, 3, 6, 10, 15, 21, …) verbally and algebraically (A-3-M) (A-4-M) (P-1-M)
19.	Use *function machines* to determine and describe the rule that generates outputs from given inputs (A-4-M) (P-3-M)
Measurement	
23.	Demonstrate an intuitive sense of comparisons between degrees Fahrenheit and Celsius in real-life situations using common reference points (M-5-M)
Data Analysis, Probability, and Discrete Math	
31.	Analyze and interpret circle graphs, and determine when a circle graph is the most appropriate type of graph to use (D-2-M)
32.	Describe data in terms of patterns, clustered data, gaps, and outliers (D-2-M)
33.	Analyze discrete and continuous data in real-life applications (D-2-M) (D-6-M)
Patterns, Relations, and Functions	
39.	Analyze and describe simple exponential number patterns (*e.g.,* 3, 9, 27 or 3^1, 3^2, 3^3) (P-1-M)
40.	Analyze and verbally describe real-life additive and multiplicative patterns involving fractions and integers (P-1-M) (P-4-M)
GLE #	**GLE Text and Benchmarks**
8th grade	
Number and Number Relations	
1.	Compare rational numbers using symbols (*i.e.,* <, ≤ , =, ≥, >) and position on a number line (N-1-M) (N-2-M)

2.	Use whole number exponents (0-3) in problem–solving contexts (N-1-M) (N-1-M)
5.	Simplify expressions involving operations on integers, grouping symbols, and whole number exponents using order of operations (N-4-M)
Algebra	
10.	Write real-life meanings of expressions and equations involving rational numbers and variables (A-1-M) (A-5-M)
11.	Translate real-life situations that can be modeled by linear or exponential relationships to algebraic expressions, equations, and inequalities (A-1-M) (A-4-M) (A-5-M)
12.	Solve and graph solutions of multi-step linear equations and inequalities (A-2-M)
13.	Switch between functions represented as tables, equations, graphs, and verbal representations, with and without technology (A-3-M) (P-2-M) (A-4-M)
14.	Construct a table of x- and y-values satisfying a linear equation and construct a graph of the line on the coordinate plane (A-3-M) (A-2-M)
15.	Describe and compare situations with constant or varying rates of change (A-4-M)
16.	Explain and formulate generalizations about how a change in one variable results in a change in another variable (A-4-M)
Measurement	
22.	Convert units of volume/capacity within systems for U.S. and metric units (M-5-M)
Data Analysis, Probability, and Discrete Math	
34.	Determine what kind of data display is appropriate for a given situation (D-1-M)
35.	Match a data set or graph to a described situation, and vice versa (D-1-M)
37.	Collect and organize data using box-and-whisker plots and use the plots to interpret quartiles and range (D-1-M) (D-2-M)
39.	Analyze and make predictions from discovered data patterns. (D-2-M)
40.	Explain factors in a data set that would affect measures of central tendency (*e.g.,* impact of extreme values) and discuss which measure is most appropriate for a given situation (D-2-M)
Patterns, Relations, and Functions	
46.	Distinguish between and explain when real-life numerical patterns are linear/arithmetic (*i.e.,* grows by addition) or exponential/geometric (*i.e.,* grows by multiplication) (P-1-M)
47.	Represent the nth term in a pattern as a formula and test the representation (P-1-M), (P-2-M), (P-3-M) (A-5-M)

Sample Activities

Activity 1: Solving Equations (GLEs: 7th – 7, 14, 16; 8th – 10, 12)

Divide the class into groups of three and give each group 50 tokens each labeled with the same letter. Make sure to use a different letter for each group when labeling the tokens (*e.g.,* if there are 9 groups, use the A, B, C, D, E, F, G, H, I and have 50 tokens of each letter). Ask each group of students to choose a value (between 1 and 10) for their tokens. Go to each group and record the value for each labeled token (*e.g*., 'A' tokens = 2, 'B' tokens = 5) so that members of a group know only the value of their own tokens. Have each group trade some of their tokens with *one* other group. Instruct each group to combine a few of their tokens (with known value) with some of the traded tokens (of unknown value) to form a new set containing tokens of two letters. Walk around the room and tell each group the value of their newly created set of tokens. Ask students to write equations representing the value of their newly created set of tokens (*e.g*., $3A+2B=16$) and to use the known value to determine the value of the unknown tokens (*e.g*., If $A=2$, find B.). Have each group write out the number of each lettered tokens they now have and substitute the values for each and solve.

Activity 2: Greater Than, Less Than (GLEs: 7th – 16, 17, 33; 8th – 12, 35)

Demonstrate the concept of greater than and less than using the chart provided or make a similar chart from information found at http://www.enchantedlearning.com/subjects/mammals/dog/index.shtml.

Present a story about a dog lover who is going on a trip with his dog or dogs. The airline on which he/she is traveling allows its passengers to transport only one animal kennel with a maximum weight limit of 120 pounds. Ask, If the kennel weighs 20 pounds, which dogs can the dog lover take? Pair students and have them choose 10 different dogs from the chart and determine the different combinations of the dogs that a person can take on the plane (*e.g*., 10 Maltese, two Cocker Spaniels or one German Shepherd if the limit is 120 pounds and the kennel is 20 pounds).

Have students use the chart and pictures from magazines or Internet sites to design their own situations with inequalities (*i.e.*, 2 Airedale Terrier ≤ 100 pounds, according to the chart + 20 pounds for the kennel) Using chart paper, have students create a situation, write an inequality for the situation, and solve it. Students should then show the solution on a number line. Allow students to present this information to the class. Ask, *Would these graphs represent continuous or discrete data? Have students explain*.

Breed of Dog	Height	Weight
Airedale Terrier	23 inches	44 to 50 pounds
Alaskan Malamute	2 feet	75 to 85 pounds
American Cocker Spaniel	14 to 15 inches	26 to 34 pounds
Basset Hound	13 to 15 inches	40 to 60 pounds
Beagles	13-15 inches	20 to 40 pounds

Collies (males)	24 to 26 inches	50 to 75 pounds
Collies (females)	22 to 24 inches	50 to 75 pounds
Chihuahua	6 to 9 inches	2 to 6 pounds
Standard Dachshunds	5 to 10 inches	16 to 32 pounds
Miniature Dachshunds	6 inches	11 pounds or less
Doberman	24-28 inches	60 to 88 pounds
German Shepherd Dog	2 ft	75-95 pounds
Irish setter	25 to 27 inches	60 to 70 pounds
Labrador Retriever (males)	22 inches	67 pounds
Labrador Retriever (females)	21 to 22 inches	63 pounds
Mastiff	27.5 to 30 inches	175 to 190 pounds
Miniature Schnauzer	12 to 14 inches	13 to 15 pounds
Old English Sheepdog	22 to 24 inches	65 to 100 pounds
Pekingese	6 to 9 inches	7 to 12 pounds
Standard poodle	over 15 inches	45 to 70 pounds
Miniature poodle	10 to 15 inches	15-17 pounds
Toy poodle	under 10 inches	6 to 9 pounds
Pug	10 to 11 inches	14 to 18 pounds
Rottweiler	22 to 27 inches	90 to 110 pounds
Scottish Terrier	14 to 15 1/2 inches	20 to 21 pounds
Siberian Husky	just under 2 feet	45 to 60 pounds
St. Bernard	24 to 28 inches	110 to 200 pounds
Yorkshire Terrier	9 inches	less than 7 pounds

Activity 3: Rate of Change (GLEs: 7th – 14, 18; 8th – 10, 14, 15, 16)

Provide students with grid paper or graphing calculators and have them create a table of at least five input/output values, including $x = 0$ and at least two values for x which are opposites, and plot coordinates for $y = x^2$, $y = 2x$, $y = x - 2$, $y = x^3$ on a coordinate grid. Have them plot and connect the points using different colors for the lines on their graphs. Pair students to create conjectures about the relationships of these equations and share conjectures with the class. Lead a class discussion about their findings.

Determine which of the equations are linear and have the students just look at these equations. Discuss $y = 2x$ and have the students determine the rate of change (remind them that they are looking at either their graph of consecutive coordinates or their tables). Next, have them look at $y = x^2$ and determine whether this rate of change is constant (they will see that it is not).

Ask the students questions such as: Do all linear equations have the same rate of change? What makes it evident, when looking at a graph, that the rate of changes are the same or different? Does one of the linear relationships look as if it changes at a faster rate than another? Discussion should evolve to determining that the slope or slant of $y = 2x$ is steeper than $y = x - 2$ and how this can be determined mathematically. Have students discover how one can get from one point on $y = 2x$ to another point on the line by going up or down first and then right or left (up 2, right

1). This can be done by identifying two points and asking students to tell what movements might be made on the grid to get from point 1 to point 2 and introduces the concept of slope as rise over run. Repeat this with $y = x - 2$. Ask questions so students see the linear equations as *additive* and the exponential equations as *multiplicative* changes? Have students look at the ordered pairs for $x = 0$ in each table and then examine the point when it is graphed for each equation. Ask students to make conjectures as to how they can tell by examining the equation where the graph of an equation will cross at the origin.

Assign groups of four students one of the four equations and ask them to create a real-life situation that could be modeled with the equation. Assist students in developing these real-life situations. It might help make sense to them if they think about lengths, areas, and volumes.

Activity 4: What's My Value? (GLEs: 7th – 12; 8th – 2, 22)

Before beginning this activity, remind student of the cubes they built in unit 3. Ask them to describe the difference in the units used for measuring surface area and volume. Have a student make a sketch either on the board or overhead and describe why the needed unit is a square or cube. Distribute 25 color tiles to each pair of students or have them sketch on grid paper. Begin with having students create a square with dimensions of 3 units and have them find the area. Have students indicate what special name we give numbers like nine (square numbers), relating the term "square number" to the picture of the square. Introduce the square root symbol by placing $\sqrt{9} = 3$ on one side of the square. Introduce the term square root and its meaning as " if $A = s^2$, then s is a square root or the length of the side of the square".

Provide a list of equations involving exponents of 2 or 3, but roots to 2. Provide another list with replacement values for the variables in these expressions. Have students determine which replacement values in the second column should be used to produce the given value for each expression in the first column. Once the students have determined the correct replacement values, they should use the solution as the length of the edge of a cube and determine the volume in cubic yards. Examples are shown in the table below.

Equations (feet)	Replacement Values (feet)	*Solution*	*Volume of cube* (yd^3)
$\sqrt{x} + 5 = 7$	a) $x = 7$	$\sqrt{4} + 5 = 7$	$7^3 = 343\ ft^3$ $343\ ft^3 = 38.11\ yd^3$
$x^2 - \sqrt{9} = 46$	b) $x = 4$		
$x^3 + \sqrt{4} = 29$	c) $x = 3$		

Have students think about the following situation:

> A new policeman had to investigate a car accident. He measured the skid marks and the skid distance was 49 feet. He remembered there was a rule for finding the speed of a car at the time of an accident and the rule was
>
> $5.5\sqrt{\textit{distance of skid} \times \textit{coefficient of friction}}$ where the coefficient of friction is 0.75.
>
> He did not have a calculator to find the square root and since 0.75 was not a square number, he used the closest square number (81) to write an equation. He wrote

$5.5(\sqrt{49 \times .81})$= speed of driver. Determine the speed of the driver that the policeman was investigating using his rule. (*Answer: Driver was traveling about 34.7 mph*)

Activity 5: Number Line Square Roots (GLEs: 7th – 2, 13; 8th – 1)

Have students sketch a number line on paper at their desk. The teacher will draw a number line on the board with numbers from -20 to 20 (or more) on which students will determine the placement of square roots. Write square roots and rational numbers on index cards and have students place them along the number line (*i.e.*, $\sqrt{9}$, $\sqrt{6}$, $\sqrt{24}$, $\sqrt{49}$, 6 ½ , 0.89, etc). Make sure to have several square roots and rational numbers. Pass out all cards to the students and have a student read his/her card aloud. All students should place a mark on the number lines on their desks when the number is read. Select one student to place the number on the number line on the board. Be sure to monitor as students are using the number lines on their desk. Allowing them to place the values on the board provides the teacher the opportunity to clarify any misconceptions. Have a class discussion about the placement of each number on the number line. Ask questions regarding the placement of the answers. Example: Is the approximation for the $\sqrt{6}$ closer to 2 or 3? Why? Continue until all problems have been read aloud and the answers placed on the number line. When all numbered card values are placed, provide three or four additional numbers to the class and have the students justify their placements in words. This is a good formative assessment piece for the teacher.

Activity 6: Name That Term! (GLEs: 7th – 39, 40; 8th – 46, 47)

Provide students with patterns and have them determine the next three or four terms of an arithmetic or geometric sequence. Have them give the value of the 10th term in the sequence. Next, have students write three or four questions about their patterns and determine the values of the answers so that they can ask these questions of students in another group. (A number of these were done in Unit 3.)

After they have completed their questions and determined the answers, have each student pair with another class member and ask the questions of each other that they have written. For example, one student may have written the question "Which term in the sequence given by –2, 3, 8, . . . would have a value of 68?" The partner would determine the solution of 'x = 63' because of the rule 'add 5 to the term number'. Have students determine whether the sequence is arithmetic (*the difference in the y values is constant – the next y value can be determined by adding a constant number to the previous y value)* or geometric (*the difference in the y values is not the same between consecutive term, but each value can be found by multiplying the previous term by same number*) and justify their choice using a table, graph or explanation.

Next, have pairs of students write a rule to illustrate both an arithmetic and geometric sequence and use the rule to create a number pattern to be solved by another pair of students. Students should write at least 2 questions that another pair of students will have to answer concerning their pattern. Students should then exchange pattern and questions with another pair and answer the questions. The teacher should plan time for students to ask clarifying questions as two groups work together. Next, have pairs of students write a real-life situation using the original pattern that could be illustrated with their pattern sequence *(Example: If the rule was multiply by 2 and add 3 – Dad told Jerry that he would double his allowance and give him three extra dollars for each week that he slept on the floor while Grandmother stayed in his room)*.

Once the real-life situations are written, have the pairs of students reform their combined groups and share their situations. Ask students to determine if the first value should be numbered as term 0 or term 1 (*i.e.*, the students will determine if the 0th term is reasonable in their sequence).

Activity 7: Patterns (GLEs: 7th – 14, 18, 19, 40; 8th – 10, 13, 15, 46)

A pattern of squares is shown below. Ask students to sketch these first three arrangements on grid paper and then follow the directions a – f.

a. Sketch the fourth and fifth figure in this pattern.
b. Determine how many squares would be in the 10th figure.
c. Make a function machine in the form of a table comparing the figure number to the number of squares it takes to make a model of the figure
d. Determine how many squares would be in the 100th figure.
e. Write an expression for the number of squares in the *n*th figure.
f. Graph the data (figure number vs. number of squares).

It is important that the students make the connection between the table of values (input/output) in terms of their equation. In unit 3 students used the difference in consecutive *y*-values to determine whether the equation was linear or not. Continue developing a deeper understanding of these connections throughout this unit.

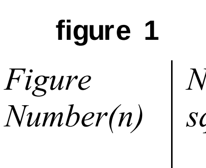


figure 1

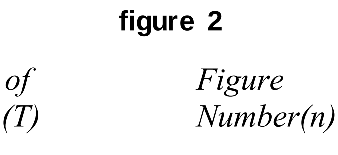


figure 2

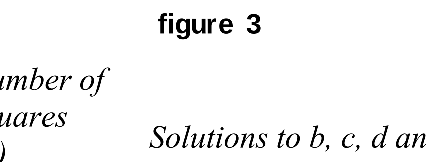


figure 3

Figure Number(n)	*Number of squares (T)*
1	*4*
2	*6*
3	*8*
4	*10*
5	*12*

Figure Number(n)	*Number of squares (T)*
10	*22*
100	*202*
n	*2n + 2*

Solutions to b, c, d and e are provided to the left.

The students may determine that each consecutive value for the number of squares is 2 times the figure number + 2, which will give them the equation: $T = 2n + 2$, where T = total number of squares and n = the figure number.

Once the students have completed a – f, ask them questions such as: Is this relationship linear? (yes) Where does the line cross the y axis? (+2) How does the place where the line crosses the y-axis relate to the rule? *(The rule is $T = 2n + 2$ and the +2 refers to those 2 squares that are in the center of the arrangement each time.*) Help students connect this question to Activity 3 in this unit in which they examined where graphs of various equations crossed the y-axis. Introduce the term y-intercept. These ideas were first introduced in Activities 4 and 5 in Unit 3. Activity 5 in this unit explores these ideas further.

Activity 8: Patterns to Investigate (GLEs: 7th – 18, 19, 39; 8th – 13, 15, 46)

In this activity, students will use the concept of a function machine to describe patterns found in charts or tables; then write an expression describing the rule for the numbers in the sequence, and provide the 100th number in the sequence using the rule described as was done with the tile pattern in Activity 4.

Start by providing students with the table 1 below in which the values for terms 1 through 4 are given. Indicate to students that these values are found with a function machine. Review with students how a function machine works (*i.e.*, when 1 is put it, the function machine follows a mathematical rule and outputs –1; inputting 2 produces 0)

Table 1

Term number	1	2	3	4	5	6	10	50	100	n
Number in the sequence	-1	0	1	2						

Have students make observations and discuss what they notice about the values in the table. Ask them to find values which are missing and describe and write the rule that generates the output from the given inputs (*i.e.*, the relationship between the term number and the value in the sequence). Values missing in the bottom row of the chart are

3	*4*	*8*	*48*	*98*	*n - 2?*

Ask students to fill in the missing values for the function machine found in Table 2 below. *Teacher Note: Do not provide the powers of 3 for the student. See below.**

Table 2

Term Number *(x)*	1	2	3	4	5	6	100	x
# in the sequence *(y)*	3 3^1	9 3^2	27 3^3					

Have students make observations and discuss what they notice about Table 2. Ask student how it is different from Table 1 and to describe and write the rule that generates the output from the given inputs. The students have looked at the differences in the y- values when determining whether or not the equation would be linear or not. They should be able to communicate why they know that this will not be linear. *(The differences are not constant between consecutive y values.)* The values missing in the chart are

81 3^4	*243* 3^5	*729* 3^6	3^{100}	3^x

*If students do not come up with the powers of '3' as part of their function, ask them to write the factors of the y-values until someone comes up with the powers as a way to determine the total number in the sequence. Challenge them to make a table of values for the equation, $y = 2^x$.

Have students graph each of these number patterns to determine whether they are linear and have them relate the results to the differences in the y-values.

Activity 9: Find that Rule! (GLEs: 7th – 32, 39, 40; 8th – 39, 46, 47)

Provide each group of no more than four students with a different pattern. (A sheet of perimeter and area patterns is provided at the end of the activity section). Each group will also need a poster-sized sheet of paper for later use during the activity. Groups will use the patterns to determine the rule for the sequence of values if *x arrangement number and y = perimeter of each arrangement* and then for sequence of values if we change *y* to *y = area of the arrangement.*

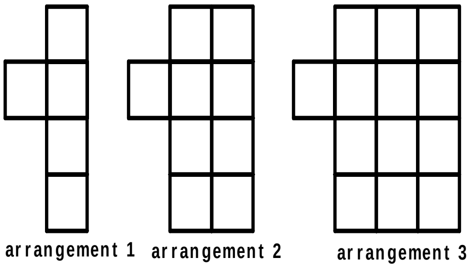
Have the groups of students create at least five terms of the sequence found in their pattern, and then write ordered pairs where *x represents* the arrangement number and *y represents* the perimeter. Ask students to examine the numbers in their ordered pairs and write the rule for generating values in the sequence and then create the graph that represents the rule on a coordinate graph.

Have students repeat the process, but have *y* represent the area of the arrangement.

Give the groups about 15 minutes to make a poster with their information from their pattern to share with the class. Have each group share their information and post their pattern on the wall.

Have the class make observations of the posters representing all of the patterns and then determine which arithmetic and geometric sequences are. (The arithmetic sequences are those were the y values increase at a constant rate and the geometric sequences are those in which the y values increase by multiplication.)

Have the students determine the table values and rule for the pattern below.

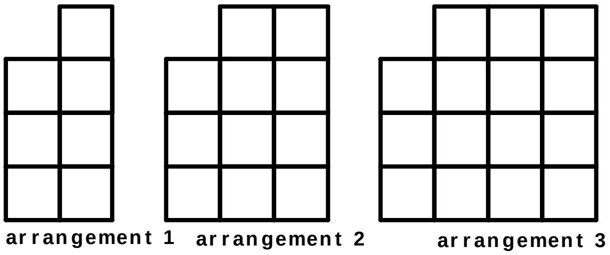


term # 'x'	# tile 'y'
	y = 4x + 1

1	5
2	9
3	13
4	?
5	?

Have them graph the values from their table on a coordinate grid.

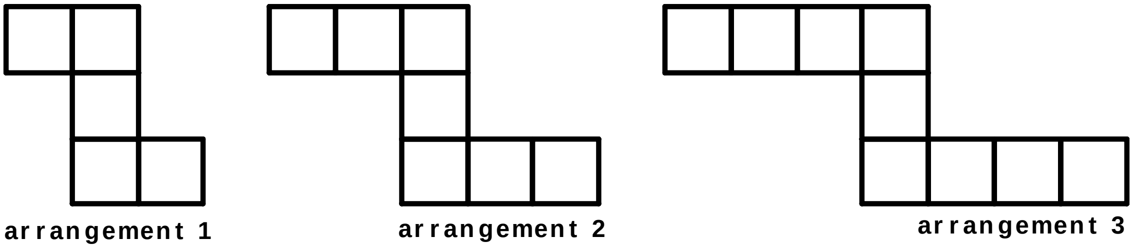
Next have students make predictions as to the effect of adding one or more squares to the pictures in each sequence above (there were 2 added to the pattern in the sequence in the example given below). Ask students to determine the effect of this addition on the rule.



term # 'x'	# tile 'y'
	y = 4x + 3

1	7
2	11
3	15
4	?
5	?

Extend this activity by giving the students a tile pattern and have them come up with an equation or rule that describes the pattern (see below).



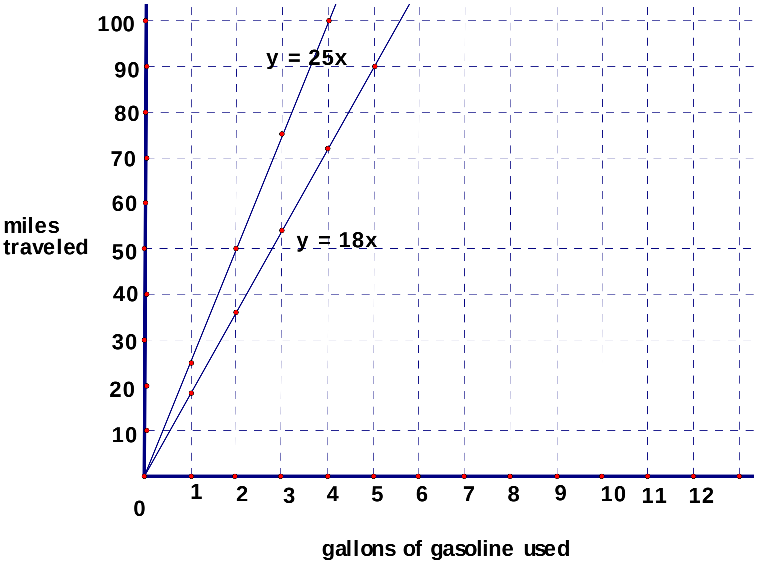
Students should determine that the simplified rule for the pattern is $y = 2x + 3$. Have students plot this linear pattern on the same graph as the earlier equation and write at least 2 observations as to the comparison of the equations of the two lines. Lead the class in a discussion to informally assess the understanding of the slope and the y-intercept and how we can determine these when the equation is written in slope-intercept form.

Activity 10: Make Up a Rule! (GLEs: 7th – 39, 40; 8th – 46, 47)

Have students work in pairs to generate an arithmetic or geometric sequence. For example, purchasing gasoline for a car is a linear/arithmetic sequence where the term number is the number of estimated miles per gallon (*i.e.,* 10, 15, 25, . . .) and the term's value is the distance the car might travel on that number of gallons of gas.

A site that gives city/hwy mileage and average cost of fuel is available at http://www.fueleconomy.gov/feg/FEG2004_GasolineVehicles.pdf. The site contains mileage estimates for most cars.

Have students select at least two of these cars and use the city/highway mileage estimate column to compare the equations which determine the distance a car can travel based on the number of gallons of gas he/she has. For example, the corvette gets 18/25 miles per gallon and the student should set up an input/output chart for the equation, $y = 18x$, which represents the distance the car can travel in the city based on the number of gallons of gas. Two gallons of gas would take the traveler, 18(2) = 36 miles. Students should find points satisfying this equation and graph them. Ask students to create a second input/output table based on the highway mileage, $y = 25x$, and graph those points. Students should be able to see the difference in the slope of the lines and begin to make a connection to the 'slope' of the line or the 'rate of change'.

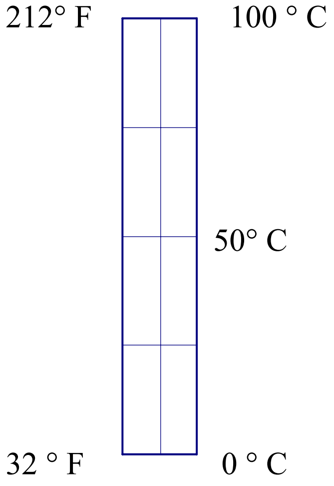


Have students create questions that could be answered by another student once the rule for their line has been determined. Such as "Will the Corvette be able to make a 320 mile trip on 16 gallons of gasoline?" Instruct students to work in pairs and discuss their graphs and questions.

Another activity using rates is an investigation of the cost of mailing letters first class in the U.S. with y representing the cost of mailing a letter or package and x representing the total number of ounces of the letter or package. The website, http://www.usps.com/consumers/domestic.htm#first, provides current U.S. post office rates. An extension might be to compare these with FedEx and UPS. Have students determine the n^{th} term of the “postal” sequence. Students should create their input/output tables and graph the equation.

Activity 11: Celsius to Fahrenheit to Formula! (GLEs: 7th – 18, 23; 8th – 15, 16)

Provide students with strips of paper about 1.5” x 8.5”. Have the students fold their strip of paper into fourths. Next, have students draw a vertical number line 8.5” long. Instruct students to label the bottom of the number line 0° C and 32° F.



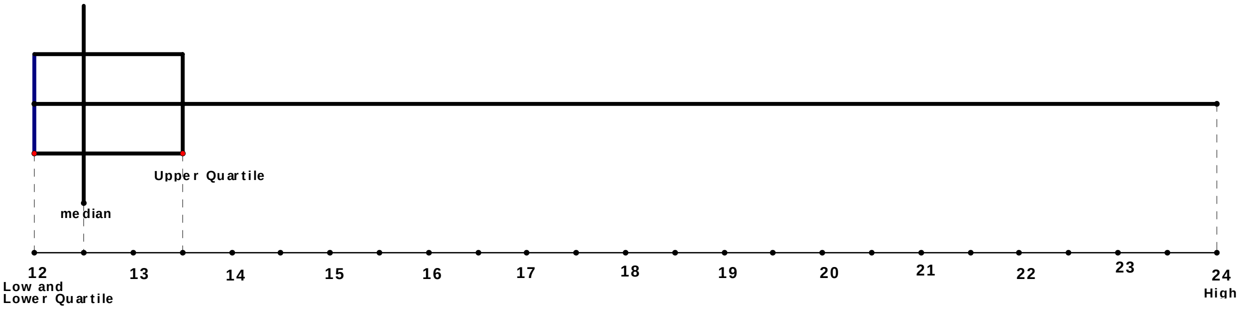
Discuss the freezing point of water and the degree measurements. Ask if students know the boiling point of water; many will remember this from science class. Have them label the top of their number line 100° C and 212° F. Since the strip of paper has been divided into fourths, the students have marks for 25° C, 50° C, and 75° C. Challenge the students to determine the equivalent degree measurement for Fahrenheit. Instruct the students to create a table of conversions from their temperature strip. Discuss student results. Have pairs or small groups of students determine equivalent measures for 5° C, 10° C, 15° C and 20° C. Once the students have determined these equivalencies, have them plot the coordinates for C and F degrees on a coordinate grid and determine whether the relationship is linear. Discuss the rate of change of Fahrenheit and Celsius degrees. Have the students determine the relationship of Fahrenheit degrees to Celsius degrees by using the table data or the graph. If the students use the midpoint of (50, 122) and the boiling point of (100, 212) the slope is $\frac{90}{50}$.) Have the students use their folded Celsius/Fahrenheit paper and make approximations for the following: 65° F to _____° C; 30° C to ______°F, 100° F to _______ °C, 70° C to ______° F.

Solutions: 65° F = 18.3° C; 30° C = 86° F; 100° F 37.8°C; 70°C = 158° F

Using the rates of change explored in this activity, have the students use the relationship given to determine $\frac{90}{50}C^\circ + 32 = F^\circ$, have them simplify this to $\frac{9}{5}C^\circ + 32 = F^\circ$, and use the equation to check their conversion estimates.

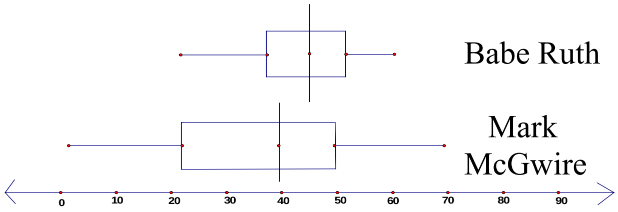
Activity 12: Representing Information (GLEs: 8th – 37, 40)

Have students collect data on the ages (in months) of all the students in their class, the teacher's age, and the principal's age, and construct stem-and-leaf plots for the entire class (including the teacher and principal), for the males, and for the females. From the stem-and-leaf plots or their list, have them generate box-and-whisker plots. Have the students find the '5 data points' that are necessary to make the plot (*low number, lower quartile or the median of the lower half of numbers, the median, the upper quartile of the median of the upper half of the numbers and the high number*). An example is given below.



The box portion of the plot begins with the lower quartile and extends to the upper quartile, and the whiskers emanate from the lower quartile to the low number and the upper quartile to the high number, extending to the extreme values of the data. The data in the box-and-whiskers plot above has the same low and lower quartile and therefore the box only has one whisker, this may happen with the ages of students in some classes. Another situation might be the number of pets that they have. Make sure they understand that each 'quarter' of the plot represents ¼ or 25% of the data used. The box represents 50% of the data. Have the students explain what this type of graph tells a person and when it would be helpful.

The box-and-whisker plots below compare the number of home runs Babe Ruth hit during his 15-year career from 1920 to 1934 with the number Mark McGwire hit during the 15 years from 1986 to 2000. Have the students determine what they can tell from these plots.



Repeat this activity with other sets of data. Have students discuss the spread of the data by examining the plots. Have students discuss the effects of the extreme values on the median and the mean. To generate a bigger spread in the original data, have students include the ages of siblings in months. Distribute various box-and-whiskers plots to students and summarize the information given. There are some sample plots for Activity 15 at the end of the activity section.

Activity 13: Promoting the Auction! (GLEs: 7^{th} – 3, 7, 12, 31; 8^{th} – 2, 5, 34)

Provide students with a chart like the one below representing data from a t-shirt Auction at Sunny Middle School. *(Students may need a reminder of their work with scientific notation in Unit 1 since the information uses exponents.)* Have the students complete a chart like the one below, and determine the total amount made at the auction if one t-shirt costs $5.50. Then have students prepare a graph representing the data to present to the Student Council for recognition from the student body. The Student Council wants to select a graph that will show how the different auctioned prices and the rule used affected the amount of money made. Lead a discussion about different data displays that might be used: bar graphs, circle graphs, box-and-whiskers plots, etc. Give groups of four students time to complete the chart and prepare a display of their data. Have the students present their representations to the class for discussion.

Rule used for Auctioned price	**Auctioned price *(p)***	**Number of T-shirts sold for this price**	**Amount made at each auctioned price *(T)***
twice the square of the price of the T-shirt		4	
one-half the cube of the price of the T-shirt		2	
5 times the cost of the T-shirt		15	
the square of the cost of the T-shirt plus $15		3	
one hundred times the cost of the T-shirt		1	
the cost of the shirt x 10^4		1	

Lead a discussion about the auctioned prices and a comparison of the effect that the exponents used had on the auctioned price.

Solutions to chart: $p = 1(10.95^2)$; $\left(\frac{10.95}{2}\right)^3$; $5(10.95)$; $10.95^2 + 15$; $100(10.95)$; $(10.95)(10^4)$ and $T = 479.61; 328.23; 821.25; 404.71; 1095; 109,500$

Activity 14: Patterns in Savings Accounts (GLEs: 7^{th} – 18, 40; 8^{th} – 11, 15, 46)

Provide groups of students with a situation similar to the following:

> Sam's grandmother opened a savings account for him with $1000. The account earns 4% interest yearly. If Sam does not add any money or take any money out of this account for 10 years, how much will he have in the account? Complete the table of values below and graph the amount in the account over the ten year period.

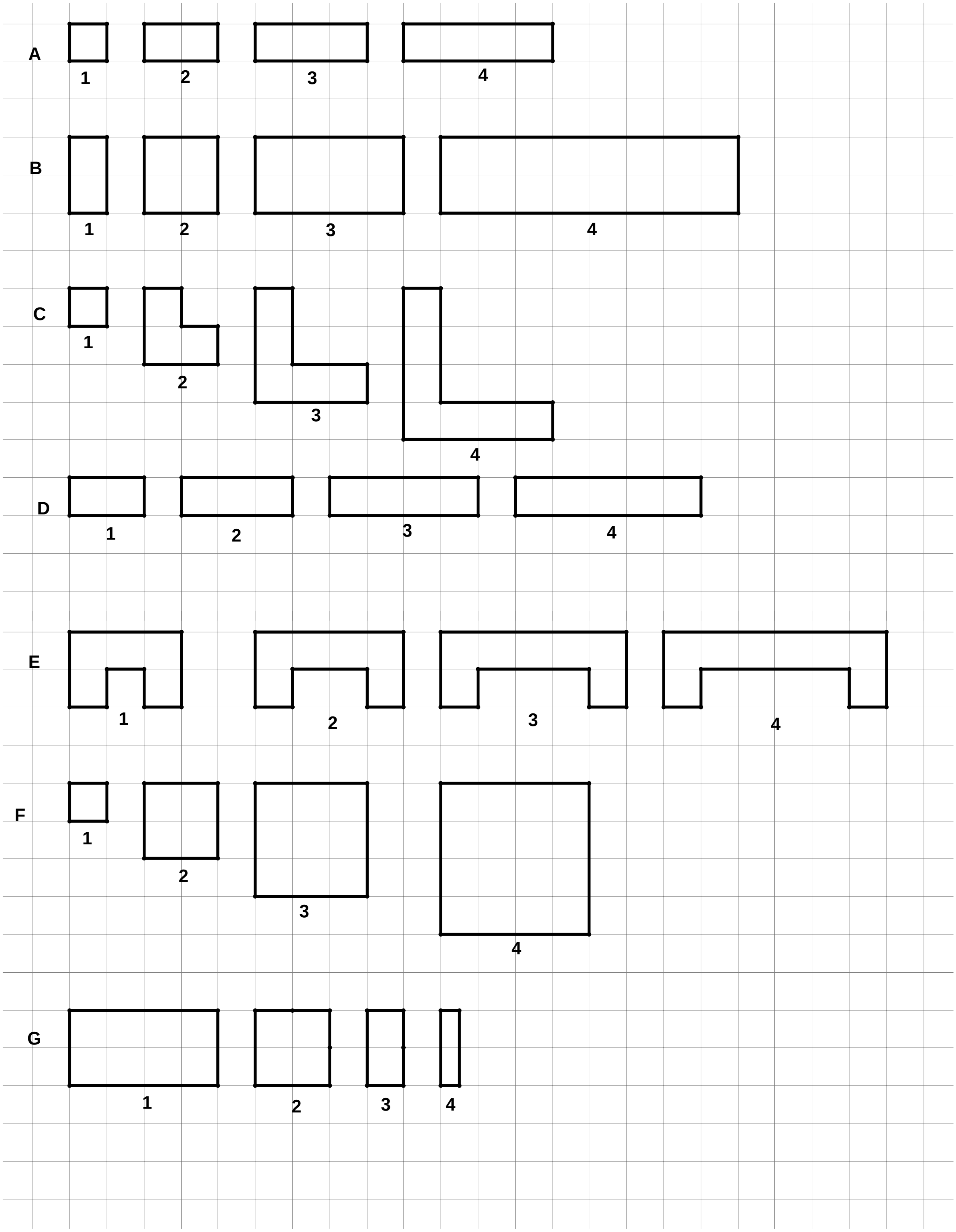
Does the pattern of balances form an arithmetic or geometric sequence? Explain.

Years	**Amount in savings**
0	$1000
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Solution: The pattern is geometric because each year's balance is multiplied by 1.04 to get the next year's balance.

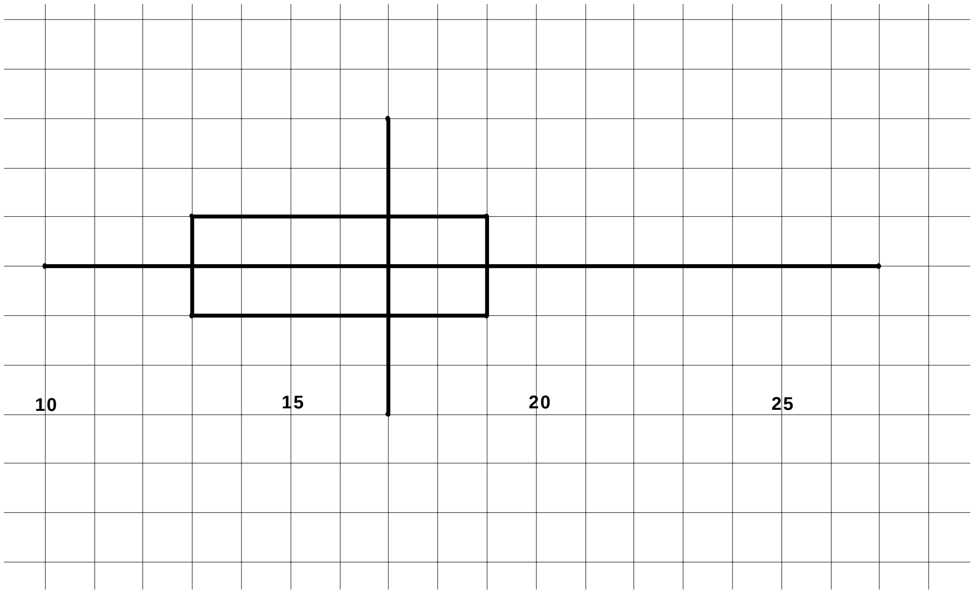
Years	**Amount in savings**
0	$1000
1	1040
2	1081.60
3	1124.86
4	1169.86
5	1216.65
6	1265.32
7	1315.93
8	1368.57
9	1423.31
10	1480.24

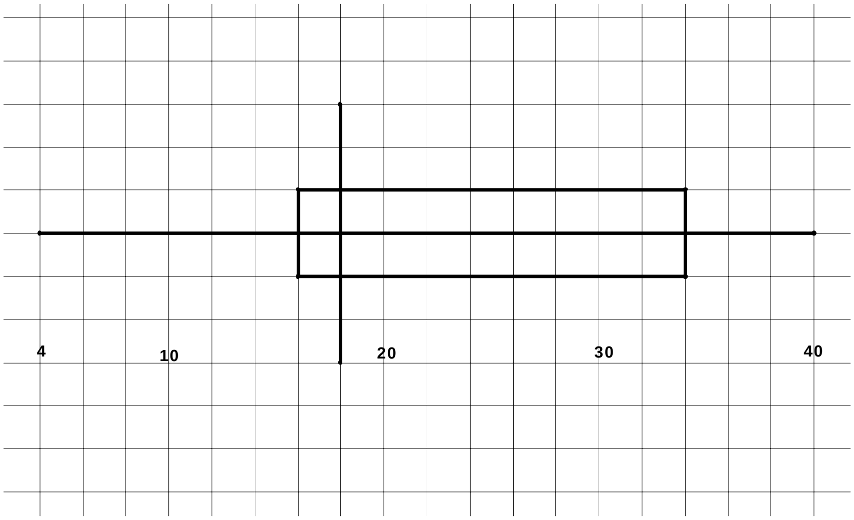
Activity 9
Patterns for Perimeter and Area



Activity 12
Box-and-Whiskers Plots

The plot at the right shows the number of questions that were correctly answered on a 30 question social studies test. Explain what is known about the results of the test from the box-and-whiskers plot.





The plot at the left shows the results of try-outs for the marathon swim team. The participants had to swim laps of the pool until they were too tired. Explain the results shown in the plot.

Sample Assessments

General Assessments

- The student will make a concentration game matching sequences and rules that describe the sequence. The student will prepare at least 15 matching sets to complete the game.
- The student will create two-variable equations to represent a given situation. Then, using given values for one of the variables, the student will solve for the other variable.
- The student will create a table of values or function machine that performs specified operations or produces a set of given outputs from a given set of inputs.
- The student will generate at least three different patterns of area and perimeter and determine the rule that describes the pattern. The student will also label each rule as either an arithmetic or geometric sequence.
- The teacher will provide the student with a table of values (input/output) and have the student explain in writing how to determine whether the list of numbers is an arithmetic sequence or a geometric sequence.
- Whenever possible, the teacher will create extensions to an activity by increasing the difficulty or by asking "what if" questions.
- The student will use the number 4 exactly four times along with any operation or combination of operations, parentheses, brackets or exponents to get answers of 1, 2, 3,…10.
- The student will complete journal entries using topics such as
  - I have a mystery number when multiplied by 4 and added to 19, the sum is 35. What is my mystery number? Set up an equation and solve for the mystery number.
  - Write 2 different two step equations that have a solution of 8. Explain your work.

Activity-Specific Assessments

- Activity 3: The student will plot a line on grid paper that has a rate of change of ¾ and make a table of values that the line represents.

- Activity 8: The student will complete the following exercise and submit to the teacher for inclusion in his/her portfolio.

  The following table shows the fees charged for campsites at one of the campgrounds at Sam Houston State Park.

C =Number of campsites	1	2	3	4	5	6	7	8
F =Total campground fees	$12.50	25.00	37.50	50.00	62.50	75.00	87.50	100.00

a. Make a coordinate graph of this data. Would it make sense to use negative numbers on this graph? Why or why not?

b. How will the x-axis and the y-axis be labeled? Why did you choose these?
c. Would it make sense to connect the points on the graph? Why or why not?
d. Using the table, describe the pattern of change in the total campground fee as the number of campsites needed increases. How is this pattern shown in the graph? Write an expression describing what is happening to C each time to get F?

- Activity 10: The student will explain whether his/her sequence is arithmetic or geometric and why. The student will should make a class presentation of the pattern and graph.

- Activity 14: Give students the first and last term in an arithmetic sequence of at least 5 numbers and have students work in pairs to determine a rule which will generate the first and last terms and also allow the students to find the missing numbers in the pattern~~s~~. For example, the first term in an arithmetic sequence is 41, and the fifth term is 77. The missing three terms are 50, 59, 68, and since the rule is 'add nine each time,' the expression will be $y = 9x + 32$.

Grade 7
Advanced Mathematics
Unit 5: Geometry and Measurement

Time Frame: Approximately four weeks

Unit Description

This unit extends the application of perimeter and area concepts learned in previous grades to irregular as well as regular polygons. An understanding of *pi* as the relationship between the circumference and the diameter of a circle and the area and circumference formulas for circles are also part of the unit. The concepts of surface area and volume are included. This unit also explores scale factors, proportional relationships, and the effects of scale on volume and area. Comparisons between metric and English measures for area, weight/mass, and temperature are made.

Student Understandings

Students apply the relationship between length of sides and perimeters and areas in similar polygons. Students determine the volume and surface area of various containers and investigate how change in dimensions affects the areas and volumes. Students begin to see the links between planar nets and their corresponding 3-D figures. Students explain relationships between vertices, edges, and faces of polyhedra. Students will determine that *pi* is the ratio of the circumference and diameter of a circle. Finally, students compare metric and U.S. measures for area, weight/mass, and temperature and convert between units within the *same system.*

Guiding Questions

1. Can students convert between measures of area within the same system of measurement?
2. Can students determine and apply the affect of scale on perimeter, area, and volume?
3. Can students compare the relative measures of area and weight/mass across different measurement systems?
4. Can students compare the relative measures of temperature across different measurement systems?
5. Can students draw and use planar nets to construct polyhedra, noting the relationships of sides, edges, and vertices?
6. Can students discuss similar and congruent figures, and make and interpret scale drawings of figures?
7. Can students use the coordinate plane to represent models of real-life problems?

Unit 5 Grade-Level Expectations (GLEs)

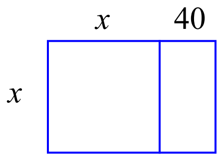
GLE #	GLE Text and Benchmarks
7th grade	
Number and Number Relations	
4.	Model and apply the distributive property in real-life applications (N-4-M)
5.	Multiply and divide positive fractions and decimals (N-5-M)
8.	Determine the reasonableness of answers involving positive fractions and decimals by comparing them to estimates (N-6-M) (N-7-M)
10.	Determine and apply rates and ratios (N-8-M)
Measurement	
20.	Determine the perimeter and area of composite plane figures by subdivision and area addition (M-1-M) (G-7-M)
21.	Compare and order measurements within and between the U.S. and metric systems in terms of common reference points (*e.g.,* weight/mass and area) (M-4-M) (G-1-M)
22.	Convert between units of area in U.S. and metric units within the same system (M-5-M)
23.	Demonstrate an intuitive sense of comparisons between degrees Fahrenheit and Celsius in real-life situations using common reference points (M-5-M)
Geometry	
24.	Identify and draw angles (using protractors), circles, diameters, radii, altitudes, and 2-dimensional figures with given specifications (G-2-M)
26.	Recognize π as the ratio between the circumference and diameter of any circle (*i.e.,* $\pi = C/d$ or $\pi = C/2r$) (G-5-M)
27.	Model and explain the relationship between perimeter and area (how scale change in a linear dimension affects perimeter and area) and between circumference and area of a circle (G-5-M)
28.	Determine the radius, diameter, circumference, and area of a circle and apply these measures in real-life problems (G-5-M) (G-7-M) (M-6-M)
29.	29. Plot points on a coordinate grid in all 4 quadrants and locate the coordinates of a missing vertex in a parallelogram (G-6-M) (A-5-M)
Patterns, Relations, and Functions	
40.	Analyze and verbally describe real-life additive and multiplicative patterns involving fractions and integers (P-1-M) (P-4-M)
41.	Illustrate patterns of change in length(s) of sides and corresponding changes in areas of polygons (P-3-M)
GLE #	**GLE Text and Benchmarks**
8th grade	
Numbers and Number Relationships	
2.	Use whole number exponents (0-3) in problem-solving contexts (N-1-M) (N-5-M)
6.	Identify missing information or suggest a strategy for solving a real-life rational-number problem (N-5-M)

Measurement	
17.	Determine the volume and surface area of prisms and cylinders (M-1-M) (G-7-M)
19.	Demonstrate an intuitive sense of the relative sizes of common units of volume in relation to real-life applications and use this sense when estimating (M-2-M) (G-1-M)
20.	Identify and select appropriate units for measuring volume (M-3-M)
21.	Compare and estimate measurements of volume and capacity within and between the U.S. and metric systems (M-4-M) (G-1-M)
22.	Convert units of volume/capacity within systems for U.S. and metric units (M-5-M)
Geometry	
24.	Demonstrate conceptual and practical understanding of symmetry, similarity, and congruence and identify similar and congruent figures (G-2-M)
26.	Predict, draw, and discuss the resulting changes in lengths, orientation, and angle measures that occur in figures under a similarity transformation (dilation) (G-3-M) (G-6-M)
27.	Construct polyhedra using 2-dimensional patterns (nets) (G-4-M)
30.	Construct, interpret, and use scale drawings in real-life situations (G-5-M) (M-6-M) (N-8-M)
32.	Model and explain the relationship between the dimensions of a rectangular prism and its volume (*i.e.,* how scale change in linear dimension(s) affects volume) (G-5-M)
33.	Graph solutions to real-life problems on the coordinate plane (G-6-M)
Patterns, Relations, and Functions	
48.	Illustrate patterns of change in dimension(s) and corresponding changes in volumes of rectangular solids (P-3-M)

Sample Activities

Activity 1: Perimeter of a Corral (GLE: 7th – 4)

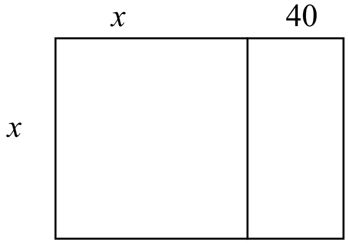
Have groups of students explore how to determine the perimeter of a rectangular corral that has a width of x feet and a length of $x+40$ feet.



Have students write the expressions for finding the perimeter different ways, explaining the method each time. Possible examples of expressions include adding all the side lengths, $x+x+(x+40)+(x+40)$, showing that there are two lengths and two widths, $2(x)+2(x+40)$, and using the distributive property, $2\left[x+(x+40)\right]$. Have a class discussion of the various methods found. Give each group a different value for x and have each student in the group use a different expression to find the perimeter. Ask group members to compare their answers with one another. Did all the expressions give the same perimeter? Why or why not? Repeat this activity using various widths and lengths.

Activity 2: Area of a Corral (GLEs: 7th – 4; 8th – 2)

Have groups of students explore how to determine the area of a rectangular corral that has a width of x feet and a length of $x + 40$ feet.



Ask each group to choose and substitute a number for x and find the area of the corral. Instruct students to show more than one method for finding the area. Have students present their methods to the class (*i.e.,* finding the areas of the two smaller rectangles and then adding them or finding the area of the large rectangle in one step).

Activity 3: Problem Solving Using the Area (GLEs: 7th – 22; 8th – 6)

Provide a real-life problem for students to solve that involves using the area to find a solution such as the following:

> Dan has a part-time job after school working for a landscaping company. He needs to purchase enough grass seed to cover the recreational soccer field. The field is 50 yards wide and 100 yards long. The instructions on a seed bag say that one bag will cover 2000 square feet. How many bags of seed does Dan need to purchase? The bags of seed cost \$11.95. How much will Dan spend on the grass seed? If Dan gets paid \$500 when he completes this job, how much will he clear after his expenses?
>
> *Solution: The field is 5000 square yards or 5000 x 9 = 45000 square feet; Dan will need to purchase 23 bags although he just needs 22.5 bags of seed. The seed cost Dan \$274.85; therefore, Dan will clear \$225.15.*

Activity 4: Area vs Perimeter (GLEs: 7th – 5, 41; 8th – 6)

Pass out 1 cm grid paper and scissors to the students. Have the students cut out rectangular shapes that have no gaps but that have an *area* of 24 cm and glue the shapes on a piece of paper. Ask them to write the dimensions of each rectangle and to find the perimeter. Have students make a chart to record the dimensions, area and perimeter. Have students make a chart to record the dimensions, area and perimeter. Have students respond to the statement: *All of these rectangular shapes have an area of 24 square cm, but the perimeters are different. How is the perimeter affected as the length of the sides changes?* Hopefully the students will begin to see that the closer to a square the rectangle becomes, the smaller the perimeter. Discuss as a group.

Rectangle dimensions	Area (u$^{2)}$	Perimeter (units)
1 x 24	24	50
2 x 12	24	28
3 x 8	24	22
4 x 6	24	20

Pass out more grid paper if needed. Have the students cut out rectangular shapes with a *perimeter* of 24 cm and glue them to a sheet of paper. Ask them to write the dimensions of each rectangle and find the area. Have students make a chart to record the dimensions, area and perimeter. Have students respond to: *All of these rectangular shapes have a perimeter of 24 cm, but the areas are different. If the perimeter remains constant, how does changing the dimensions of the rectangle affect its area?*

Rectangle dimensions	Area (u$^{2)}$	Perimeter (units)
1 x 11	11	24
2 x 10	20	24
3 x 9	27	24
4 x 8	32	24
5 x 7	35	24
6 x 8	48	24
7 x 7	49	24

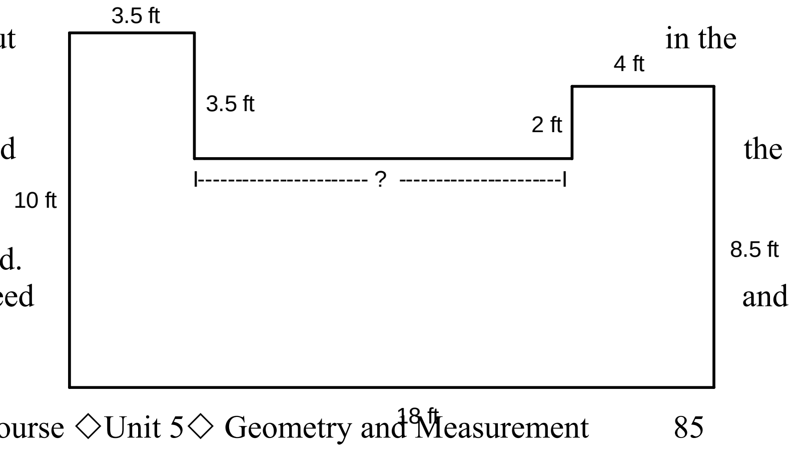
The students should discuss the idea that the largest area is a square with a perimeter of 24. Ask the students if it is possible to get an area less than 11 square units. *(This is possible if one dimension is a fraction less than one.)* Discuss as a group.

If students do not come up with the observations, lead the students to recognize that when areas vary—the most elongated rectangle has the smallest area and the rectangle closest to a square has the largest area. Have a class discussion of the importance of knowing how to compute largest area from smallest perimeter (building a house or garden) or the smallest area with the largest perimeter (dinner party serving guests around the tables) .

Activity 5: More Problem Solving Using Area and Perimeter (GLEs: 7th – 20, 22; 8th – 17)

Pose this situation and have the class work in groups of four to come up with a solution to the different parts of the problem provided below. This problem is one that the teacher can assign different parts to different groups. One group may find the cost of the cover for the pool one group may find the cost of the bricks around the pool, a third group could find the cost of covering the hot tub, and the fourth group the cost of the bricks around the hot tub. Require students to record their steps and show all work.

The swimming pool that is to be put back yard has an irregular shape as shown. A pool cover is needed to keep the leaves out this winter. Find area of the top of the pool. All corners are 90º. Pool covering material costs $4.95 per square yard. How many square yards will we need



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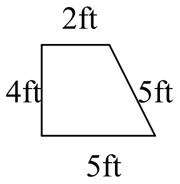
how much will the pool cover cost? Explain how to determine both the area and the cover cost. In order to brick around the edge, the perimeter of the pool is also needed. Find the perimeter. The bricklayer charges $20 per yard of perimeter to install the brick. How much will the bricklayer charge? Bricks are 6 inches long and are $0.60 each. How many bricks will we need to buy to put one row of bricks end to end around the pool? Explain and show all steps in each part of the problem.

A hot tub in the shape of a trapezoid has the dimensions shown in the diagram. The hot tub will be built along the right side of the pool and adjacent to the bricks. (Teacher Note: Students may solve the problem by subdividing the area of the trapezoid into a rectangle and triangle if the formula has not been taught.) A top view of the hot tub is shown. Decide where to place the hot tub. Find the cost of making a cover for the hot tub. Since the hot tub will be placed next to the swimming pool, the side with length 4 ft. will not be bricked. Find the cost of bricking the remaining three sides. Show all work for determining the cost of the cover and the bricks.



Solution: The area of the pool is 137.25 square feet, these needs to be converted to yards for the cover which is 137.25/9=15.25 square yards. 15.25 square yards x $4.95 = $75.49. The perimeter of the pool is 60 feet. 60 feet x $.60 = $36.00.

The area of the trapezoid is ½ (4)(2 + 5) = 14 square feet. Converted to square yards this is a little more than 1.55 square yards. 1.55 x 4.95 = $7.67 for the cover. The three sides to brick are 12 feet long. 12 x $.60 = $7.20. The perimeter of the entire area will be 60' + 12' = 72'. The bricklayer charges $20 a yard which will be (72 ÷ 3)20 = $480.

Activity 6: Discovering π! (GLEs: 7th – 5, 8, 10, 26, 28, 40)

The purpose of this activity is to have students investigate or discover π. A great resource to use to introduce or reinforce the terms that are used in this activity is *Sir Cumference and the Dragon of Pi* by Cindy Neuschwander. If the book is available, read the story aloud to the class and discuss how the terms are used in the story. Name all parts of the circle making sure to include the term *chord*.

Have students work in pairs for this activity. Have a variety of sizes of cans (or plastic tops for cans) that are numbered for this activity. Also provide rulers and tape measures. (Note: If tape measures are not available, students may use twine to measure the lengths and then find the length of the twine with a yard stick or ruler.) Each group should determine and record the circumference and diameter of at least 5 of these cans in chart similar to the one below.

Can Number	Diameter	Circumference	Ratio of Circumference/Diameter	Decimal Equivalent to ratio

Once the students have completed the chart, lead a discussion about the findings. Have students look at their ratios and make observations about their results. *These should all be close to 22/7 when simplified or a little more than 3 whole units. The student's decimal equivalents should be close to 3.14 or pi.* Discuss any measurements that are not close approximations and why they think this is so. Lead students to understand that averaging their decimal equivalents may be helpful in getting a more accurate estimate of the relationship. A larger data set may improve this estimate of the ratio, so if time allows, have the class find a class average.

Activity 7 - Circumference and Area (GLEs: 7th – 24, 27, 28)

Break the students into groups of 4. Give each group rulers and three measurements. Review the formula for the area of a circle. Refer to the book *Sir Cumference and the Dragon of Pi* (if read in the previous activity) to help the students relate this formula to the parts of the circle. Have the students draw three different circles with the given radii. Give $\frac{1}{3}$ of the groups the measurements 4 cm., 8 cm., 16 cm., another third the measurements 2cm., 4cm., 8cm., and the last third of the groups the measurements 3cm., 6cm., 12cm. Have students make a chart with titles radius, area, diameter, and circumference. Ask students to find the areas of the circles. Have the groups discuss how the radii in their three circles compare. (The radii double each time.) How do the areas compare? (The areas are 4 times bigger each time the radii doubles.) Discuss as a class the three different sets of numbers used and ask if the patterns are the same for all sets of numbers?

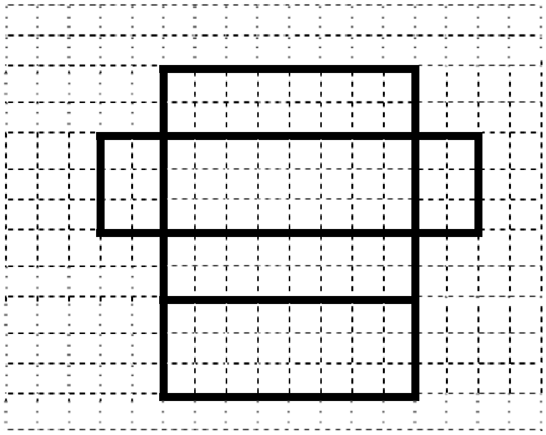
Have the students find the circumference of the circles. How do the circumferences compare? (The circumferences double each time the radii are doubled.) How do the diameters compare? (The circumferences double each time the diameters are doubled.)

An example of the chart with some answers is given. Provide students with practice measuring and calculating area, diameter, radius, and circumference of various circles.

Radius	Area	Diameter	Circumference
2 cm	12.56cm²	4cm	12.56cm
4 cm	50.24cm²	8 cm	25.12cm
8 cm	200.96cm²	16 cm	50.24cm

An interactive math lesson for calculating the circumference of a circle can be found at http://www.aaamath.com/geo612-circumference-circle.html.
An interactive math lesson to teach the finding the area of a circle can be found at http://www.aaamath.com/geo612-area-circle.html.

Activity 8: Volume of a Prism (GLEs: 7th – 20, 41; 8th – 17, 27, 32, 48)



Have student pairs create a rectangular prism using grid paper or give them the pattern like the one on the right and have them cut it out. They should fold the prism along the solid lines to form a rectangular prism with dimensions of 2 units x 8 units x 3 units. Have students identify and describe the *faces, edges,* and *vertices* of 3-D objects.

Discuss volume and surface area. (*Review the concept of both with the class and the proper units for measuring each.)* Next, have them use the prism to locate the dimensions to find volume. Once they have found the volume, have them find the area of each face and add these together to get the Surface Area of the rectangular prism. Define Total Surface Area as the sum of the areas of all the faces in a three-dimensional figure and explain that the formula used to find the area of a face depends on the shape of the face.

Have students create a second prism that is similar to the one made earlier using a scale factor of ½. Have students explore the effects the changes in the dimensions have on the volume and the surface area. Have the students construct a third rectangular prism similar to the original rectangular prism using a scale factor of 2. Lead a discussion about the rate of change of volume and surface area with respect to (1) the changes made in the dimensions and (2) the effects on the volume and surface area. Two charts have been provided to help students summarize the information learned about the effect of scale factor on surface area and volume. Give students the charts with only the first two columns filled in and have them complete the other charts. They may need help with recognizing and rewriting the ratios in column 4 as powers of 2.

Dimensions	**Scale Factor of Dimensions to Original**	**Surface Area**	**Ratio of Surface Area to Original Surface area**
8 x 2 x 3	1	92	92/92 = 1
16 x 4 x 6	2	368	368/92 = 4 (2^2)
24 x 6 x 9	3	828	828/92 = 9 (3^2)
4 x 1x 1 ½	½	23	23/92 = ¼ ($½^2$)

Dimensions	**Scale Factor of Dimensions to Original**	**Volume (V)**	**Ratio of Volume to Original Volume**
8 x 2 x 3	1	48	48/48 = 1
16 x 4 x 6	2	384	384/48 = 8 (2^3)
24 x 6 x 9	3	1296	1296/48 = 27 (3^3)
4 x 1x 1 ½	½	6	6/48 = 1/8 ($½^3$)

Solution: The big idea for the student to understand is that when the original dimensions are multiplied ½, the original SA is multiplied by 1/4 and the original V is multiplied by 1/8; when *dimensions are multiplied by 2, the original SA is multiplied by 4 and the original V is multiplied by 8.*

Next, have students repeat this activity using other types of prisms, including triangular, hexagonal prisms and pentagonal prisms.

Activity 9: Problem Solving—Comparing Changes in Surface area and Volume (GLEs: 8th – 20, 32, 33, 48)

Provide the students with the following situation.

> Sally's father can provide pieces of cardboard to make gift boxes for her friends at Christmas. He will make the cardboard boxes using a net and sketched the one shown for Sally to consider. What are the dimensions of the box that can be formed by the net? *(8" x 1" x 3").*
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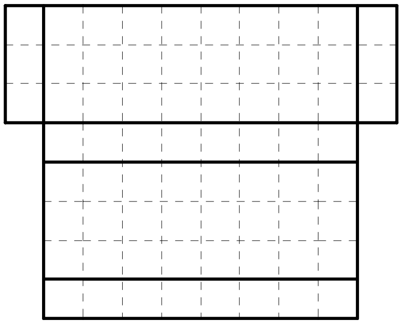
> 
>
> Sally's dad told her she could change no more than two of the dimensions of the original model, but she must give him all three dimensions before he cuts the nets for her. If Sally wants to increase the volume of the original box by a factor of 8 (*i.e.,* have a box which 8 times the volume of the original), what should the volume of the new box be? (*24 x 8 = 192 cubic inches*) Find two ways in which the dimensions of the box can be modified in order to meet this condition.

Have students test various dimensions and record the dimensions of any box which has 8 times as much volume as the original. As students work, make sure that they have modified only one or two of the original side lengths.

When students have completed their tasks, record the dimensions of the original box on the board or overhead and then list the new dimensions that students found. It may be easier to ask students who changed only one dimension to provide their dimensions first, asking each student to state which dimension they changed and how they changed it. (*One dimension should have been multiplied by 8.*) Repeat the process for those students who changed two dimensions. (*One dimension could have been multiplied by 4 and a second dimension multiplied by 2; one dimension could have been multiplied by 16 and a second dimension multiplied by ½ .*) Students may not think about increasing one factor and decreasing the second factor, so lead them to that conclusion, if necessary.

Example of dimension change:

Original box	Factor used to change volume	New dimensions	New volume
1" x 8" x 3" V = 24 in^3	2 x 4 = 8 so one dimension is multiplied by2 and another by 4.	2"x 8" x 12"	192 in^3
	Multiply one dimension by a factor of 8	8" x 8" x 3"	192 in^3

Ask students what might have been done to the original dimensions had Sally wanted to increase the volume by a factor of 10. Repeat as needed until students see that the product of the factors used determines the factor of change in the volume of the box. This correlates to the idea that doubling each of the sides of a prism (Activity 8) increases the volume of the prism by 2^3 or 2 x 2 x 2. Should all dimensions not be changed by the same factor, then the resulting change in volume is the product of the factors used to change the sides. See if students can extend this concept to area by asking, *How would the area of a rectangle change if the length is multiplied by 1/3 and the width is multiplied by 6?*

Ask students to use the dimensions that they found to determine which would have the greatest surface area. *(Answers will vary, but answers for the examples shown the chart are:*
2 x 8 x 12 = 272 in^2 *or 4 x 8 x 6 = 208* in^2*)*

Activity 10: Cylinders and Volume (GLEs: 7th – 20; 8th – 17, 19, 22)

Provide the students with the following situation.

> Lois is planning to make cylinders to decorate for candy at Christmas. She wants to determine what effect increasing the height has on the volume of the box if the base has a diameter of 6 inches. Lois needs a cylinder that will hold at least 135 in^3. She decided that she would make a table to compare the change that the height of the cylinder has on the volume. Determine the minimum height that she will need to construct her cylinder. Would the volume of this cylinder be more accurately reported in cubic feet? What is the surface area of the cylinder that Lois needs? Explain. *(She would need a cylinder with a height of 5 inches to have a volume of at least 135* in^3. *The cylinder with a height of 5 inches would have a volume of 141.3 in*^{3}. *No, the volume would not be more accurately reported in cubic feet because it has a little less that one cubic foot of volume.)* What everyday object might fit into a cylinder this size? *(a softball would fit, some cereal bowls would fit, but these might not touch the sides.)*

Area of the circular base ($\pi = 3.14$)	Height (inches)	Volume (in^3)
	1	
	2	
	3	

Have students complete a table like the one above and determine how changes in the height of an object affect the volume. Ask if the resulting values for volume are linear or non-linear (*linear).* Next, have students calculate the cost of covering the cylinder if the paper used costs \$0.36/square inch.

As an extension, have the students predict whether a box with dimensions of 6” x 3” x 5” would have more or less volume than the cylinder with a base diameter of 6” and a height of 5”. *(The rectangular prism has a volume of 90* in^3 *and the cylinder has a volume of 141.3* in^3 *)* Ask students which will have the greatest surface area? *(SA =* $2\pi r^2 + 2\pi rh = 87.92\ in^2$ *is the SA of the cylinder; 126* in^2 *is the surface area of the rectangular prism)* .

Activity 11: Common Containers (GLEs: 7th – 22; 8th – 17, 19, 20, 21, 22)

Provide student pairs with several common containers (rectangular solids and cylinders) found in the grocery or hardware store (with the labels removed or volume information covered). Have students estimate the volume of each container and arrange the containers in order from smallest to largest. Next, ask the students to determine the volume of each container using U.S. units. Finally, have students repeat the process using a metric measurement tool. Lead a discussion comparing measurement between systems. Once the volumes of the containers have been determined, have students convert their answers to another unit in the same system (*e.g.,* convert from cubic inches to cubic feet and vice versa—include conversions with metric units, also). Have students uncover the volume information on labels of the containers and discuss why the unit used is appropriate for the container. *Solutions: These answers will vary depending on the containers used. Be sure to provide some cylinders in the group so that the students have to find the area of the circular base and multiply by the height.*

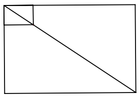
Repeat the activity with larger containers. For example, show a picture of a silo to the students. Explain to them that silos have been used for many years to store grain. Provide the dimensions of a silo and have students determine its volume. Repeat using a cylindrical underground gasoline holding tank. Have students determine the amount of gasoline one such container holds. Buildings can also be used as examples. The Superdome is in a cylindrical shape with a domed roof. Find the actual dimensions of the Superdome at www.superdome.com. Go to the *About Us* section and scroll down to the table of facts. Assuming the roof is flat, have students approximate the volume of the Superdome.

A basketball gym is typically a rectangular solid. Have students determine the volume of their school’s gym. In each case, have students express their answers in both U.S. and metric units. Discuss selecting appropriate units for measuring volume and capacity.

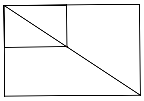
Activity 12: Similar Objects. (GLEs: 8th – 17, 22, 24, 26, 29, 30)

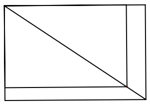
Provide students a sheet like the one after the activity pages that has a picture of similar objects with labeled vertices, a ruler and protractor. Instruct the students to work with a partner and gather the information. Lead a discussion to make sure the students see that similar figures are proportional and angle measurements are equal.

Provide students with grid paper and straight edges. Have the students use the grid paper to sketch a scale model of a 10 foot by 16 foot room. Have students draw a diagonal through the scale model of the room. Ask the students if the diagonal *bisects* the angles. *(No, the two angles* *are not equal)* What does *bisect* mean? Ask the students to determine the total number of degrees in the triangle. *Using the idea that all four of the angles in the rectangle are right angles, the triangle is ½ of this and would have 180°.*



Ask the students to draw a smaller rectangle inside the first rectangle, using the diagonal drawn through the scale model of the room as the diagonal of the new rectangle (examples at right).





Have students work in small groups to determine whether the rectangles are similar and repeat the actions with different sized rectangles to determine if their conjectures hold true. (*As in the diagram, the students should determine that they are similar if the same diagonal travels through opposite vertices of both rectangles because the sides are proportional. Make sure the students set up proportions to show the proportions are the same).* Have students determine at least two smaller rectangles that are similar to the original rectangle. Have students find the actual dimensions of the new rooms, using the scale established using the original rectangle. Have groups share and justify their conjectures.

Have students draw a length of a rectangle that is 7 inches and a width of 3 inches. If the original rectangle is the base of a rectangular prism, construct a net that will form a rectangular prism with a height of 2 inches. What is the volume of the rectangular prism? (30 in^3) What is the surface area of the rectangular prism? (82 in^2)

Activity 14: How Big Is This Room Anyway? (GLEs: 8th – 30)

Tell students that the class will make a classroom blueprint. Assign different groups of students the task of measuring the classroom dimensions. Have the class determine a scale that would fit on a piece of newsprint or poster board and then have someone draw the room dimensions to scale on the poster.

Divide students into groups of three to five. Assign each group a different object in the classroom to measure (file cabinets, book shelves, trash can, etc. - remember only length and width of the top of the object is needed for the blueprint). Have students convert actual measurements using the scale measurements determined earlier. Instruct students to measure, draw, and cut out models from an index card. Have each student measure his/her own desktop and make a scale model for the classroom blueprint. Remind students to write their names on the desktop model. Ask, What is the actual area of your desktop? What is the area of your scaled desktop on the blueprint? What comparisons do you see as you make observations of the areas of your room and desktop? List your observations.

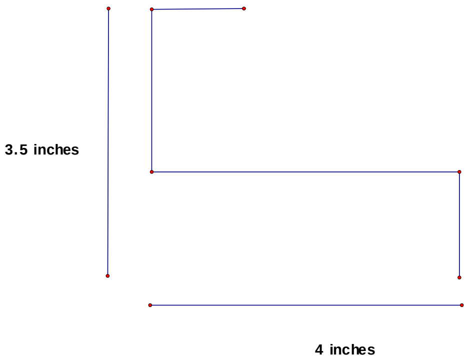
Have groups submit their scale models of the classroom objects (not desks at this time) for the blueprint. Discuss methods used to determine the measurements of the models and then glue the models in the correct position on the classroom blueprint. Have students, one group at a time, place their desktop models on the classroom blueprint, working so that those who sit in the center of the room can add their models first. Post blueprints/scale models on the wall for all classes to compare.

Activity 15: Mapping My Way! (GLEs: 7th – 29; 8th – 30)

Provide the students with grid paper and the following information: Sandy was given the assignment during a summer job to draw a map from the city recreational complex to the high school. Sandy started from the recreational complex and walked north 3.5 miles, west 10 miles, north 5.25 miles, and then east 3 miles. Sandy was told by the teacher to plot each point on a coordinate grid with the starting point at (15, 8). Sandy had to label each point with the correct coordinates.

The last part of the assignment was to assume that there was only a space $3\frac{1}{2}$ inches x 4 inches to sketch the route on a brochure being made by the staff at the complex. The teacher would select the correct sketches to send to the director of the recreational complex for their brochure. Determine a scale that Sandy will be able to use and draw a map that can be used in the space provided. Explain how the scale was determined.

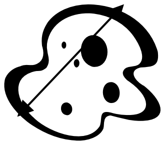
Solution: The scale factor must be divided by 2.5 or $\frac{1}{2.5}$ *to fit in the area given, therefore the scale should be 1 inch represents – 2.5 miles. The student should see the east-west direction is maximum 10 units, the north-south direction is 8.8 units*



Activity 16: How Big Was It Anyway? (GLEs: 8th – 30)

Provide groups of four students with situations like the ones below. Lead the class in a discussion of these scaled drawing situations after the groups have had time to complete the problems.

1. Draw a diagram of a rectangular bedroom with dimensions of 24 feet by 15 feet. Use a scale of $\frac{1}{2}$ inch = 6 feet.
2. The picture of the amoeba at the right shows a width of 2 centimeters. If the actual ameba's length is 0.005 millimeter, what is the scale of the drawing?



Activity 17: Measuring Scavenger Hunt (GLEs: 7th – 22; 8th – 22)

Make a list of measurements describing various objects found in the classroom or in a specified area outside on the school grounds. Provide measures in the U.S. and metric systems with angle measurements included. Give each pair of students a list of objects, a yard stick (or tape measure), a meter stick, and a protractor. Have students hunt for each object described, measuring objects to find the ones on the list and writing the name of the object found on paper. Specify a time limit for completion of the hunt. (Example of description of objects: This object is 6 inches off the ground, and its dimensions are 12 inches by 4 inches or this object has an angle that measures 60° and the sides that form the angle each have a length of 15cm. You bring this object to class and it is around 14 cm in length and weighs about 4 grams*(pencil)*). When the students return to the classroom, have them convert specified measurements within the same system (*e.g.,* 6 inches = _____ foot, 12 in = ____ft, 4 in = _____ foot). Let students use the site http://www.onlineconversion.com/ for help with conversions in length, weight and volume.

Activity 18: Metric Madness (GLEs: 7th – 21, 22, 40; 8th – 21, 22)

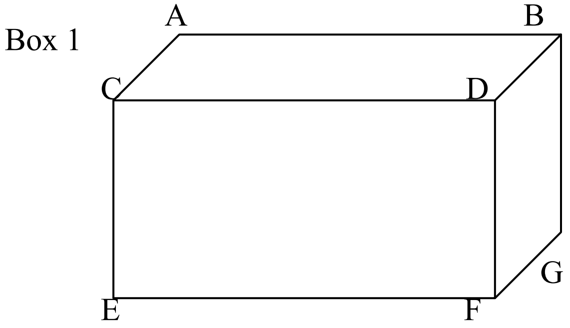
Pass out metric rulers and meter sticks to group of students. Have the students study the rulers and meter sticks and write down observations about the relationships between units. Lead a class discussion to help students develop an understanding of equivalencies. Discuss how the metric system is based on 10, the same as the place-value system. Help the students connect the metric prefixes with the decimal place value system. Place metric prefixes with decimal names on a place value chart to help students remember their values (*i.e.,* thousands–kilo, hundreds–hecto, tens–deca, ones–meter, tenths–deci, hundredths–centi, thousandths–milli). Discuss this chart emphasizing the most needed units–kilo, centi and milli. Using the meter stick, have the students convert millimeters to centimeters and kilometers and vice versa. Record each conversion on the board, study the conversions, and discuss how converting from a smaller unit to a larger unit requires division and converting from a larger unit to a smaller unit requires multiplication. Make sure that students are aware that the prefixes work the same with grams and liters. Present real-life problems in which students convert between measurements.

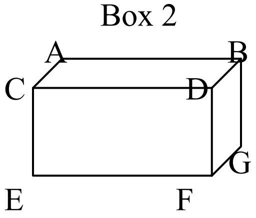
Activity 19: Temperature in the Newspaper (GLE: 7th – 23)

Bring several weather pages from the newspaper (or have the students bring them) to class. Give each group of four students a weather page. Have the students record as many observations as they can in 8 minutes. Discuss the students' observations--the temperatures and all the other information found on the weather page. Ask, *Are these temperatures in Celsius or Fahrenheit? How do you know?* Have students place and order the Fahrenheit temperatures given on the weather page on the temperature strip used in Activity 11, Unit 4. Have the students determine the approximate Celsius temperature for each Fahrenheit temperature using the temperature strip. Have students look at temperatures for cities, find the highest temperature and lowest temperature, place them on the temperature strip, and determine the approximate Celsius temperature for each. Have students explain methods of determining the Celsius temperatures to the whole class.

Activity 12
Possible Activity Sheet

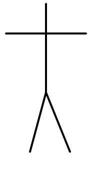
Box 1 is an enlargement of **Box 2**. Use a ruler to measure the lengths of the sides to the nearest mm in the chart below.





Measurement	Box 1	Box 2	Ratio
AB			
DF			
DE			
∠ACD			
∠ECD			

1. Side AB in Box 1 is a corresponding side to side AB in Box 2. What is the ratio of the sides?
2. Side DF in Box 1 is also corresponding to side DF in Box 2 . What is the ratio of the sides?
3. What conjecture can you make about the ratios of sides of similar figures?
4. What is the ratio of the angles you measured in Box 1 and Box 2? How do these measures relate?
5. Suppose side EC in Box 1 measures 2.8 cm. What would be the measure of EC in Box 2? Explain.
6. The stick figure below is similar to a figure that is twice as large. The figure below is 3.6 cm tall. What would be the height of the enlargement? Explain how you determined this.



Sample Assessments

General Assessments

- The teacher will create and use checklists to determine the student's understanding of measurement concepts.
- Whenever possible, the teacher will create extensions to an activity by increasing the difficulty or by asking "what if" questions.
- The student will be encouraged to create his/her own questions to evaluate his/her understanding of measurement concepts.
- The student will accurately measure different objects using a variety of measurement tools.
- The student will draw a scale model of his/her bedroom and create a makeover of the room. The makeover will include the purchase of flooring (carpet, tiles, etc), paint for the walls and relocation of furniture based on scaled drawings of the pieces of furniture. The student will show all mathematical steps for the work. The teacher will place the work in the student's portfolio.
- The student will complete journal entries using such topics as:
  1. Explain the difference between measurements made in ft, ft^2 and ft^3.
  2. Explain how it is possible to get two different numbers when measuring the same item. For example, one person measures a board as 18 feet and another measures it to be 6 yards.
- The student will prepare a brochure comparing US and metric volume measurement, using pictures of objects and listing a US volume measure and an approximate metric volume measurement or a metric volume measurement and an approximate US volume measure.
- The teacher will provide the student with a set of rectangular solids. The student will measure dimensions and find the volumes.
- The teacher will provide the student with two similar solids. The student will measure dimensions and find the volume of one solid, determine the scale factor, and use the scale factor to help calculate the volume of the other solid.

Activity-Specific Assessments

- Activity 2: The student will solve the following problem correctly:
  Your backyard is a rectangular shape that is 100 feet by 40 feet. The patio in the backyard is 18 feet by 20 feet. How much of the backyard is not covered by the patio?

- Activity 7: The students will solve problems such as the following:
  A rectangle has an area of 64 $in.^2$ and a side length of 8 in. Find the perimeter.
  A circle has the radius of 3.6 ft. Find the area.
  A circle has the area of 10 m^2. What is the diameter?
  Estimate the circumference of a circle with a radius of 1 ½ inches. (*use the ratio we determined in activity 6)*

- Activity 8: The students will work in small groups to prepare presentations to prove the groups' conjectures or rules on how the doubling or taking ½ of the dimensions of a rectangular prism affect volume of a rectangular prism to the class.

  Solution: The student should determine through these constructions that the volume of a rectangular prism is 8 times the volume each time the dimensions are doubled. The volume is divided by 8 when the dimensions are halved.

- Activity 11: The student will determine the volume of several containers in both U.S. and metric units.

- Activity 17: The student will determine an appropriate metric unit for measuring each of the following and explain his/her choice:
  a) The capacity of soda can
  b) The length of a person's foot
  c) The parking lot of a school
  d) The weight of a small box of raisins
  e) The thickness of a fingernail
  f) A bag of potato chips
  g) The area of a classroom
  h) The area of a leaf
  i) A single dose of liquid medicine

Grade 7
Advanced Mathematics
Unit 6: Measurement and Geometry

Time Frame: Approximately four weeks

Unit Description
This unit explores the properties of transformations on the coordinate grid; the relationships among angles formed by parallel lines; the use of nets to help students visualize three-dimensional solids; and applications of the Pythagorean Theorem and its converse. Elementary logic is explored through the use of Venn diagrams. The Counting Principle and probability are also introduced in this unit.

Student Understandings

Students grasp the meaning of congruence and measurement. They can apply transformations and identify properties that remain the same as figures undergo transformations in the plane. Students see the links between planar nets and their corresponding 3-D figures between vertices, edges, and faces of polyhedra. Students can provide one justification of the Pythagorean theorem and its converse and apply both in real-life applications. Students explore experimental versus theoretical probability and use the counting principle to solve simple problems.

Guiding Questions

1. Can students identify and apply the angle-sum relationship for a triangle in problem-solving situations?
2. Can students use their geometric knowledge in generalized problem solving?
3. Can students use elementary logic to solve problems?
4. Can students use transformations (reflections, translations, rotations) to match figures and note the properties of the figures that remain invariant under transformations?
5. Can students define and apply the terms *measure*, *distance*, *bisector*, *angle bisector*, and *perpendicular bisector* appropriately and use them in discussing figures synthetically and with reference to coordinates as well?
6. Can students draw and use planar nets to construct polyhedra?
7. Can students discuss similar and congruent figures, and make and interpret scale drawings of figures?
8. Can students state and apply the Pythagorean theorem and its converse in finding the lengths of missing sides of right triangles and showing triangles are right respectively?
9. Can students use the coordinate plane to represent models of real-life problems?

Unit 6 Grade-Level Expectations (GLEs)

GLE #	**7th grade GLE Text and Benchmarks**
7th grade	
Algebra	
12.	Evaluate algebraic expressions containing exponents (especially 2 and 3) and square roots, using substitution (A-1-M)
15.	Match algebraic inequalities with equivalent verbal statements and vice versa (A-1-M)
19.	Use *function machines* to determine and describe the rule that generates outputs from given inputs (A-4-M) (P-3-M)
Geometry	
24.	Identify and draw angles (using protractors), circles, diameters, radii, altitudes and 2-dimensional figures with given specifications (G-2-M)
25.	Draw the results of reflections and translations of geometric shapes on a coordinate grid (G-3-M)
27.	Model and explain the relationship between perimeter and area (how scale change in a linear dimension affects perimeter and area) and between circumference and area of a circle (G-5-M)
29.	Plot points on a coordinate grid in all 4 quadrants and locate the coordinates of a missing vertex in a parallelogram (G-6-M) (A-5-M)
30.	Apply the knowledge that the measures of the interior angles in a triangle add up to 180 degrees (G-7-M)
Data, Statistics, Probability and Discrete Math	
34.	Create and use Venn diagrams with three overlapping categories to solve counting logic problems (D-3-M)
36.	Apply the fundamental counting principle in real-life situations (D-4-M)
37.	Determine probability from experiments and from data displayed in tables and graphs (D-5-M)
38.	Compare theoretical and experimental probability in real-life situations (D-5-M)
Patterns, Relations, and Functions	
41.	Illustrate patterns of change in length(s) of sides and corresponding changes in areas of polygons (P-3-M)

GLE #	**GLE Text and Benchmarks**
8th grade	
Number and Number Relations	
2.	Compare positive fractions, decimals, percents, and integers using symbols (*i.e.,* <, ≤, =, ≥, >) and position on a number line (N-2-M)
Algebra	
11.	Use proportions involving whole numbers to solve real-life problems (N-8-M)
13.	Determine the square root of perfect squares and mentally approximate other square roots by identifying the two whole numbers between which they fall (A-1-M)

15.	Describe and compare situations with constant or varying rates of change (A-4-M)
16.	Explain and formulate generalizations about how a change in one variable results in a change in another variable (A-4-M)
Measurement	
17.	Determine the volume and surface area of prisms and cylinders (M-1-M) (G-7-M)
Geometry	
23.	Define and apply the terms *measure, distance, midpoint, bisect, bisector*, and *perpendicular bisector* (G-2-M)
24.	Demonstrate conceptual and practical understanding of symmetry, similarity, and congruence and identify similar and congruent figures (G-2-M)
25.	Predict, draw, and discuss the resulting changes in lengths, orientation, angle measures, and coordinates when figures are translated, reflected across horizontal or vertical lines, and rotated on a grid (G-3-M) (G-6-M)
26.	Predict, draw, and discuss the resulting changes in lengths, orientation, and angle measures that occur in figures under a similarity transformation (dilation) (G-3-M) (G-6-M)
27.	Construct polyhedra using 2-dimensional patterns (nets) (G-4-M)
28.	Apply concepts, properties, and relationships of adjacent, corresponding, vertical, alternate interior, complementary, and supplementary angles (G-5-M)
31.	Use area to justify the Pythagorean theorem and apply the Pythagorean theorem and its converse in real-life problems (G-5-M) (G-7-M)
33.	Graph solutions to real-life problems on the coordinate plane (G-6-M)
Data, Statistics, Probability and Discrete Math	
43.	Use lists and tables to apply the concept of combinations to represent the number of possible ways a set of objects can be selected from a group (D-4-M)
44.	Use experimental data presented in tables and graphs to make outcome predictions of independent events (D-5-M)
45.	Calculate, illustrate, and apply single- and multiple-event probabilities, including mutually exclusive, independent events and non-mutually exclusive, dependent events (D-5-M)

Sample Activities

Activity 1: Triangle Fun! (GLEs: 7th – 30)

Have students discover that the sum of the interior angles of a triangle is 180 degrees. Divide students into teams of three students each. Ask students to create as many different triangles as possible and to measure and label the angles in a 10-minute period. Lead a discussion of the students' conjectures concerning the sum of the angles in a triangle.

Have students cut out one of the triangles that they drew and then tear off the angles of a triangle. Instruct students to place the angles adjacent to one another with vertices touching to help them see that the sides of two of the angles form a straight line and that the sum of the three angles is 180 degrees. Software programs such as *The Geometer's Sketchpad* can also be used to allow students to investigate this property.

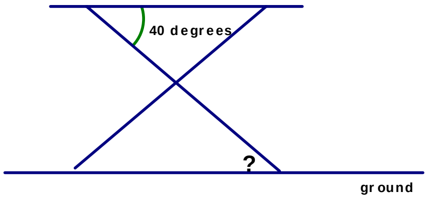
Have student teams create a triangle on colored paper or construction paper, attach it to a card, and label two of the angles labeled. The third angle should be marked with a question mark. On the back of the card is the numerical value of the missing angle. Have teams exchange their triangles with another team and play a game of finding the missing angle.

Activity 2: Angle Relationships (GLE: 8th – 28)

Have students to trace 2 parallel lines on a sheet of notebook paper. The lines should be about 2 inches apart. Instruct the students to place a point on the top parallel line and use their protractors to draw a transversal at a 30 degree angle through the parallel lines. Students should number the four angles formed around the top line of the parallel lines as angles 1 through 4 and those angles around the bottom parallel line as angles 5 through 8. For later discussion, it is better if all students number the angles in the same manner. Have students investigate the relationship among the angles that are formed by intersecting two parallel line segments with a transversal. This is a good time for the students to practice measuring angles with a protractor so that they can identify the congruent angles.

Have pairs of students determine pairs of angles that are supplementary, congruent, corresponding, adjacent, vertical, and alternate interior. Challenge the students to draw a second transversal through the parallel lines that would allow them to identify complementary angles. (A transversal with an angle measuring 45° would allow them to have two angles that are complementary)

As an application, pose the following problem to students: As a class project, you are going to build a picnic table with legs that form an "X." Of course, the top of the table must be parallel to the floor. If one of the legs is attached so that it forms a 40^{o} angle with the top of the table, draw a sketch to determine the measure of the angle formed between the leg and the ground to ensure the tabletop is parallel to the floor. Ask students to explain their reasoning.



Solution: As in the diagram, these are alternate interior angles and the angle with the ground should measure 40^{o}.

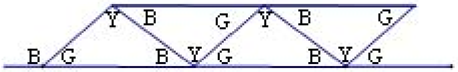
Activity 3: More and More Angles (GLEs: 7th – 30; 8th – 26, 28)

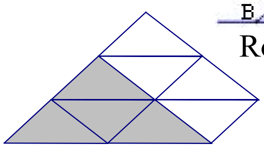
Give students a triangle cut from stiff paper and have them color each angle of the triangle a different color. In the diagram below, Y is for the color yellow, B is for the color blue and G is for the color green (since colors do not show up on the directions). Instruct students not to label the angles of their triangles. Make sure that the longest side of the triangle used is no longer than

3 inches for this activity. If each student has a different triangle, the activity becomes more powerful.

On a sheet of unlined paper, have the students draw a line near the bottom third of the paper. Take the triangle and place a base of the triangle on the line with the altitude pointing to the top of the paper. Trace it. Color each angle of the triangle drawn on the unlined paper the same as it is on the cut out triangle.

Slide the cut out triangle to the right on the line and trace it. Color each angle of the triangle. Slide the triangle over to the left and/or right until the page runs out, trace it and color in the angles of the triangle that show. Rotate the triangle so that the yellow vertex fits snuggly into the vertex formed by the blue and green angles. Trace this triangle. Color in the angles of the drawn triangle. Slide the triangle over until yellow fits snugly into the vertex formed by the blue and green angles; trace and color in the angles.





Rotate the triangle for the next level and trace the triangle, color the angles, slide the triangle, trace the triangle, then color the angles. Continue this until there is no more room at the top of the page.

In groups, have the students study the figure and list as many conclusions as they can. Have students outline or highlight certain parts of the figure, or cover up certain parts to make their conclusions. Some possible conclusions are: the total degrees of a triangle is 180° (all three different colors along the bottom and the sides come together to form a straight angle or 180°); the sum of the angles in 4-sided figures (squares, parallelograms, rectangles, trapezoids, and rhombii) is 360°; the sum of the angles around a point (a circle) is 360°; doubling the sides of the triangle quadruples the area; doubling the sides of the triangle doubles the perimeter while the altitudes are twice the original altitude; and a doubled triangle is similar to the original triangle. If students were able to make enough copies of the triangle to create a tripled triangle, discuss the effect of tripling on area, perimeter, and altitude with the class. Make sure that students show proof of their findings by showing where the property is illustrated in the drawing. Use probing questions to help students draw conclusions that were not addressed. Refer to the activity in Unit 5 where these comparisons were made with rectangular regions, surface area, and volume. Have students recall the relationships and make comparisons of the effect on the area of the triangle when the dimensions are doubled and halved.

This activity also provides students an opportunity to review the angle relationships when parallel lines are cut by a transversal. Have students identify parallel lines and *transversals that appear in the diagram.* The diagram also provides a good visual to identify *interior and exterior angles, vertical or opposite angles, supplementary angles and terms such as congruent and similar.*

Activity 4: Developing the Pythagorean Theorem (GLE: 8th – 31)

Have students draw a right triangle on grid paper with the two perpendicular sides having lengths of 3 and 4 units. Have students draw a square using one of the legs of the triangle as the side of the square (*i.e.,* draw a 3 x 3 square). Repeat, using the other leg as a side of a square (*i.e.,* draw a 4 x 4 square). Have students find the area of each square. Ask students to determine a method for finding the length of the hypotenuse, the area of the square which can be formed by the side of the hypotenuse, and how the areas of the three squares relate to one another. (Some students may remember the Pythagorean theorem from previous years and use that information to determine the length of the hypotenuse. Others may compare the length of the hypotenuse to the units on the grid paper. The process used is not important, but all students should eventually see that the hypotenuse length is 5 and the area of the corresponding square is 25 square units.) Have students show that the sum of the areas of the two smaller squares is the same as the area of the square formed by the hypotenuse by cutting and rearranging the small squares inside the larger squares. (Many texts and websites show how to do this. Two websites which use animations to develop the Pythagorean theorem are:
http://www.nadn.navy.mil/MathDept/mdm/pyth.html
http://www.pbs.org/wgbh/nova/proof/puzzle/theorem.html.)

Have students draw a triangle that is not a right triangle on the grid and determine whether they get the same results. Discuss conjectures that students develop about the results of their explorations.

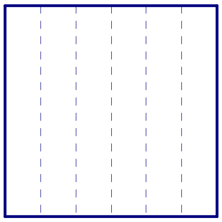
Activity 5: The Converse of the Pythagorean Theorem (GLE: 8th – 31)

Have student pairs cut out squares from grid paper that are 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, and 169 square units and create triangles using any three of the squares as sides. Have students use a protractor to determine the measures of each angle. Next, have students determine the relationship between the sum of the areas of the two smaller squares and the area of the largest square (e.g., are they the same or different?) Have students make a conjecture about the relationship between the areas of the squares when one of the angle measures of the triangle is 90 degrees. Remind students that these relationships are those of the Pythagorean theorem and its converse (studied in earlier activities). Lead a discussion of applications of the converse of the Pythagorean theorem to real-life situations. For example: A carpenter goes to the corner of a frame wall that he is building and marks off a 3 foot length on one board and a 4 foot length on the adjacent board. He then nails a 5 foot brace to connect the two marks. What is the purpose of his work? (*He is making sure that the two boards are perpendicular (that his wall is 'square') because a triangle with sides of 3-4-5 is a right triangle*.)

Activity 6: Squares and Square Roots (GLEs: 7th – 13)

Provide the students with a situation such as:

> Billy and Joseph were hiking in the mountains close to their home and discovered a tomato garden. At the side of the garden was a



scarecrow holding a sign that said “39 square feet.” This seemed odd and the boys decided that the sign must be the area of the garden. Joseph walked around the garden and determined that it was in the shape of a square. The boys decided that they wanted to find the length of each row to determine the unit rate of tomato plants per row. Find the length of the rows and explain how this can be determined.

Lead a discussion about using square roots to find the area of squares. The students should give the square root of 36 as 6 and the square root of 49 as 7 and justify that the square root of 39 is closer to 6 because the difference in 39 and 36 is 3 and the difference in 49 and 39 is 10. Since 3< 10, the square root of 39 is closer to 6. On a number line this would be between 6 and 6.5, since students just estimate the square roots. Challenge the students to find approximate square roots of other numbers, some of which are perfect squares and some not.

Activity 7: The Net! (GLEs: 8th – 27, 31)

Provide students with a 2-dimensional pattern (net) for a cube and have them fold and tape it together to form a cube. (A model is provided after the Activity pages). Discuss any other methods that the students used to create a net that formed a cube. Have students determine methods other than the ones discussed to make a net for a cube. Discuss any conjectures that the students can make about the nets. (*i.e.*, the squares must be arranged so that there are 4 that will cover the sides and one that will fold to form the bottom and another to form the top.) Have students find the area of one face of the cube and determine the surface area of the cube and then discuss. Lead a discussion about the number of faces, vertices, and edges each of these figures contains. Next, provide a net for a triangular prism and have students construct the prism by appropriately folding and taping it together. Have students discuss shapes that make up each face of the triangular prism. Determine a method of finding the area of each face. Provide rulers for measuring lengths so that the groups can find the areas.

Distribute grid paper and challenge the students to form a triangular prism is a right-triangular prism, then have students use the Pythagorean theorem to determine the area of the right triangular ends of the prism.

Activity 8: Graphing Ordered Pairs (GLEs: 7th – 25, 27, 29; 8th – 26, 33)

Have the students plot the points $A(1,3)$, $B(5,3)$, $C(5,5)$, $D(13,5)$ $E(13,7)$, $F(10,7)$, $G(10,11)$, $H(1,11)$ on a coordinate grid. Have the students label and connect each point as they are plotted to form a polygon. Ask students to:

- find the area and perimeter of the polygon
- reflect the polygon over the *x*-axis, then provide the new coordinates
- translate the *original* figure to the right 2 and down 1, then provide the new coordinates

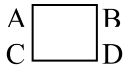
Have the students cut each original ordered pair in $\frac{1}{2}$ and then plot the new shape using these ordered pairs. Introduce the terms *reduction* and *dilation*. Ask students to find the area and perimeter of this figure and the area and perimeter of the original polygon. Discuss the students' findings. Ask if the process result in a reduction or dilation of the original figure.

Have students create their own polygons on a coordinate grid then dilate/reduce the originals by doubling and then halving the coordinates. Have students compare the areas and perimeters of the three figures.

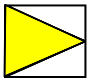
Activity 9: Transformations! (GLEs: 7th – 25; 8th – 23, 24, 25)

Have students work in cooperative groups of 4 and give each student a sheet of 1-inch grid paper. Have students in each group cut off the edges of their grid paper and tape the four sheets together to form a large coordinate grid with each sheet representing one quadrant of the coordinate plane. Have students label the point at which all four sheets meet as the *origin*. Ask them to label both the *x*- and *y*-axes, indicating the locations of –10 to 10 on each axis. Distribute four, 3" x 5" index cards to each group. Have students follow the steps listed below:

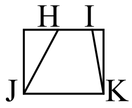
1. Index card #1 - Label the *vertices* of the index card with A, B, C, and D.



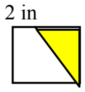
2. Index card #2 - Mark the *midpoint* of one of the 3" sides. Draw segments connecting this *midpoint* to each of the vertices on the opposite side. Cut out the *isosceles triangle* that is formed. Label the vertices of the triangle E, F, and G.



3. Index card #3 – On one of the 5 inch sides (say $\overline{AB}$), locate two points – one which is 2 inches from vertex A and one that is 1 inch from vertex B. . The segment between these two points forms the top of a *trapezoid*. Connect these points to the vertices the on the opposite side. Cut out the *trapezoid.* Label the vertices of the trapezoid H, I, J, and K.



4. Index card #4 – Measure 2 inches along one of the 5" sides and mark a point. Connect this point to the vertex on the opposite side to form an *isosceles right triangle.* Cut out this *triangle*. Label the vertices of the triangle formed L, M, and N.



Post a large sheet of newsprint on the wall for the new vocabulary used. As each new geometry term is discussed, have a student add the word to the *word wall poster*.

Ask students to create a chart as shown below:

Shape	Original Coordinates	Translation Coordinates	Rotation Coordinates	Reflection coordinates *x*-axis	Reflection coordinates *y*-axis
rectangle	A: B: C: D:	A: B: C: D:	A: B: C: D:	A: B: C: D:	A: B: C: D:

isosceles triangle	E: F: G:	E: F: G:	E: F: G:	E: F: G:	E: F: G:
trapezoid	H: I: J: K:	H: I: J: K:	H: I: J: K:	H: I: J: K:	H: I: J: K:
isosceles right triangle	L: M: N:	L: M: N:	L: M: N:	L: M: N:	L: M: N:

- Have students place the rectangle in the first quadrant and record the coordinates of all four vertices of the rectangle in its original position in column one of the table.
- Have the students *translate* the rectangle up (or down) and right (or left), and then record the new coordinates in column two.
- Have students return the rectangle to its original location and record coordinates of each vertex after a 180° clockwise *rotation*. Discuss *rotational symmetry* as students begin to rotate their shapes. (If this is new to the students, it works well if students put a small piece of tape on the rectangle to hold the rectangle in its original place on the grid, trace the figure, and then *rotate the traced figure 180° around the origin***.**) Have students discuss the new coordinates and identify the quadrant in which the rotated rectangle lies.
- Have students return the rectangle to its original location and then perform a *reflection* of the rectangle across the *x*-axis. Be sure to discuss *line of symmetry* as the rectangle is reflected. Model lifting the rectangle from the plane and flipping the triangle over the *x*-axis, if needed. Have students record coordinates of the four vertices.
- Have students return the rectangle to its original position, perform a reflection across the *y*-axis, and then record the new coordinates.
- Have the students complete the same actions using their trapezoid, right triangle, and isosceles triangle, recording all of the new coordinates on the chart. Remind them always to return their shapes to the original position before making a *transformation.*

After the class has had time to complete the transformations of all four shapes, have the groups make some conjectures about how they might be able to determine the positions of polygons after a transformation from the information in the chart. Have the groups share their conjectures with the class. At this point, encourage discussion about coordinate relationships with each transformation.

Activity 10: Reflections and Translations (GLEs: 7th – 25, 29; 8th – 25)

Provide students with three coordinates for the vertices of a parallelogram and have them plot the points on a coordinate grid. Have the students find and plot the fourth vertex in order to complete the parallelogram. Have students predict where the vertices of the parallelogram will be when reflected across the *x*-axis. Repeat with the predictions with a reflection across the *y*-axis. Next, have students reflect the polygon across the *x*-axis or *y*-axis. Ask students to determine the coordinates of the vertices of the reflected polygon. Repeat this activity with several different polygons. Have students make predictions as to the coordinates of their polygons after a reflection across the *x*-or *y*-axis. Use mirrors to help students justify their predictions for the reflections across the *x*- and *y*-axis.

Provide students with a set of coordinates for the vertices of a polygon that they are to plot on a grid. Again, have them predict the vertices of the polygon after the translation before the manipulation of the polygon. (This is reinforcing the conjectures formed in Activity 8.) Next, have students translate the polygon a fixed number of units and the coordinates of the vertices of the translated polygon. Repeat this activity with several different polygons. If students are having a difficult time translating the figures, provide them with polygon shapes to trace on grid paper, translate, and trace in the new position to determine the coordinates.

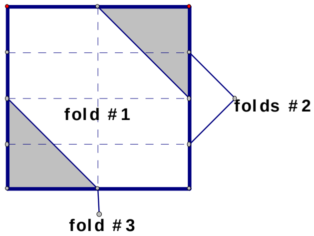
Activity 11: Transform Me! (GLEs: 7th – 25, 30; 8th – 23, 24, 25, 26)

Provide students with several sheets of grid paper. Have students plot the vertices of a predetermined polygon on a coordinate system and then connect the vertices to form the polygon. Have students find the measure of each angle, locate the *midpoint* of each side, and find the distance from vertex to vertex (*i.e.,* length) for each side. (*Teacher Note: Have students use rulers to measure lengths of sides that are not vertical or horizontal.*) Next, have students vertically translate the polygon a given number of units. Ask students to provide the new coordinates of the vertices, the measure of each angle, the location of midpoint of each side, and the side lengths. Discuss which (if any) of the properties of the polygon changed and which (if any) remained the same. Repeat this activity using reflections, rotations, and dilations with various polygons placed in various locations in the Cartesian plane. Ask questions about whether the original polygon is congruent or only similar to the transformed polygon.

Activity 12: From Table to Graph to Conjecture (GLEs: 7th – 12, 15, 19; 8th – 13, 15, 16)

Have students create a table of values for the area of a circle. The table should contain a column for the radius and a column for area (πr^2). Help students plot the ordered pairs $\left(r, \pi r^2\right)$ and compare the formula for finding the area of a circle with the graph of the points from the table of points. Remind students of the idea of a function machine and relate it to the use of the ordered pair, $\left(r, \pi r^2\right)$, with *r* being the input and the area being the output. Lead students to make a conjecture about the effects of doubling, tripling, etc., the radius on the area of the circle by examining the table of values and the graph. Have students share their conjectures and justify their reasoning as to why they think their conjectures are true. Discuss the shape of the graph and whether the relationship is linear or not.

Activity 13: Folding squares (GLEs: 8th – 23, 24, 28)



Provide square sheets of paper to each student. Have the students fold the paper in half with a *horizontal fold* (fold 1), make a good crease, and open the paper up again. Then instruct students to fold each *half* in *half again* using a second *horizontal fold* (fold 2), make a good crease, and open the paper up again. Have students make a *vertical fold* (fold 3), make a good crease, and open the paper up again. Ask students to make observations about the relationships of length of the line segments formed by the folds. Have students identify the vertical fold as a *bisector* and the *perpendicular bisector* of the horizontal segments. Instruct students to take the top right corner and fold it so that the vertex meets the *intersection* of their center folds, rotate their paper 180° and repeat this fold with the opposite corner. Have them open their paper and in their groups determine the measures of all angles formed by the different folds. Have groups prepare a presentation to the class and justify their angle measurements (*e.g.,* complementary, supplementary, vertical angles). Ask students to find similar triangles and similar rectangles and to provide a justification as to how they know the figures are similar.

Activity 14: Venn Diagrams, Parallelograms, Triangles, Rhombuses, Squares, Rectangles (GLEs: 7th – 24, 34)

Distribute grid paper and protractors. Have pairs of students cut the following shapes with the given specifications.

- A square with an area of 9 square units
- A parallelogram with one angle measuring 60°
- A scalene triangle
- A rhombus with sides 3 units
- A rectangle with the perimeter that is double the perimeter of the square that was made

Once the pairs of students have constructed and cut the shapes, have them draw a three circle Venn diagram. Working in pairs, the students should determine a method of classifying their shapes and place them in appropriate circles. Circulate and make sure the students are able to classify their shapes. Ask questions of the students to help them determine a classification method. Have each pair of students travel to another pair of students and study their Venn diagram and determine how the shapes were classified.

Activity 15: Probability Using Markers (GLEs: 7th – 37, 38; 8th – 43, 44, 45)

Make bags of 10 color tiles, markers, or plastic chips (*e.g.,* 5 blue, 3 red, and 2 green). Have students determine the number of tiles/markers there are of each color in the bag by randomly pulling items from the bag.

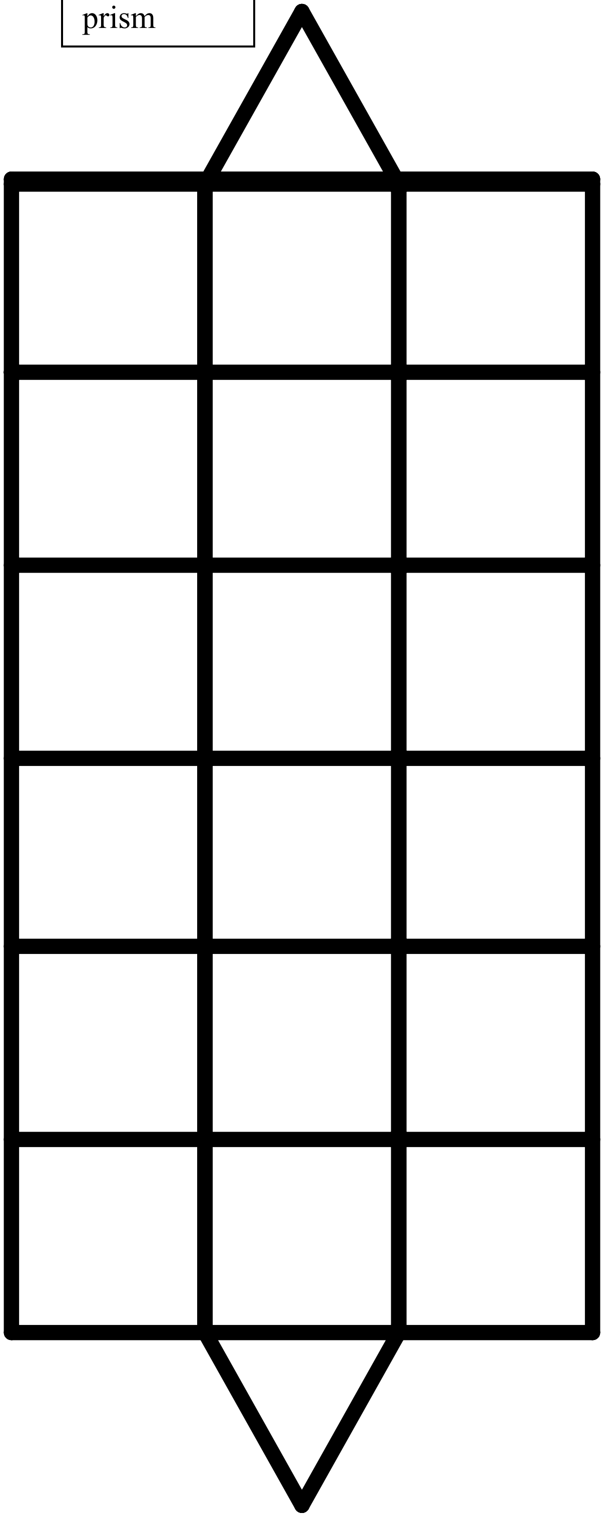
Have students work in pairs. Instruct student 1 to draw 1 tile/marker from the bag (without looking), record the color, and then replace it in the bag. Repeat this process 10 times. Have student 2 complete the same process. Ask, *Using your data, which color tile is most prevalent*? Discuss the students' predictions. Ask, *Do you think it is possible to have a color in the bag and never draw that color?* Have the students predict all the colors in the bag based on their data. Discuss their predictions. Have students open the bag, then count and record the number of each color. Ask students to compute the experimental probability of drawing each color in the bag based on their data and then to compute the theoretical probability. Ask, How do these compare?

Activity 16: Using the Counting Principle (GLEs: 7th – 36)

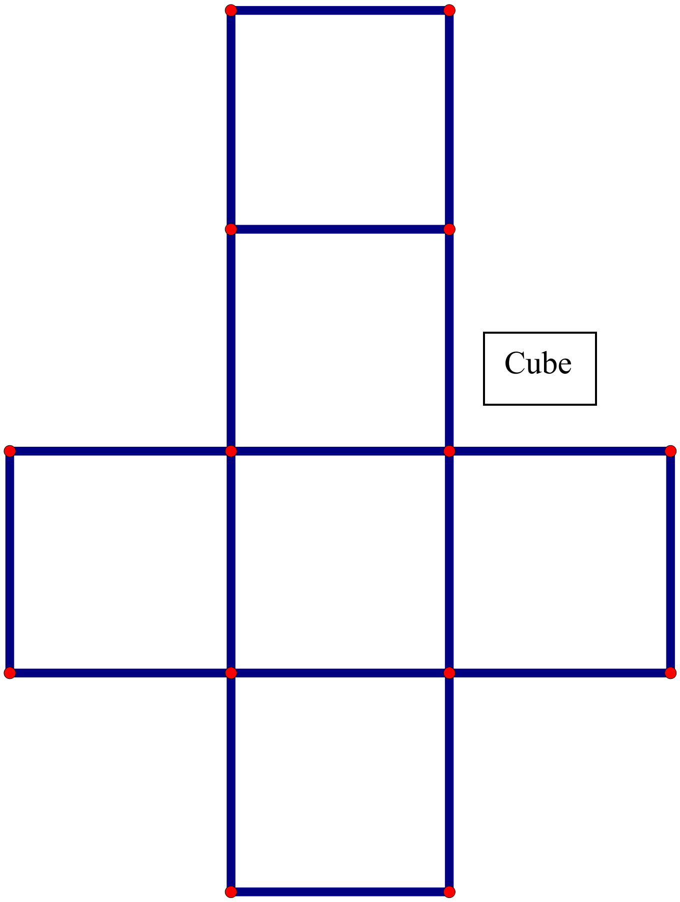
Provide students with real-life situations that require the use of the counting principle to find a solution. Some possible situations might be:

- Your aunt just bought a sewing machine that will make monograms at her store. She wants to make patterns for the customers to see. She wants to know how many two-letter monograms are possible.
- There are two new airlines that connect Houston, Shreveport, and Baton Rouge. They are the Blue Bird Line and Cajun Airways. How many choices are there for the three connecting cities and the two airlines?
- A restaurant has 4 different muffins and 5 types of spread. How many choices do are there?
- Barbara took a blue, a red, and a yellow shirt and a pair of green shorts, blue shorts and white shorts on a weekend trip to the beach. How many outfits could she make?

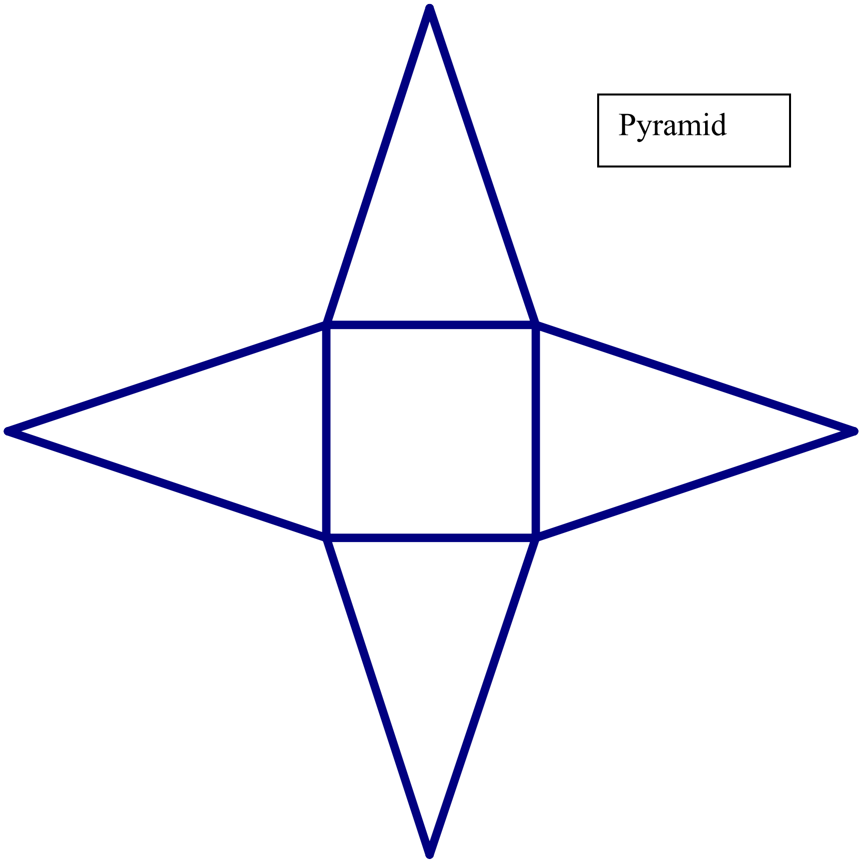
Activity 7 Nets



Triangular prism



Cube



Pyramid

Sample Assessments

General Assessments

- The teacher will determine student understanding as the student engages in the various activities.
- Whenever possible, the teacher will create extensions to an activity by increasing the difficulty or by asking "what if" questions.
- The student will be encouraged to create his/her own questions.
- The student will use a model to demonstrate an understanding of surface area.
- The student will create a portfolio containing samples to demonstrate understanding of transformations and where these transformations can be observed in real life.
- The student will create a geometry vocabulary foldable with illustrations of the terms that have been included in this unit of study. The illustrations should be pictures from magazines, computer articles, etc. Try to limit the number of hand-drawn pictures because the students are not as likely to make the real-life connection without finding examples.

Activity Specific Assessments:

- Activity 2: The teacher will provide the student with a sketch of an ironing board. In a journal entry, the student will explain the relationships of the angles formed by the legs of the ironing board.

- Activity 9: The teacher will provide the students with coordinates for a triangle in quadrant 2 of the coordinate grid. The students should be able to give the new coordinates after a reflection across *x*-axis, *y*-axis, a translation that will put only one vertex of the translated triangle into the 4th quadrant and explain the translation, and the new coordinates when the original triangle is rotated clockwise 90 degrees.

- Activity 13: The teacher will provide specification to students about certain geometric shapes. Students will construct these shapes using protractors and rulers.

  Possible specification:
  - A rectangle with an area of 98 square units and the length is twice the width
  - An isosceles triangle with a height of 3 units and an area of 12 square units
  - A rectangle with an area that is half of the square root of one hundred
  - A triangle with the hypotenuse of 4 units and one leg measuring 3 units.

Grade 7
Advanced Mathematics
Unit 7: Data and Statistics

Time Frame: Approximately four weeks

Unit Description

This unit focuses on the analysis and interpretation of data. Reasoning numerical or logical problems will be expanded to include three-circle Venn diagrams. Representations of data using appropriate graphs and displays and concepts of range, quartiles and shapes of distributions are explored as appropriate graphic displays are explored.

Student Understandings

Students can distinguish between discrete and continuous data, make appropriate choices in displaying data, and understand the impact this has on interpretations of the graphs. Further, they are able to identify and discuss the significance of gaps, clusters, and outliers as they examine information from surveys and experiments. Students can discuss variability in data through the nature of its spread, using range and quartiles, and illustrate the data with stem-and-leaf and box-and-whisker plots. In discussing distributions, students should be able to note the effect that the extremes of different distributions have on measures of central tendency (mean, median, and mode). Students can reason both logically and numerically in situations represented by three-circle Venn diagrams. Students can compare data sets by graphing data on the coordinate plane and draw and interpret trend lines for the data set. Finally, students should be able to analyze the validity of projections and generalizations made about patterns in different data sets.

Guiding Questions

1. Can students distinguish between discrete and continuous data and choose appropriate graphical representations?
2. Can students talk about clusters, gaps, and outliers in data and their meanings?
3. Can the students determine probability from experiments and from data displayed in tables and graphs?
4. Can students select and defend their choice of graphs to represent data sets for single-variable data or two-variable (bivariate) data? Can students discuss the nature of variability and graphically illustrate it with stem-and-leaf and box-and-whisker plots, as well as through the use of range and quartiles?
5. Can students graph two-variable data on a coordinate graph and draw and discuss trend lines for its pattern, if any?
6. Can students describe the effect that extremes in data distributions have on the values of their mean, median, and mode(s)?

GLE #	**GLE Text and Benchmarks**
7th grade	
Geometry	
24.	Identify and draw angles (using protractors), circles, diameters, radii, altitudes, and 2-dimensional figures with given specifications (G-2-M)
Data Analysis, Probability, and Discrete Math	
31.	Analyze and interpret circle graphs, and determine when a circle graph is the most appropriate type of graph to use (D-2-M)
32.	Describe data in terms of patterns, clustered data, gaps, and outliers (D-2-M)
33.	Analyze discrete and continuous data in real-life applications (D-2-M) (D-6-M)
34.	Create and use Venn diagrams with three overlapping categories to solve counting logic problems (D-3-M)
35.	Use informal thinking procedures of elementary logic involving *if/then* statements (D-3-M)
8th grade	
Numbers and Number Relations	
7.	Use proportional reasoning to model and solve real-life problems (N-8-M)
Measurement	
19.	Demonstrate an intuitive sense of the relative sizes of common units of volume in relation to real-life applications and use this sense when estimating (M-2-M) (G-1-M)
Data Analysis, Probability, and Discrete Math	
34.	Determine what kind of data display is appropriate for a given situation (D-1-M)
35.	Match a data set or graph to a described situation, and vice versa (D-1-M)
36.	Organize and display data using circle graphs (D-1-M)
37.	Collect and organize data using box-and-whisker plots and use the plots to interpret quartiles and range (D-1-M) (D-2-M)
38.	Sketch and interpret a trend line (*i.e.,* line of best fit) on a scatterplot (D-2-M) (A-4-M) (A-5-M)
39.	Analyze and make predictions from discovered data patterns (D-2-M)
40.	Explain factors in a data set that would affect measures of central tendency (*e.g.,* impact of extreme values) and discuss which measure is most appropriate for a given situation (D-2-M)

Sample Activities

Activity 1: Circle Graphs All Around (GLEs: 7th – 31, 32; 8th – 34, 39, 41)

Review the concept of surveys such as television rating surveys. Conduct a survey in the classroom such as which of 5 network television shows students in the class prefer. Lead a discussion about what the students remember about circle and bar graphs. Make sure someone points out that the percentages used in a circle graph must be set as the percent of the 360 degrees in the circle. Have the students create circle graphs and bar graphs of these favorite television programs by gender and lead a discussion about what they show. Discuss how to interpret a circle graph and determine when it is appropriate to make circle graphs. Example discussion questions: Does this type of graph lend itself to presenting clusters, outliers, or gaps? Are there any "outliers" or "gaps"? Where does the data seem to cluster? *(no)* When and why are circle graphs used? (*to compare parts of something as compared to the entire population)* Find circle graphs from magazines and newspapers and give each group of 4 students a different circle graph to study and interpret. Have students write at least 5 mathematical statements about their graph. After 10 minutes, have each group present their interpretation of the circle graph to the class. When all presentations have been presented, facilitate a discussion of the differences and similarities of the circle graphs presented.

Activity 2: Circle Graph *M & M's*® (GLEs: 7th – 31; 8th – 7, 34)

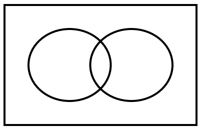
Give each student an individual pack of *M & M's*® and a chart which has the headings: color, prediction, actual number, fraction, decimal, and percent. Before opening the bag of *M & M's*®, have the students write on the chart a prediction of the number of each color of in the bag. Open the bag of candy and count out the actual number of each color and write the actual number in the chart. DON'T let the students eat the candy yet. In the fraction column, have the students make a ratio in fraction form of each color to the total number of candies they have in the bag. Ask them to change this to a decimal for the decimal column and then a percent for the percent column. Have the students use the *M & M's* ® to form a circle on a sheet of paper keeping the colors together and the candies touching. Draw a circle either around the outside edge of the *M & M's* ® or on the inside edge of the *M & M's* ® with a pencil. Ask them to make a mark for the center point of the circle and draw a line from the center point to the edge of the circle where two different colors meet. Instruct students to write in the color and percent for each of the sections. Make sure the students are using the correct angle measurements for the sections of the circle graph. If graphing calculators are available, have students make a circle graph with the data in the chart and compare it to the circle graph they made on their paper.

Once the students have their circle graphs drawn, allow them to eat their *M & M's* ® . Have a class discussion on the differences in colors and number of *M & M's* ® in each bag. In advance of the class, prepare a poster board by gluing *M & M's* ® found in a large bag in the same circular shape described earlier. Lead a discussion to determine how the percents and colors in the large bag compare to those of the smaller bags.

Activity 3: Venn Diagrams (GLE: 7th – 34)

Students have already been introduced to Venn Diagrams in the earlier grades. In the 6th grade, students created Venn diagrams with two over-lapping circles. This activity expands this skill to the use of three overlapping circles.

Examples of Venn diagrams are below. Enlarge and place these around the room.



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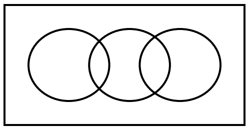

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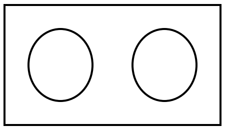

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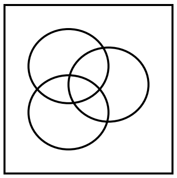

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Lead a discussion of the meaning of each poster. (Change meaning if needed to fit students in the class.) The first Venn diagram shows two categories that overlap such as *I am wearing blue jeans* and *I ate breakfast this morning*. Those students who meet both criteria would be listed in the overlapping section. Students who are wearing red and didn't eat breakfast would not be placed in either circle, but would lie in the Universe (inside the rectangle, but not in either circle). The second Venn diagram shows three categories that overlap: *I ride a bus to school*, *I take more than 10 minutes to get to school*, and *I walk to school*. Have students tell where a student would be placed if he/she walks to school in 8 minutes or rides 12 minutes in a car. In Venn diagram three, two categories that have no connection are shown such as the statements: *I am at least 70 inches tall* and *I am less than 64 inches tall*. Ask where to locate a student who is 68 inches tall, 56 inches tall, or 70 inches tall. The fourth Venn diagram shows *popular pets for students—dogs, cats, and fish*. Ask why each circle would overlap each of the other circles.

With the class, properly label each Venn diagram. Pass out 4 different colored sticky dots to each student and have them place their dots on each Venn diagram. Allow them to put their initials on the dots if they wish. Discuss the placement of the dots to check for understanding. Lead a discussion regarding trends noticed in the Venn diagrams. Example of the questions that may be asked on the fourth Venn diagram include: How many people own a dog? How many people own only a dog? What is the most or least popular pet? Have students create their own surveys (favorite type of movie, favorite type of music, etc.) and make a Venn diagram to show the results.

Activity 4: Describing Data (GLEs: 7th – 32; 8th – 38, 39)

This activity explores the relationship between grams of fat and grams of protein in menu items at various fast food restaurants, allowing students to explore real-life data by creating scatter plots. Go to http://www.mcdonalds.com/app_controller.nutrition.index1.html or http://www.wendys.com/food/pdf/us/nutrition.pdf (gives all menu items in chart form), or http://www.bk.com/Nutrition/PDFs/brochure.pdf (choose each menu item individually) and print information from at least two restaurants prior to class. Have groups of four students pick at least 10 items listed as individual servings for lunch or dinner and plot them on a coordinate graph. Have students determine if their group data has any 'clusters' by determining if there are any places on the graph that have a greater number of points; 'outliers' where one point is an extreme; or 'trends'. Have them think about a question such as, "Is there a relationship between the number of fat grams and the number of protein grams?" Have students place a pencil or uncooked piece of spaghetti diagonally on their group's graph so that about ½ of the points are on either side of the pencil. This gives them an approximation for the line of best fit. Have them draw this line on their scatter plot. Allow time for groups to discuss any trends that their graphs show.

Provide a transparency grid on which the scale has been pre-determined and have each group of students plot their points on the transparency to form a class scatter plot. (This will go faster if the teacher provides a duplicate copy of the transparency for each group and then overlays the transparencies on the overhead.)

Lead a class discussion about how the patterns in the data are more easily determined with a larger set of data. Are there any "outliers" or "gaps"? Is this different than their group's graph? Why or why not? Where does the class data seem to cluster? Is there a trend? Place a piece of uncooked spaghetti on the graph to develop the concept of line of best fit. Is the trend more easily determined with the larger set of data? Will this always be true? Why or why not?

Have students collect data on another topic, create a scatter plot, and discuss any patterns observed in the data

Activity 5: Data Patterns (GLEs: 8th – 19, 39, 41)

Lead a class discussion about data patterns and about when the students remember seeing some kind of data pattern. A student graph and question sheet that illustrates a pattern of the number of documented alligator bites versus the length of the gators that bit humans is included at the end of this unit. Have the students work in pairs to determine the pattern and make a prediction from the data. A picture of an alligator has been included to allow students to determine the scale used in the picture and then give dimensions of a rectangular solid that would contain the gator pictured.

Activity 6: Unbiased Sampling and Appropriate Graphs (GLEs: 7th – 24, 31, 32; 8th – 34, 36, 38, 39, 41)

Discuss **sampling.** Be sure to discuss the importance of the sample population on the results of the sample. Inform students that whether a sample is unbiased or biased depends in part on the purpose of the survey. Have students get into groups of four and determine current issue that they would be interested in conducting a survey. Some suggested topics might help the students with their surveys. Examples: Would a talent show or a softball game be a better fund raiser for the 7th grade class field trip? Would a caution light or a stop light be the better solution for the traffic congestion at _____________ intersection? Would red, white, & silver, red, white & gold or just red & white be the preferred color background for the Valentine Dance picture backdrop? Would the Sadie Hawkins Dance or a Homecoming Dance be the better fund raiser for the 7th grade class? etc.

Have students turn in their survey topics for approval prior to developing survey and indicate the sample population that will be used to collect their data. Encourage the students to select a topic that gives them results that would interest them and a sample population that will result in unbiased data.

Have students make a prediction of what the results of their survey will be. Tell them to pay attention to the number of people they survey and the sample group that they survey. Student groups should develop their survey, and ***conduct the survey and bring results of their surveys to the next class.*** Ask students to present their survey results in a display that is appropriate for the type of data collected. Students should be ready to present survey question, sample population surveyed, and display to the class. When appropriate, have students make predictions using their data and displays. If data are displayed as a line graph, have students project future values by finding points on the line of best fit. Encourage the use of a spreadsheet to enhance the ability to display the data. Remind the students to:

- Display data that are partitioned so that a ratio or percentage of each part to the whole is of interest in a circle graph.
- Display data that show a trend over time as a scatter plot with a trend line sketched.
- Display data that have no numeric ordering as a bar graph.
- Use histograms when the data are grouped in categories along a numeric scale (*e.g.,* ages of presidents when elected).

Activity 7: Discrete and Continuous Data (GLEs: 7th – 33; 8th – 35)

A set of data is said to be discrete if the values / items belonging to it are distinct and separate, *i.e.*, they can be counted (1,2,3,...). Some examples of discrete data might be: the heights of each of the students in the class; the number of female bus drivers at each school; etc.

A set of data is said to be continuous if the values / items belonging to it may take on any value within a finite or infinite interval. Examples of continuous data might be: a student's height over time; the distance traveled over time; etc.

Discrete data when graphed are not connected by intermediate values while the graph of continuous data has no gaps, jumps or holes. The graph of a straight line is continuous, while the graph of the cost of n first-class postage stamps is not.

Have students count the number of vehicles in the school parking lot along with the make and color of each vehicle in chart format. (An alternative is to provide students with a chart of the same information.) Discuss the fact that this is an example of *discrete* data. Lead a discussion about discrete data, emphasizing the inability to have a fraction of a car or truck. Have students work in groups draw a graph of a situation in which discrete data would be appropriate. Emphasize that dots are never connected on discrete data graph.

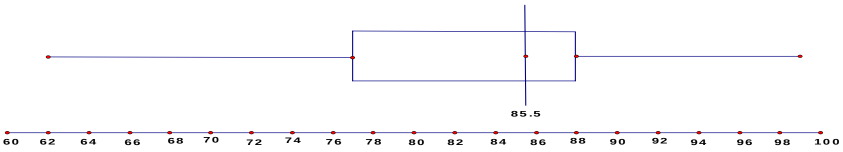
Discuss the concept of *continuous* data in using examples such as the temperature during the day, miles per hour, fuel consumption, etc. Display a continuous graph and lead a discussion that emphasizes the importance of the connection of the data points and the information that can be found by inspecting the graph between the data points. Have students work in groups to create and display different continuous situations to the class.

Review the water cycle. Place beakers (well-marked with metric measures) of water and thermometers around the room in both sunlit and shaded areas. Assign a group of students to monitor each beaker and to record the level of the water in the beaker and temperature of the water each day. On the fifth day, have students create a chart, graph the data, and explain their findings to the class. Lead a discussion of the differences in the findings and explore possible hypotheses about the temperatures based on locations of the beakers. (This activity will only work for those classrooms with windows.) The students should include in their presentations whether or not the data that was collected is continuous or discrete. *Since both temperature and water levels are changing over time and the levels of the water and temperatures are connected by intermediate values, then both would be continuous data.*

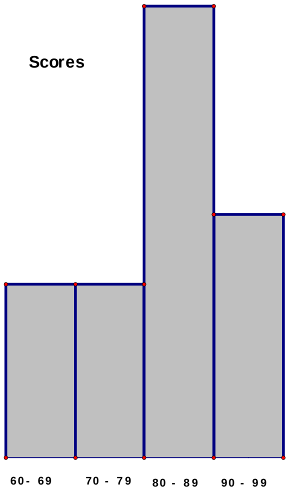
Activity 8: Data Displays (GLEs: 8th – 37, 40)

Put data like that on the right on the board or overhead for student use. Instruct the students to create a box-and-whiskers plot to show the range of the scores on the test. Remind the students that we did a box-and whiskers plot in Unit 4 and they must find the 5 data points to make the plot. Have students review what these points are and have someone write them on the board. Give students 10 – 15 minutes to work in pairs and create their box-and-whiskers plot. A completed plot should look similar to the one below.

Student Number	Score
1	77
2	65
3	88
4	98
5	78
6	86
7	88
8	93
9	91
10	88
11	83
12	81
13	74
14	62
15	86
16	67
17	81
18	85
19	95
20	99
21	78
22	68
23	87
24	71
25	68
26	98
27	95
28	88
29	86
30	81



Next, have the students make a stem-and-leaf plot and a histogram with the same data. Discuss what the intervals might be for the histogram (most will probably suggest 60 – 69, 70 – 79, 80 – 89, and 90 – 99). Make sure the students understand that the intervals have to be the same size, and in this case, the intervals are each 10 points.



stem	leaf
6	2, 5, 7, 8, 8
7	1, 4, 7, 8, 8
8	1, 1, 1, 3, 5, 6, 6, 6, 7, 8, 8, 8, 8
9	1, 3, 5, 5, 8, 8, 9

6/2 represents 62

Once the pairs of students have constructed the three displays, ask them which of the displays makes it possible to determine the mean (stem/leaf). Ask them why the other displays do not make this possible (all data is not entered). Ask them if it is possible to find the median with all of the plots (box-and-whiskers and stem/leaf only), and the mode (only the stem/leaf).

Ask the students what information the histogram shows (it shows that more than 60% of the class scored above 80%). What other data might be displayed in a histogram? (Students need to be sure it is data than can be recorded within intervals).

Activity 9: A Stable Measure (GLE: 8th – 40)

Provide students with a set of numerical data containing some extreme values. Numerical data that involve test scores provides real-life meaning to students. Have students determine the mean, median, and mode for the data. Next, have students throw out the extreme values and recalculate the mean, median, and mode. Have them discover that the mean is most affected by the extreme values and the median is most stable. Have students discuss which measure should be used to report such things as a class-average test score, average salaries, etc. Lead discussion about how different measures of central tendency can lead to different conclusions. For example: The store manager kept a record of the sizes of dresses sold last month in the formal dress department. The sizes were 8, 8, 10, 12, 14, 16, 8, 14, 12, 10, 8, and 6. He found the mean of the dress sizes sold last month was 10.5, the median was 10 and the mode was 8. Explain why these measures of central tendency show different results. Which would best represent the data set? Explain.

Another example: The cheerleaders were buying new tennis shoes for their next season. The sizes needed were: 4, 8, 10, 9, 11, 12, 10 and 9. Explain how the 4 in the list will affect the mean, median and mode.

Activity 10: How Good of a Guesser Are You? (GLEs: 7th – 35; 8th – 38, 39)

Gather pictures of celebrities and their birthdates prior to class. They are easily found on the Internet with almost any search engine. One site that you might want to search is http://dir.yahoo.com/Society_and_Culture/People/Celebrities/. Show the students pictures of these people one at a time and have students guess their ages, writing the ages in a column labeled ‘guess’. A PowerPoint® presentation is an easy way to view pictures of the celebrities. Next, provide students with the celebrities’ birthdates and have them calculate the ages and write these ages in a column labeled ‘actual’. Have the students make a scatter plot with their guesses along the x-axis and the actual ages of the celebrities along the y- axis. Have the students examine their scatter plot to see if a trend or relationship seems to exist. Students should then approximate a line of best fit. Students should place the line of best fit so that there is approximately the same number of points on either side of the *line of best fit.* Make sure the line crosses the origin.

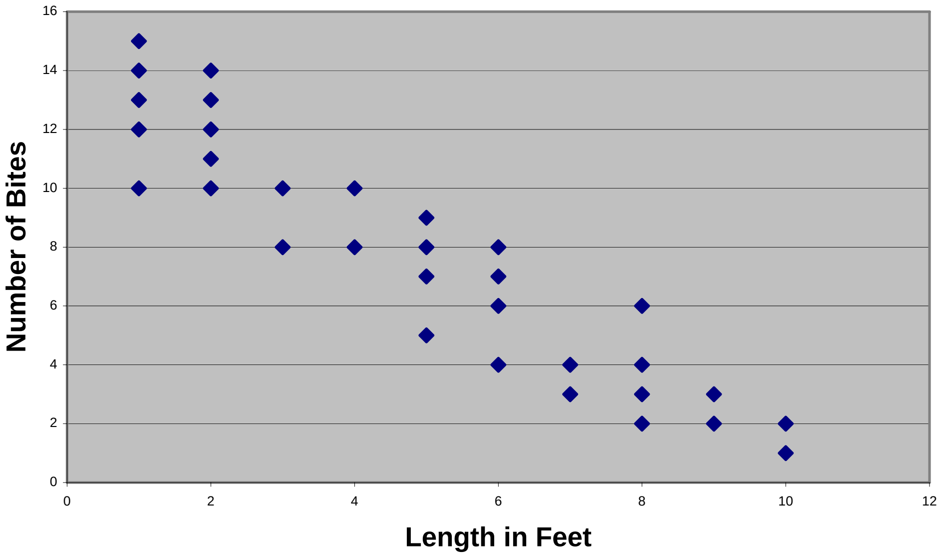
Ask if/then questions such as*: If all of your guesses were exactly right then where would your line of best fit be located?* Place a piece of uncooked spaghetti or a pencil along this ***perfect guess line.*** Have the students use their pencil and draw the 'perfect guess line' on their paper. Ask what the equation would look like for this *perfect guess line* ($y = x$)*.* Questions to elicit discussion might include:

- How many of your points are along the *perfect guess line?*
- What do the points above the *perfect guess line* tell you about your guesses? (*your guess was lower than the actual age*)
- What do the points below the *perfect guess line* tell you about your guesses? (*your guess was higher than the actual age*)
- How close to the perfect guess line is your line of best fit? Explain why you think this is so.
- Suppose someone had a line of best fit that was almost parallel to the y-axis. What might this tell you about the person's guesses? (*The person guessed that everyone presented was the same age. This might be something that very young child who has no concept of age might do.*)

Activity 5
Possible Graph

Directions: In Louisiana, there are many alligators. Use the information in the graph below to write a paragraph describing whether or not there is a relationship between the length of an alligator and the number of documented bites by alligators of each length. Justify your conclusion with any information from the graph. Make a prediction as to the number of times an alligator that is about five feet long bites and explain why you think your prediction is correct.

Number of Alligator bites
(each point represents one alligator)



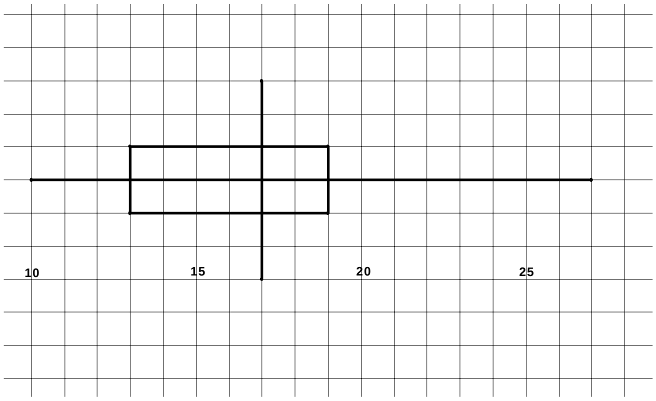
The alligator at the left is a 10-foot alligator. What is the scale used in the drawing? Explain.

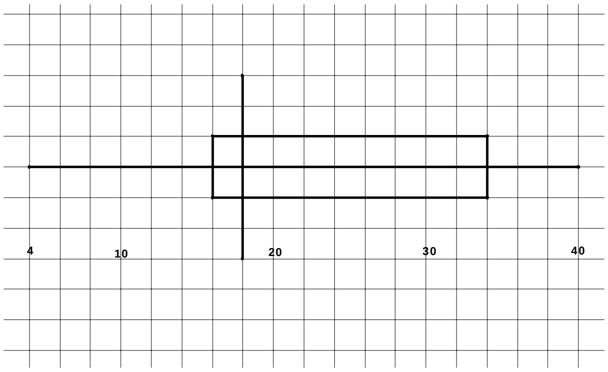
Place a point on the graph above to represent the number of projected bites from an alligator of this length.

Give approximate dimensions of a rectangular prism or solid that could be used to transport this gator. Explain why your dimensions will create a box to contain this gator.

Activity 8
Box-and-Whiskers Plots

The plot at the right shows the number of questions that were correctly answered on a 30 question social studies test. Explain what you know about the results of the test from the box-and-whiskers plot.





The plot at the left shows the results of try-outs for the marathon swim team. The participants had to swim laps of the pool until they were too tired. Explain the results shown in the plot.

Sample Assessments

General Assessments

- The teacher will determine student understanding as the student engages in the various activities.
- Whenever possible, the teacher will create extensions to an activity by increasing the difficulty or by asking "what if" questions.
- The student will be encouraged to create his/her own questions.
- The teacher will provide the student with a mean, median and mode of a set of data. The student will create a set of data that would result in the given mean, median and mode.
- The student will find an example of a graph in the newspaper or a magazine and explain in his/her journal what information is gained from the graph.
- The teacher will provide the student with data and have him/her develop all possible appropriate displays for the data. Rubrics will be used to assess the displays.
- The teacher will provide the student with a data display and have him/her interpret it.
- The teacher will provide the student with a scatter plot of data and have him/her provide the line of best fit and then interpret what that line represents.
- Whenever possible, the teacher will create extensions to an activity by increasing the difficulty or by asking "what if" questions.
- The student will create portfolios containing samples of their experiments and activities.
- The student will complete a data project (collection, organization, conclusions/predictions) and present the results on a poster to be displayed in the classroom.
- The student will complete journal entries by responding to prompts such as:
  - Explain the purpose and usefulness of circle graphs.
  - Explain how an outlier affects the mean in a set of data.
  - Explain how doubling or tripling the sides of a square affect the area.

Activity-Specific Assessments

- Activity 2: The student will explain why the following information cannot be represented in a circle graph.

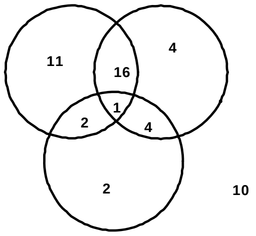
  Responses to: What is your favorite color?

  Blue: 52%
  Red: 32%
  Yellow: 14.5%
  Green: 8.5%
  Purple: 4%
  Orange: 3%

  Solution: The percents add up to 114%. The percentages in a circle graph have a sum of 100%.

- Activity 3: The student will create a Venn diagram and answer the question. A survey of 50 students found that 30 had cats, 25 had dogs, 9 had white mice, 16 had both dogs and cats, 4 had both dogs and mice, and 2 had both cats and mice, and only 1 had all three kinds of pets. How many students had no pets of these types? (10)

  Solution:



- Activity 7: Provide the students with a list of situation to be graphed and have them determine if the data would be discrete or continuous. Some possible situations might be:
  1. The number of buses needed for a field trip (discrete)
  2. The weight of a baby from birth to the first birthday (continuous)
  3. The length of an earthworm for one month (continuous)
  4. The height of each flower in a sunflower garden at 5:00 p.m. on Monday (discrete)
  5. The number of coins in your pocket (discrete)
  6. The amount of tea that someone drinks during dinner (continuous)

- Activity 10: Have students determine whether a scatter plot of the data for situations like the following might show a *positive, negative,* or *no* relationship.
  1. length of a side of a square and the perimeter of the square *(positive)*
  2. day of the week and amount of rain *(no relationship)*
  3. grade in school and number of pets *(no relationship)*
  4. length of time for a shower and amount of water used *(positive)*
  5. outside temperature and amount of heating bill *(negative)*
  6. age and expected number of years a person has yet to live *(negative)*
  7. playing time and points scored in a basketball game *(positive)*
  8. pages in a book and copies sold *(no relationship)*

Grade 7
Advanced Mathematics
Unit 8: Understanding Probability

Time Frame: Approximately four weeks

Unit Description:

This unit solidifies the fundamental counting principle while involving students in computing probabilities from collected data and recording the data in the form of tables and charts to help analyze the outcomes of experiments. These experiments are both theoretical and experimental in nature. Students will collect and analyze data to make predictions. Students will develop an understanding of how sampling with and without replacement effects the outcome and the need for randomness in statistical situations. Permutations and combinations are used in situations that describe counts for elementary ordering and grouping. Single- and multiple-event probability situations explore the role of mutually exclusive, independent, and non-mutually exclusive, dependent events.

Student Understandings

Students' understanding of choices and chances extends from understanding the difference in theoretical and experimental probability to include the role of randomness in sampling and surveys, as well as for games of chance. They can analyze the nature of independent, mutually exclusive and dependent, non-mutually exclusive events. They can apply permutations to analyze orderings with and without replacements and combinations and to examine the number of *r*-sized groups that can be formed from *n*-objects or individuals. They can calculate, illustrate, and apply single- and multiple-event probabilities for a wide variety of events.

Guiding Questions

1. Can students recognize and discuss ways that randomness contributes to surveys, experiments, and games of chance?
2. Can students determine the number of orderings (permutations) or combinations (groupings) that can occur under given conditions?
3. Can students calculate and interpret single- and multiple-event probabilities in a wide variety of situations, including independent, mutually exclusive, and dependent, non-mutually exclusive settings?
4. Can students suggest ways of minimizing bias in sampling or surveys through the use of random samples?
5. Can students make basic counts by tables and charts of possible outcomes and use this information to isolate successes and failures for experiments and theoretical analyses of problems, determining the probabilities of various events?
6. Can students compare and contrast the outcomes associated with theoretical and experimental analyses of the same question?

Unit 8 Grade-Level Expectations (GLEs)

GLE #	GLE Text and Benchmarks
7th Grade	
Data Analysis, Probability, and Discrete Math	
36.	Apply the fundamental counting principle in real-life situations (D-4-M)
37.	Determine probability from experiments and from data displayed in tables and graphs (D-5-M)
38.	Compare theoretical and experimental probability in real-life situations (D-5-M)
8th grade	
Measurement	
17.	Determine the volume and surface area of prisms and cylinders (M-1-M) (G-7-M)
Data Analysis, Probability, and Discrete Math	
41.	Select random samples that are representative of the population, including sampling with and without replacement, and explain the effect of sampling on bias (D-2-M) (D-4-M)
42.	Use lists, tree diagrams, and tables to apply the concept of permutations to represent an ordering with and without replacement (D-4-M)
43.	Use lists and tables to apply the concept of combinations to represent the number of possible ways a set of objects can be selected from a group (D-4-M)
44.	Use experimental data presented in tables and graphs to make outcome predictions of independent events (D-5-M)
45.	Calculate, illustrate, and apply single- and multiple-event probabilities, including mutually exclusive, independent events and non-mutually exclusive, dependent events (D-5-M)

Sample Activities

Activity 1: Probability Using Spinners (GLEs: 7th – 37, 38; 8th – 44, 45)

Make a fair spinner (spinner with all sections exactly the same size) with numbers 1 through 9 and duplicate one for each pair of students using card stock. Have students cut out the spinner. Show them how to spin a paper clip around a pencil point to make the spinning device. (If available, a graphing calculator can be used to make a fair spinner with the numbers 1 through 9.) Ask students to study the numbers on the spinner and have them predict the probability of spinning a multiple of 3. Have students discuss how they determined the probability. Someone should be able to explain that there are 9 equal parts and there are 3 multiples of 3 making the ratio of 3/9. Discuss theoretical probability.

Have the students discuss in their groups how they would collect data to determine the accuracy of their predictions. Have students record the steps in data collection that their group will follow to collect the data. Ask students to conduct the experiment and record the data. Have each group find the probability for spinning a multiple of 3 from the data they collected. Discuss *experimental* (data collected and probability figured from collected data) and *theoretical* (the possibilities of each event happening in theory) *probability.* Be sure to include a discussion of why the probability might be different when data is collected through an experiment.

Activity 2: Uses of Experimental Probabilities (GLEs: 7th – 38; 8th – 44)

Introduce the National Park Service and its importance to the American people and environment. The website (http://www.usatoday.com/weather/resources/climate/wparkrec.htm) provides research that can be used. Assign each student to report the average temperatures on a monthly and yearly basis for one of the parks. Based on these findings, have students develop an experimental probability for various weather situations that might occur during a vacation given a specific month or time of year for the vacation. Example: If there was snow on 20 of the 30 days in January at the National Park in Wisconsin, would it be safe to put the statement in a brochure that states "If you come visit our park in January, you will enjoy the snow!"?

Activity 3: Who Did It? (GLEs: 7th – 37, 38; 8th – 41, 44)

Provide each group of four students with four brown lunch bags filled with the following (unknown to them) tile or cube combinations: Bags should be labeled A – D and should each contain a total of 10 tiles or cubes of four different colors. Let the students know that there are 10 tiles in each bag and four different colors. Make Bag A and one of the other bags identical. Tell the students that the sample in Bag A was found at the scene of a crime and the CSI investigators divided the contents of Bag A and put the identical contents into a second bag but forgot to label the second bag.

A new CSI trainee came into the lab and mixed up the bags so that now there are four bags, and only Bag A and one other are from the current crime scene. It is very important that the contents of the bags not be touched any more than necessary. Challenge the students to devise a plan to sample contents of the bags without replacement so that they can make the best prediction based on experimental probability without looking at the contents of the bags.

When samples are examined without replacement, the sample size is constantly changing. Suppose a red tile is selected from Bag A on the first selection, a red tile from Bag B on the first selection, a green tile from Bag 3 on the first selection and a red tile from Bag 4 on the first selection, based on the information collected so far, can a good prediction be made as to the matching bags? Now there are only nine tiles in each bag to select from. Have students record their results and make a prediction after the 6th selection from each bag, justifying their selection of a particular bag. Lead a discussion about whether the predictions give enough information to make the prediction. Ask students to stop collecting data when they are ready to make a good prediction. Have them explain what information helped them make their prediction. Have them explain their thinking and their results.

Teacher Note: Student results will be different, and they will have to use some logical reasoning as they compare the results they gather.

Activity 4: Let Me Count the Ways! (GLEs: 8th – 42, 43)

Divide the students into groups of 4 for the following activity. Have the students determine the number of ways they can line up in order 1st, 2nd, 3rd, etc. Tell them to make a list of the different ways each one of them can be 1st in line. Their chart should show 24 different orders of students in 1st, 2nd, 3rd and 4th in line. Have students make observations about the data in their list. The students should see that there are 6 different ways to arrange themselves in different order in the line. The students might be seeing the number of choice for the 1st place, 2nd place, 3rd place and the number of students in the 4th place. 4 choices for first, 3 choices left for 2nd, 2 choices left for 3rd and 1 for 4th giving P(4!) = 24. A **permutation** is an arrangement or listing in which order is important. A **factorial** is a '!' after a number that means the product of all the counting numbers beginning with that number and counting down to 1. 0! is defined as 1.

1	2	3	4
1	2	4	3
1	3	2	4
1	3	4	2
1	4	2	3
1	4	3	2
2	1	3	4
2	1	4	3
2	3	1	4
2	3	4	1
2	4	1	3
2	4	3	1
3	1	2	4
3	1	4	2
3	2	1	4
3	2	4	1
3	4	1	2
3	4	2	1
4	1	2	3
4	1	3	2
4	2	1	3
4	2	3	1
4	3	1	2
4	3	2	1

Next, have them determine the number of ways a person could list pizza combinations if cheese, hamburger, sausage, olives, and jalapenos can be used. Will there be room on a menu to list each different combination? How many ways are there?

Lead discussion about any similarities that they observe between the number of ways their group could form a single-file line and the number of ways to organize the types of pizza on a menu. In this problem, adding cheese, hamburger and olives is the same as adding cheese, olives and hamburger. This is a combination.

Allow students to use lists, tables, or tree diagrams to aid them in determining the number of permutations. This website can be used as an introduction to probability and has a interactive spinner, die, and a collection of colored marbles:
http://www.mathgoodies.com/lessons/vol6/intro_probability.html

Activity 5: How Many Choices! (GLE: 7th – 36)

Break students into groups of 3 or 4 and have them collaborate on making posters that illustrate the counting principle. Have them write each situation on a strip of paper with each group choosing a strip and illustrating the situation. Instruct students to be ready to share with the class in 15 minutes, including an explanation of how their problem illustrates the counting principle.

Examples of situations:

- There are 4 flavors of yogurt and 10 toppings. How many choices are available if only one topping is allowed?
- A pizza parlor offers 3 sizes of pizza, 2 types of sauces, and 3 types of cheese, how many choices of pizza are there?

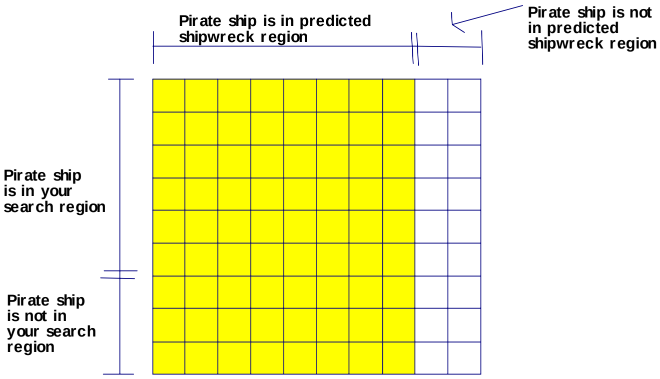
- To make a peanut butter and jelly sandwich from 5 types of bread, one kind of peanut butter and 4 flavors of jellies, how many choices are there if only one flavor of jelly is used?

Activity 6: Odd Volumes: Is It Fair? (GLEs: 7th – 37; 8th – 17, 43)

Have students work in pairs to create a circular spinner with 4 equal sections and 1 paper clip. Once the students have created their spinner, have them place the digits 2, 3, 4 and 5 on different sections of the spinner. Have students make a table with headings of *length, width, height*, and *volume* to record their spins. Instruct them to spin three times and record each of the three spins as a dimension of a rectangular prism. Instruct students to determine the volume of each rectangular prism. Player 1 gets a point if the volume is odd and Player 2 gets a point if the volume is even. Have students continue to spin until each has determined the volumes of 5 rectangular prisms. Once the students have gone through the activity, challenge them to determine the number of possible combinations of odd and even volumes using a chart or table. Have the students determine the probability that player 1 will get the point, using a tree diagram, graph or table. (There are only 8 out of 64 products that will give an odd volume, so the probability is 12.5%) Discuss as a class whether the rules of the game were fair.

Activity 7: Determining Multiple Event Probability (GLEs: 7th 37; 8th – 43, 44, 45)

Divide the students into groups of three to four. Explain to the students that underwater explorers have been searching for a pirate ship. A researcher has given them a shipwreck region on the map which they will search. There is an 80% probability that there is a pirate ship in the region. Instruct the students use a 10 x 10 grid and represent the 80% probability by shading this area with one color.



Once the students have shaded the 80% region have them shade to represent that 70% of the shipwreck region has been searched using a second color.

Ask questions about the model such as:
- What does the uncolored region represent?
- What does the area that is only shaded with color #1 represent?
- What does the area that is shaded with the two colors represent?

Explain that the area with two colors represents the probability that that the ship lies in the expected region that has been searched. (56 squares or 56% probability)

The part of the grid that is shaded with only color #1 (24 squares) represents the probability that the pirate is in the unsearched shipwreck region. When this number is divided by 100 the students will have the probability of 24%.

Activity 8: Probability with *Jumanji* (GLEs:7th – 37, 38; 8th – 44, 45)

Read the book *Jumanji* to the students. While reading the book, stop at different points in the book to ask mathematical questions. For example, page 6 (where the two children are sitting at the table with the game board) say, “To play the game of *Jumanji,* the children rolled two die and found the sum. If there are 48 spaces on the game board, what is the least number of plays it would take one person to win the game? Explain.” Continue reading the book and asking questions periodically. After reading the entire book, give each pair of students a pair of die. Roll the pair of die 12 times and record the sum of the roll each time. Have the students compare experimental and theoretical probability from the roll of their die using the following information:

Suppose the sums and events were those listed below.

Spiders get inside the backpack = sum of 2	Volcano erupts = sum of 3
Monsoon season = sum of 4	Guide gets lost = sum of 5
Tsetse fly bites = sum of 6	Lion attacks = sum of 7
Monkeys eat all food = sum of 8	Rhinoceros stampede = sum of 9
Quicksand on trail = sum of 10	Python sneaks into camp = sum of 11
You find a short cut = sum of 12	

Have students write the *theoretical probability* that each misfortune will happen during the course of one game. Using the *experimental probability* found by rolling the 2 die, ask students to check to see which of the misfortunes they would experience. How did this compare to the *theoretical probability*? Why do you think the results were like this?

Activity 9: Sums Game (GLEs: 7th – 37, 38; 8th – 43, 44, 45)

Make sacks containing 8 same color markers or plastic chips for each pair of students. (These should be marked A-1, B-1, C-2, D-2, E-3, F-3, G-4, H-4). Distribute sacks to each pair of students. **Tell them not to look inside the sack.** Tell them they will play a game involving random draws from the sack, replacing the markers after each draw. Have a whole class discussion about ways to ensure that draws are random. Write these ideas on the board or on chart paper. Go over the directions with the students.

Directions: Player 1 randomly draws 2 markers from the sack, computes the sum of the marker numbers, tallies the sum on the score card, replaces the markers in the sack, and shakes the sack. Player 2 repeats this procedure, recording on a separate score card. Players continue alternating turns. The winner of this game is the *first person* to obtain each different sum at least once *or* to obtain any single sum 6 times.

Example of score card:

2	3	4	5	6	7	8

Have students play the game and record their sums. Have the pairs of students post their data on the wall. Give students 5 minutes to walk around the room and make observations from the score cards posted. Have the students determine a method of determining the *theoretical probabilities.* Ask, Theoretically, on any draw from the sacks what sum is most likely to occur? Least likely? Make an organized chart showing all the possible sums and how each can occur. Then make one or more graphs showing all the possible sums. Given below are two types of charts and a bar graph showing the possible sums. Students should come up with something similar to these.

Sums:	2	3	4	5	6	7	8
Combinations:	A,B	A,C	A,E	A,G	C,G	E,G	G,H
		A,D	A,F	A,H	C,H	E,H	
		B,C	B,E	B,G	D,G	F,G	
		B,D	B,F	B,H	D,H	F,H	
			C,D	C,E	E,F		
				C,F			
				D,E			
				D,F			

The above chart can easily be seen as a bar graph which is shown below.

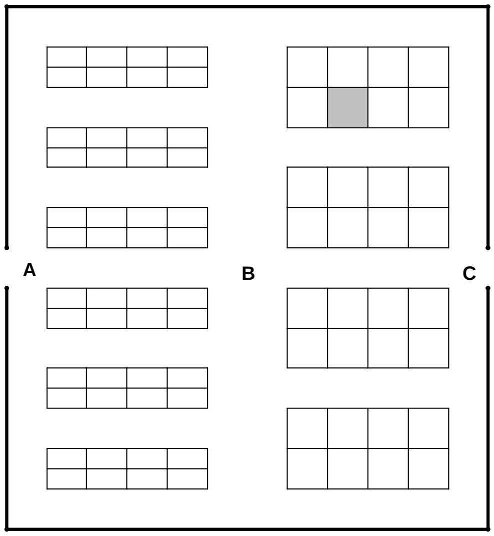
Combinations (Sums)						
A,B (2)						
A,C (3)	B,C (3)					
A,D (3)	B,D (3)	C,D (4)				
A,E (4)	B,E (4)	C,E (5)	D,E (5)			
A,F (4)	B,F (4)	C,F (5)	D,F (5)	E,F (6)		
A,G (5)	B,G (5)	C,G (6)	D,G (6)	E,G (7)	F,G (7)	
A,H (5)	B,H (5)	C,H (5)	D,H (6)	E,H (7)	F,H (7)	G,H (8)

Activity 10: Mr. Oval Head Combinations (GLEs: 7th – 37; 8th – 42, 43)

Provide groups of four students with a copy of Mr. Oval Head and pieces that are attached after the Sample Activities. Have students find all possible outfits and combinations, recording them in some kind of order. Give students time to create their favorite Mr. Oval Head and give the probability of selecting each of those pieces if all of each type of piece were in separate containers and they could select only one piece from each container. It might be fun to have some kind of contest with finished Mr. Oval Heads.

Activity 11: (GLEs: 7th – 36, 37; 8th – 43, 44)

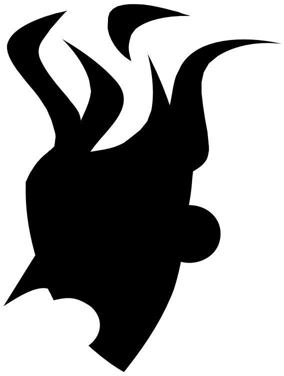
Provide the students with as situation similar to the counting principal problem below. Have the students work in pairs to complete the questions about the situation.

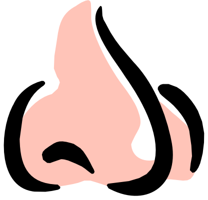


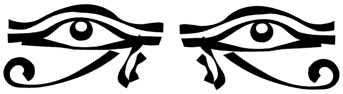
- A construction company was putting new lockers in the student area of the school. The following diagram shows the student area and the lockers. Susan was going to meet Julie at point B. If she enters the student area at point A, how many ways are there for her to get to point B and meet Julie? (7 ways) How many ways are there for Susan and Julie to get to point C from point B? (5 ways)
- Suppose Samantha was hoping to meet Susan and Julie at her locker (shaded square represents Samantha's locker) but had forgotten to tell the girls to meet her. What is the probability that Susan and Julie will pass by Samantha's locker? (1 out of 35 chances or about 3%)

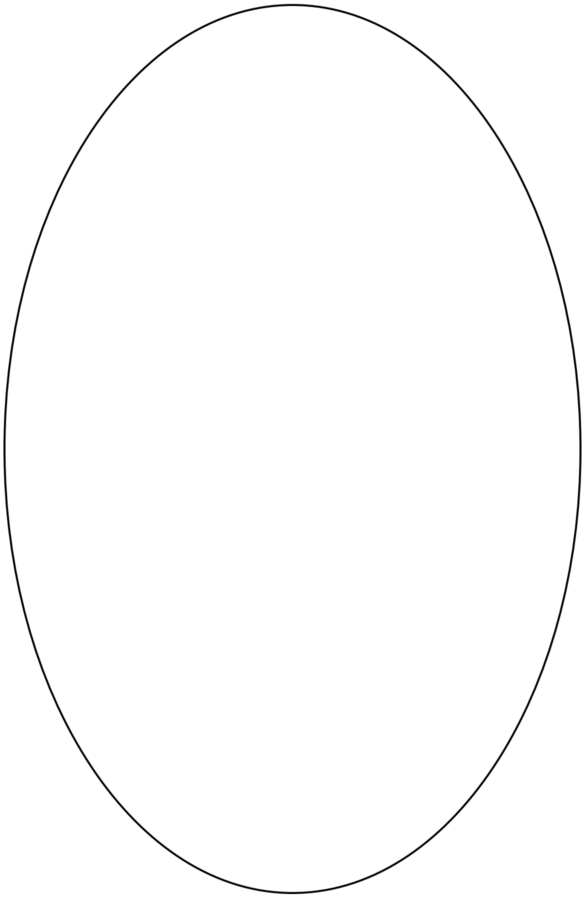
Lead a discussion about the situation having the students justify their thinking about the situation. Ask the students a question that uses an if/then statement such as: "If Julie is waiting for Susan between the first two sets of small lockers, then would the probability be different that if she stands at point B?"

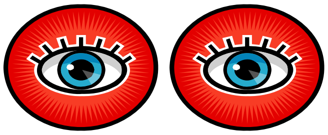
Activity 11
Mr. Oval Head

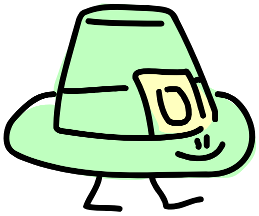


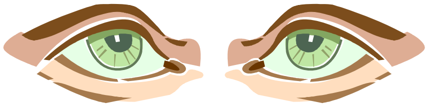


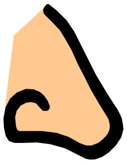


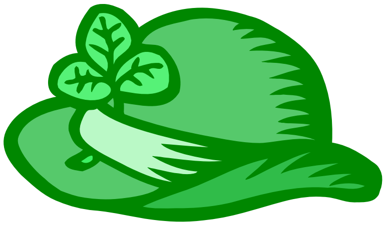


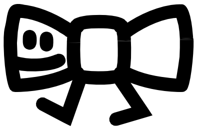


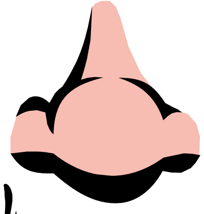


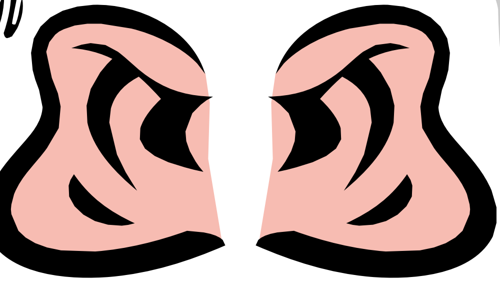








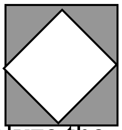




Sample Assessments

General Assessments:

- The student will create a game of chance in which player 1 has twice the chance of winning as player 2.
- The student will prepare a presentation to explain how theoretical probability is used to make predictions like the weather forecast.
- The student will make four different sketches of polygons with a shaded area inside or outside of the polygon that would illustrate a 25%, 50%, 75% and 60% probability of an object falling randomly on each figure and landing on the shaded area. *Example: the figure at the right would represent a 50% probability of a randomly dropped object that would fall on the figure landing on the shaded area.*



- The student will play several different games of chance and then analyze the probabilities of winning.
- The student will develop an experiment and then determine the experimental probability associated with the event taking place.
- Whenever possible, the teacher will create extensions to an activity by increasing the difficulty or by asking "what if" questions.
- The student will create portfolios containing samples of their experiments and activities.
- The student will complete a probability project assessed by a teacher-created rubric.
- The student will provide a portfolio item to show understanding of one of the concepts in the unit, such as:
    - The student will create and write a situation that illustrates the counting principle and explain the work.
- The student will complete journal entries using such topics as:
    - Compare and contrast *theoretical* and *experimental* probability
    - Explain whether or not order matters when finding arrangements for specific situations

Activity-Specific Assessments:

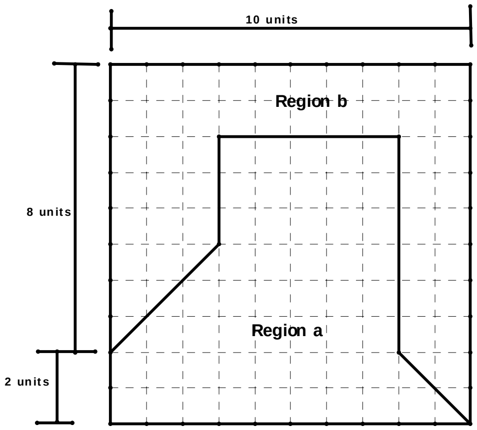
- Activity 2: The student will solve the following problem:
  You have a fair spinner divided into ten equal portions numbered 1 through 10.
    1. What is the probability of spinning an even number? (*Solution: 5 out of 10 or* $\frac{1}{2}$)
    2. What is the probability of spinning a 3? *(Solution: 1 out of 10 or 1/10)*
    3. What is the probability of spinning an even number followed by an odd number? (*Solution: 1 out of 4 or* $\frac{1}{4}$)
    4. What is the probability of spinning a number greater than 8? *(Solution: 2 out of 10 or 1/5)*

- Activity 3: The student will prepare a poster proving that his/her prediction is based on experimental probability after the 6th selection. The student will also use the actual contents of the bag to compare the theoretical probability of his/her prediction after the 6th selection. The student will include an explanation of how sampling without replacement affected their prediction.

- Activity5 : The teacher will secure menus from a restaurant that advertises several ways its product can be purchased (*e.g.,* Burger King, Baskin-Robbins Ice Cream), and the student will determine the validity of the claim.

- Activity 7: Provide students with a situation such as the following that involves real-life multiple-event probability.

  In the 1950s, there was an outbreak of a disease called malaria on the island of Borneo. This disease killed many animals, including dogs and cats. The rats were not killed by the disease and therefore they began parachuting cats into Borneo to help kill the rats. Suppose the cats were parachuted into a region like the one below. What is the probability that the 1st cat landed in region a.

- Activity 8: The student will prepare directions and make a game that involves dependent events. The student will describe the game using the theoretical probability of outcomes to describe how the game is won.